Complete One–loop Corrections to $A^0 \to \tilde{f}_1\,\bar{\tilde{f}}_2$

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Amsterdam, 1–4 April 2003

Outline of the calculation

- Short review on sfermion sector of MSSM
- Full electroweak one–loop corrections to $A^0 \to \tilde{f}_1 \bar{\tilde{f}}_2$ in pure on–shell scheme
- Numerical examples for decay into stops
  Decay into sbottoms and staus: PT breaks down
- Improvement:
  redefine tree–level using
  $\overline{\text{DR}}$ running values for $m_{f\downarrow}$ and $A_{f\downarrow}$
- Conclusions

Sfermion masses and mixing

$$\mathcal{M}^2_{\tilde f} = \begin{pmatrix} m^2_{\tilde f_L} & a_f\, m_f \\ a_f\, m_f & m^2_{\tilde f_R} \end{pmatrix}$$

$$\begin{array}{ccc} \text{soft } \cancel{\text{SUSY}} \text{ mass} & D\text{–term} & F\text{–term} \\ \Downarrow & \Downarrow & \Downarrow \end{array}$$

$$\begin{aligned} m^2_{\tilde f_L} &= M^2_{\{\tilde Q, \tilde L\}} + (I^{3L}_f - e_f \sin^2\theta_W) \cos 2\beta\, m^2_Z + m^2_f \\ m^2_{\tilde f_R} &= M^2_{\{\tilde U, \tilde D, \tilde E\}} - e_f \sin^2\theta_W \cos 2\beta\, m^2_Z + m^2_f \end{aligned}$$

$$a_f = \underset{\substack{\Uparrow \\ \text{cubic} \\ \text{coupling}}}{A_f} - \mu \left\{ \begin{matrix} \cot\beta & \ldots & \text{up} \\ \underset{\substack{\Uparrow \\ F\text{–term}}}{\tan\beta} & \ldots & \text{down} \end{matrix} \right. \text{–type sfermions}$$

$$\begin{pmatrix} \tilde f_1 \\ \tilde f_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde f} & \sin\theta_{\tilde f} \\ -\sin\theta_{\tilde f} & \cos\theta_{\tilde f} \end{pmatrix} \begin{pmatrix} \tilde f_L \\ \tilde f_R \end{pmatrix}$$

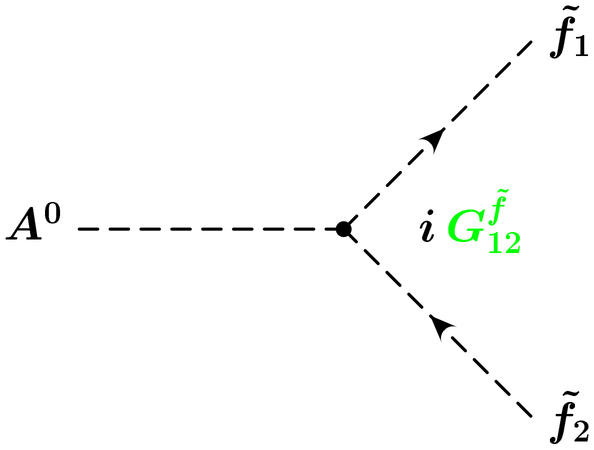
$$m^2_{\tilde f_{1,2}} = \frac{1}{2}\left(m^2_{\tilde f_L} + m^2_{\tilde f_R} \mp \sqrt{(m^2_{\tilde f_L} - m^2_{\tilde f_R})^2 + 4 a^2_f m^2_f} \right)$$

Convention: $m_{\tilde f_1} \le m_{\tilde f_2}$

$$\cos\theta_{\tilde f} = \frac{-a_f\, m_f}{\sqrt{(m^2_{\tilde f_L} - m^2_{\tilde f_1})^2 + a^2_f m^2_f}} \qquad (0 \le \theta_{\tilde f} < \pi)$$

$$m_f A_f = \frac{1}{2}\left(m^2_{\tilde f_1} - m^2_{\tilde f_2} \right) \sin 2\theta_{\tilde f} + m_f\, \mu \left\{ \begin{matrix} \cot\beta \\ \tan\beta \end{matrix} \right\} \qquad (1)$$

Tree–level



$$G^{\tilde{f}}_{12} = \frac{i}{\sqrt{2}}\, h_f \left(A_f \left\{ \begin{matrix} \cos\beta \\ \sin\beta \end{matrix} \right\} + \mu \left\{ \begin{matrix} \sin\beta \\ \cos\beta \end{matrix} \right\} \right)$$

for up– and down–type sfermions.

Tree–level width:

$$\Gamma^{\rm tree}(A^0 \to \tilde{f}_1 \bar{\tilde{f}}_2) \;=\; \frac{N_C^f\, \kappa(m^2_{A^0}, m^2_1, m^2_2)}{16\,\pi\, m^3_{A^0}}\; |G^{\tilde{f}}_{12}|^2$$

with

$$\kappa(x,y,z) \;=\; \sqrt{(x-y-z)^2 - 4yz}$$

$$N_C^f \;=\; 3 \qquad \text{for squarks}$$

$$N_C^f \;=\; 1 \qquad \text{for sleptons}$$

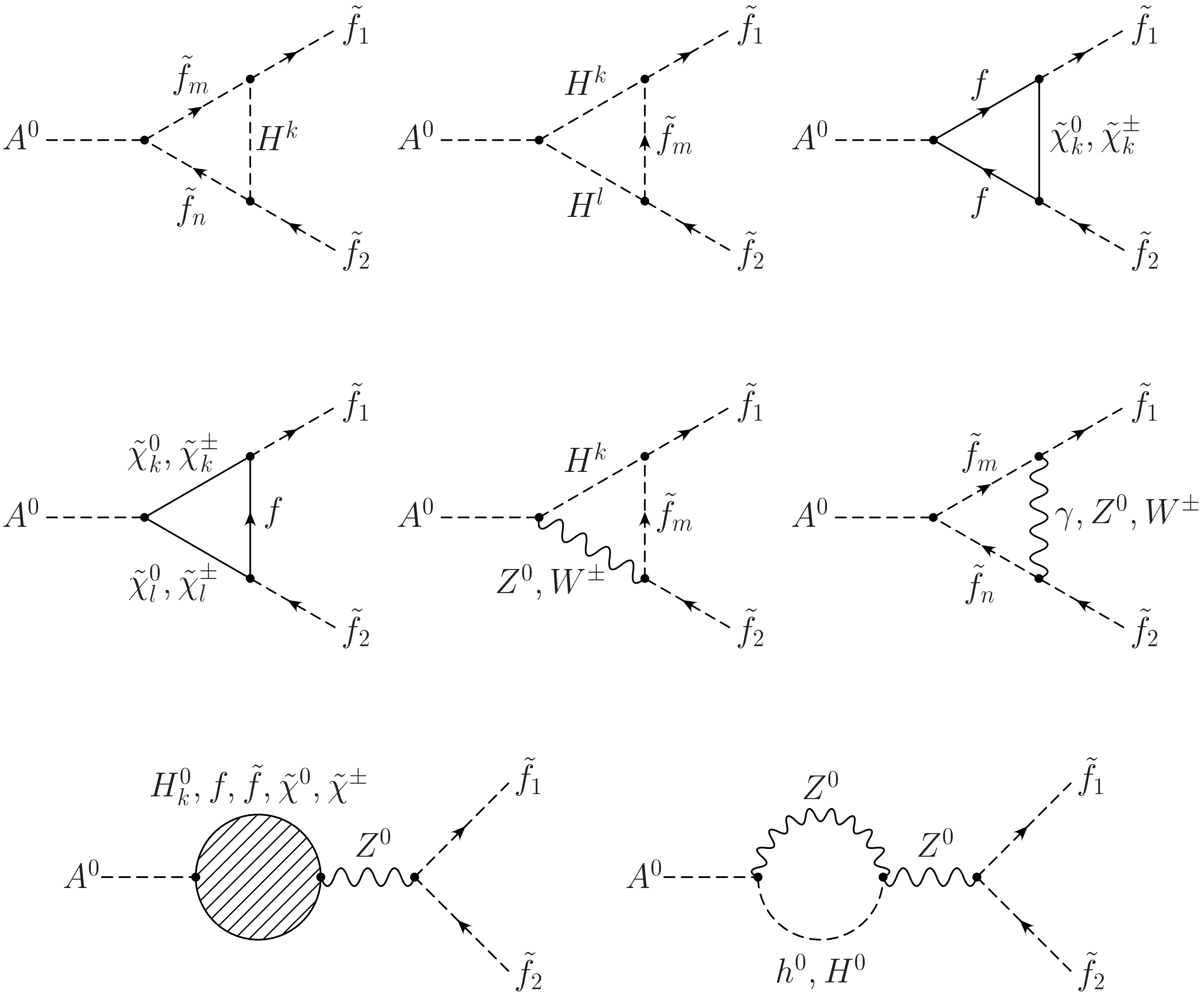
One–loop corrections

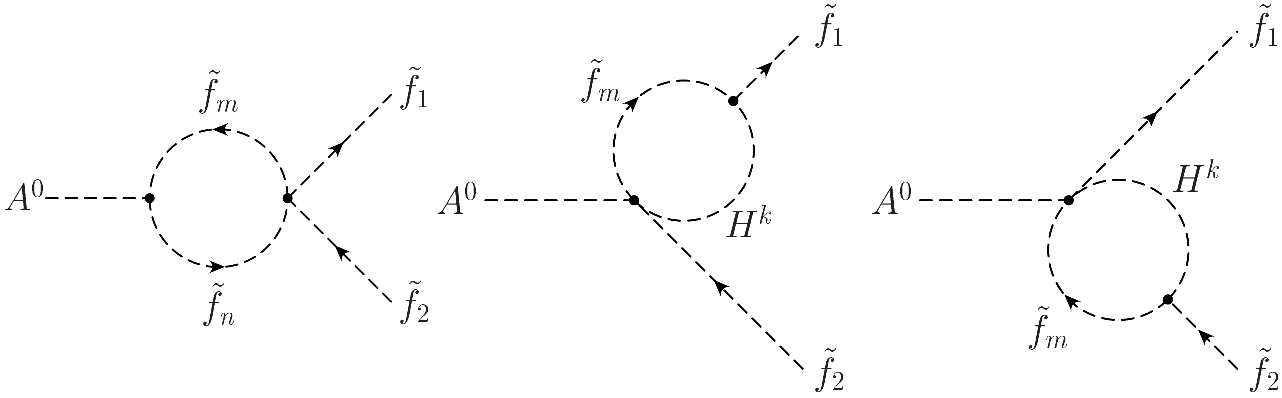
We write the renormalized coupling as

$$\boldsymbol{G_{12}^{\tilde{f}\,\mathrm{ren}} = G_{12}^{\tilde{f}} + \delta G_{12}^{\tilde{f}(v)} + \delta G_{12}^{\tilde{f}(w)} + \delta G_{12}^{\tilde{f}(c)}}$$

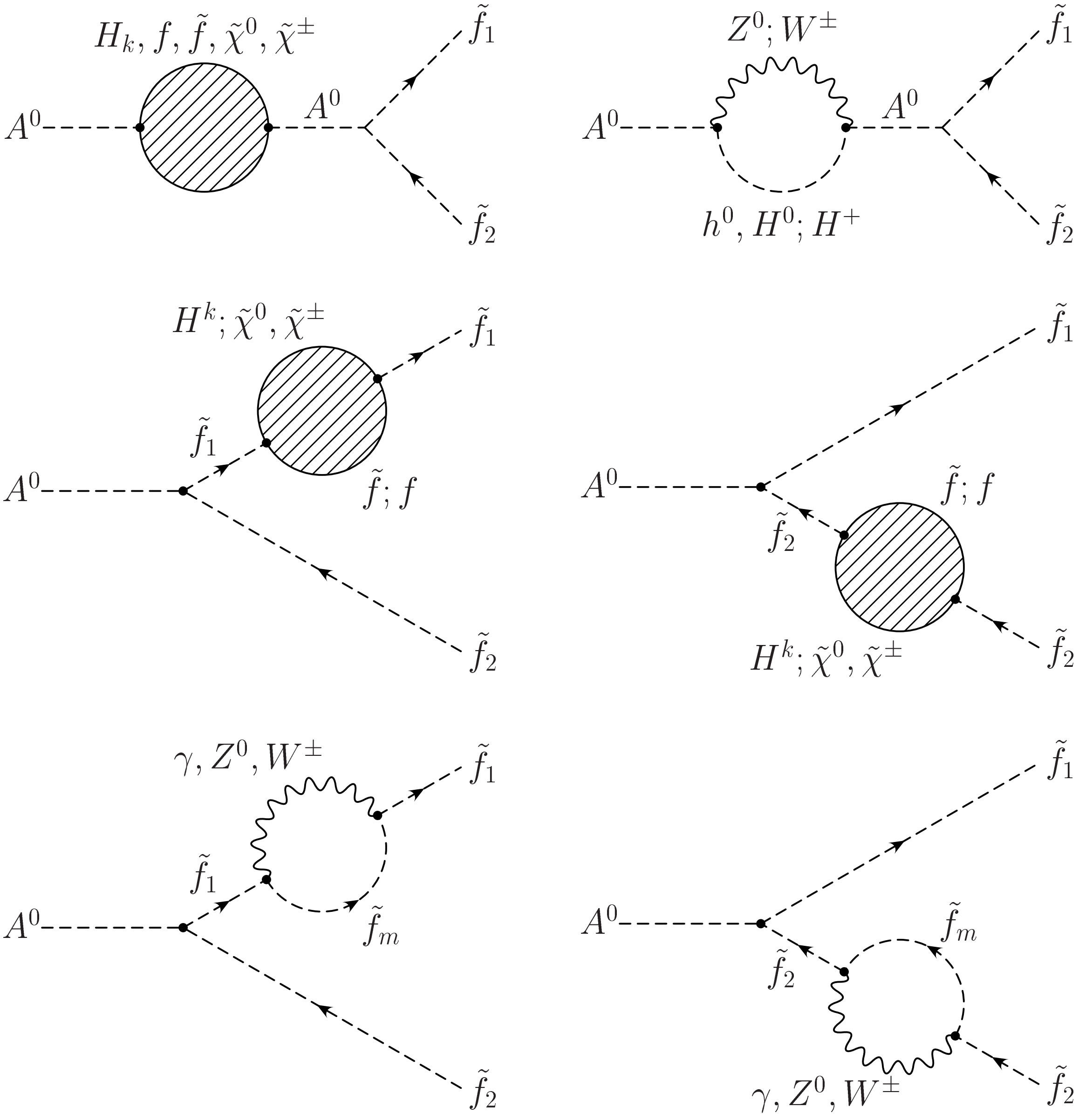
$$= \textbf{tree} + \textbf{vertex} + \textbf{wave fct.} + \textbf{coupl. c.t.}$$

Vertex diagrams

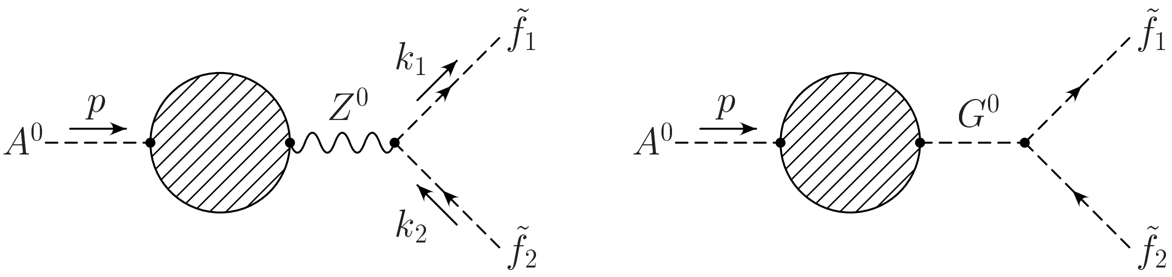




Diagrams for wave–function corrections



Combining A^0–Z^0 and A^0–G^0 corrections



$$\mathcal{M}^Z = \left(-ip^\mu \Pi_{AZ}(p^2)\right) \frac{-ig_{\mu\nu}}{p^2 - m_Z^2} \left(-\, ig_Z\, z_{12}^{\tilde{f}}\right)(k_1 + k_2)^\nu$$

$$\mathcal{M}^G = \left(i\Pi_{AG}(p^2)\right) \frac{i}{p^2 - m_Z^2}\, i\widetilde{G}_{12}^{\tilde{f}}$$

ST and coupling identity:

$$\boldsymbol{p^2 \Pi_{AZ}(p^2) + im_Z \Pi_{AG}(p^2) = 0}$$

$$\boldsymbol{g_Z\, z_{12}^{\tilde{f}}\left(m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2\right) = im_Z\, \widetilde{G}_{12}^{\tilde{f}}}$$

$$\mathcal{M}^{Z+G} = \frac{\Pi_{AZ}(p^2)}{p^2 - m_Z^2} \left[i\, g_Z\, z_{12}^{\tilde{f}}\left(m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2\right) + \frac{p^2}{m_Z}\, \widetilde{G}_{12}^{\tilde{f}} \right]$$

$$\boldsymbol{\delta G_{12}^{\tilde{f}(Z+G)} = -i\mathcal{M}^{Z+G}(p^2 \to m_{A^0}^2)}$$

$$\boldsymbol{= -\frac{i}{m_Z}\, \Pi_{AZ}(m_{A^0}^2)\, \widetilde{G}_{12}^{\tilde{f}}}$$

Coupling counter terms

The variation with respect to all parameters in the coupling $G^{\tilde{f}}_{12}$ reads

$$\delta G^{\tilde{f}(c)}_{12} = \frac{\delta h_f}{h_f} G^{\tilde{f}}_{12} + \frac{i}{\sqrt{2}} h_f \, \delta\left(A_f \begin{Bmatrix} \cos\beta \\ \sin\beta \end{Bmatrix} + \mu \begin{Bmatrix} \sin\beta \\ \cos\beta \end{Bmatrix} \right)$$

for up– and down–type sfermions resp..

From the explicit form of the Yukawa couplings,

$$h_t = \frac{g\, m_t}{\sqrt{2}\, m_W \sin\beta}\,, \qquad h_b = \frac{g\, m_b}{\sqrt{2}\, m_W \cos\beta}\,,$$

we find the Yukawa coupling counter term

$$\frac{\delta h_f}{h_f} = \frac{\delta g}{g} + \frac{\delta m_f}{m_f} - \frac{\delta m_W}{m_W} + \begin{Bmatrix} -\cos^2\beta \\ \sin^2\beta \end{Bmatrix} \frac{\delta \tan\beta}{\tan\beta}\,.$$

The counter term for the trilinear coupling A_f we get from eq. (1),

$$\frac{\delta A_f}{A_f} = \frac{\delta(m_f A_f)}{m_f A_f} - \frac{\delta m_f}{m_f}$$

$$\delta(m_f A_f) = \delta\left(m_f \mu \begin{Bmatrix} \cot\beta \\ \tan\beta \end{Bmatrix} \right) + \frac{1}{2}\left(\delta m^2_{\tilde{f}_1} - \delta m^2_{\tilde{f}_2} \right) \sin 2\theta_{\tilde{f}}$$

$$+ \left(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2} \right) \cos 2\theta_{\tilde{f}}\, \delta\theta_{\tilde{f}}$$

Renormalization conditions

- g fixed in electroweak gauge boson sector

$$\frac{\delta g}{g} = \frac{\delta e}{e} + \frac{1}{\tan^2\theta_W}\left(\frac{\delta m_W}{m_W} - \frac{\delta m_Z}{m_Z}\right)$$

- m_f, $m_{\tilde{f}_i}$ and $m_{W,Z}$ are fixed as the physical (pole) masses

- $\tan\beta$ by the condition $\mathrm{Im}\hat{\Pi}_{A^0Z^0}(m_A^2) = 0$

$$\frac{\delta\tan\beta}{\tan\beta} = \frac{1}{m_Z\sin 2\beta}\mathrm{Im}\Pi_{A^0Z^0}(m_{A^0}^2).$$

[e.g. Dabelstein 1995]

- Renormalization of μ in chargino sector

$$X = \begin{pmatrix} M & \sqrt{2}m_W\sin\beta \\ \sqrt{2}m_W\cos\beta & \mu \end{pmatrix}$$

$$\rightarrow \delta\mu = (\delta X)_{22}$$

[Eberl, Kincel, Majerotto, Yamada '01]

- $\delta\theta_{\tilde{f}}$ fixed in sfermion sector

$$\delta\theta_{\tilde{f}} = \frac{1}{4}\left(\delta Z_{12}^{\tilde{f}} - \delta Z_{21}^{\tilde{f}}\right)$$

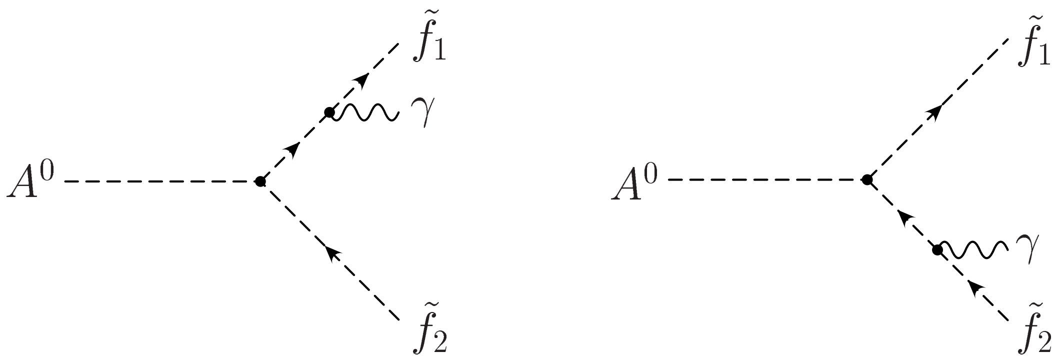
[Guasch, Solà, Hollik 1998]

[Eberl, Kraml, Majerotto 1999]

One–loop corrected width:

$$\Gamma(A^0 \to \tilde{f}_1 \bar{\tilde{f}}_2) \; = \; \frac{N_C^f\, \kappa(m_{A^0}^2, m_1^2, m_2^2)}{16\,\pi\, m_{A^0}^3} \left[|G_{12}^{\tilde{f}}|^2 + 2\,\Re\!\left(G_{12}^{\tilde{f}} \cdot \delta G_{12}^{\tilde{f}} \right) \right]$$

IR divergencies $\to$ include real photon emission



Corrected decay width:

$$\Gamma^{\mathrm{corr}}(A^0 \to \tilde{f}_1 \bar{\tilde{f}}_2) \; \equiv \; \Gamma(A^0 \to \tilde{f}_1 \bar{\tilde{f}}_2) + \Gamma(A^0 \to \tilde{f}_1 \bar{\tilde{f}}_2\, \gamma)$$

Simplifications for (on–shell) numerics:

$$M_{\tilde{Q}_{1,2}} \;=\; M_{\tilde{U}_{1,2}} \;=\; M_{\tilde{D}_{1,2}} \;=\; M_{\tilde{L}_{1,2}} \;=\; M_{\tilde{E}_{1,2}}$$

$$M_{\tilde{Q}_3} \;=\; \tfrac{9}{8}\, M_{\tilde{U}_3} \;=\; \tfrac{9}{10}\, M_{\tilde{D}_3} \;=\; M_{\tilde{L}_3} \;=\; M_{\tilde{E}_3} \;=\; M_{\tilde{Q}}$$

$$A_t \;=\; A_b \;=\; A_\tau \;=\; A$$

$m_t = 174.3$ GeV, $m_b = 4.7$ GeV, $m_Z = 91.1876$ GeV, $m_W = 80.423$ GeV

$M' = \dfrac{5}{3}\, \tan^2\theta_W\, M$ (gaugino unification)

$\to$ Input set is:

$$\left\{ m_{A^0},\ \tan\beta,\ M_{\tilde{Q}_1},\ M_{\tilde{Q}},\ A,\ M,\ \mu \right\}$$

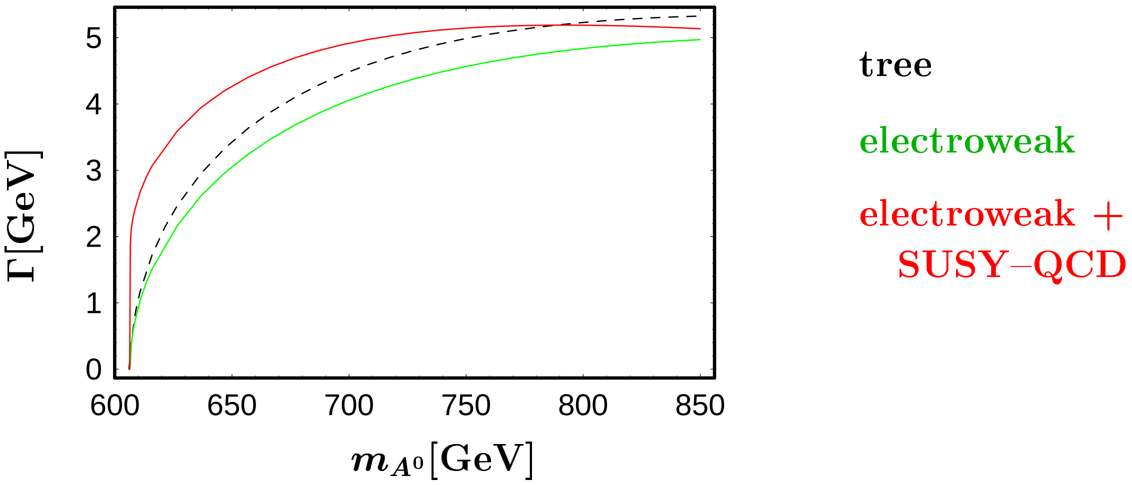
Numerical examples (on–shell scheme)

$A^0 \to \tilde{t}_1 \bar{\tilde{t}}_2$

$\{M_{\tilde{Q}_1}, M_{\tilde{Q}}, A, M, \mu\} = \{1500, 300, -500, 120, -260\}$ GeV

$\tan\beta = 7$

(preliminary)

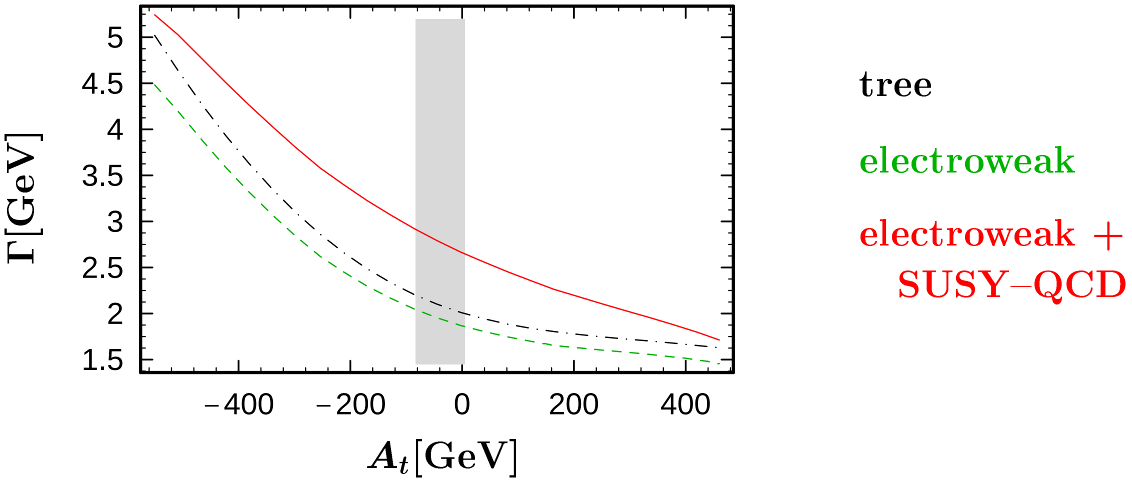


$A^0 \to \tilde{t}_1 \bar{\tilde{t}}_2$

$\{M_{\tilde{Q}_1}, M_{\tilde{Q}}, A_{b,\tau}, M, \mu\} = \{1500, 300, -500, 120, -260\}$ GeV

$m_{A^0} = 700$, $\tan\beta = 7$

(preliminary)

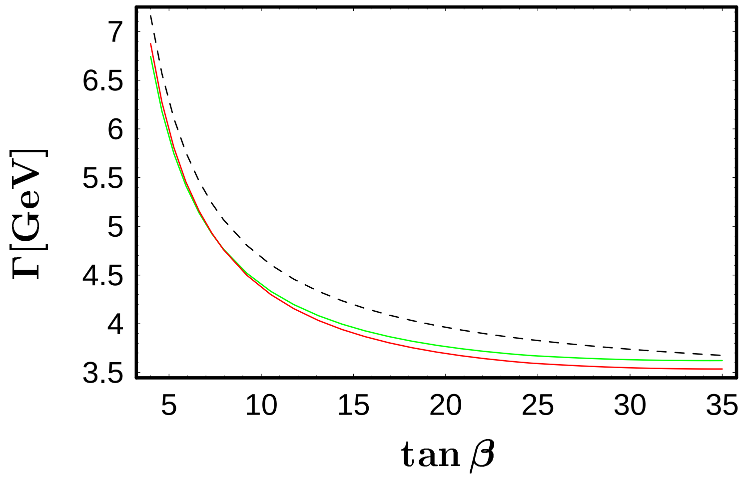


$A^0 \to \tilde{t}_1 \bar{\tilde{t}}_2$

$\{M_{\tilde{Q}_1}, M_{\tilde{Q}}, A, M, \mu\} = \{1500, 300, -500, 120, -260\}$ GeV

$m_{A^0} = 900$ GeV

(preliminary)



tree

electroweak

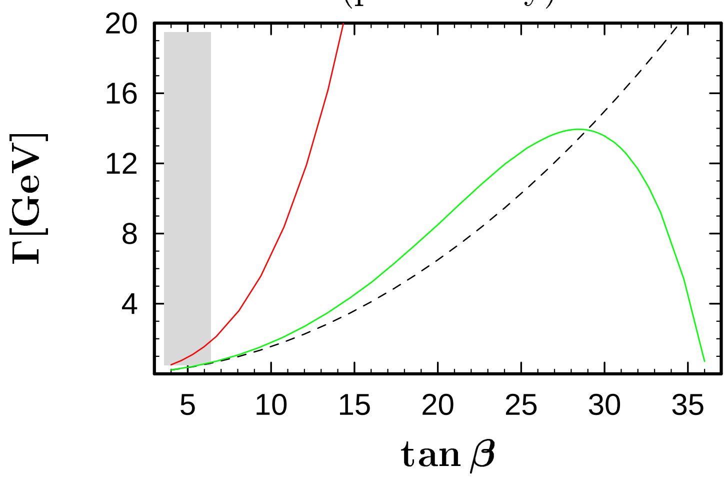
electroweak +
SUSY–QCD

$A^0 \to \tilde{b}_1 \bar{\tilde{b}}_2$

$\{M_{\tilde{Q}_1}, M_{\tilde{Q}}, A_t, A_b, A_\tau\} = \{1500, 300, 150, -700, -500\}$ GeV

$\{m_{A^0}, M, \mu\} = \{800, 120, 260\}$ GeV

(preliminary)



tree

electroweak

electroweak +
SUSY–QCD

$\to$ PT breaks down due to large counterterms for the Higgs–sfermion coupling, escpecially δm_b and δA_b

Improvement

Absorb large counterterms for m_b and A_b into tree–level Higgs– sfermion couplings by using $\overline{\text{DR}}$ running m_b and A_b

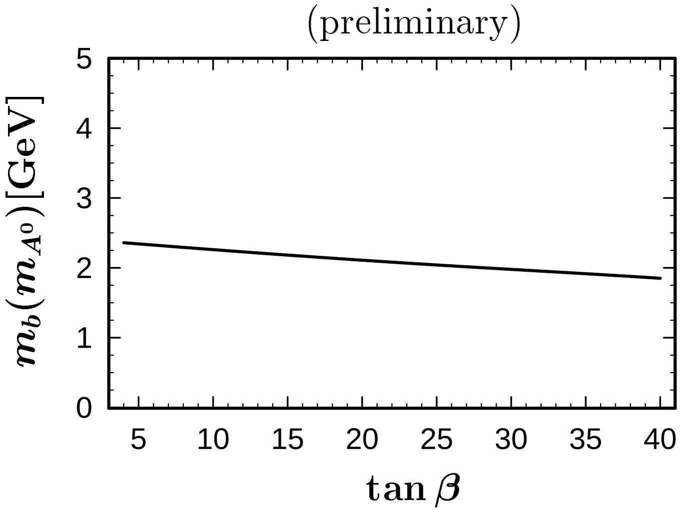
[Eberl et al. '00]

$\rightarrow \delta m_b = \delta A_b = 0$

running m_b

— SM two–loop RGEs (gluon loop)

— contributions from gluinos and all EW interacting particles



Input parameters in numerical calculation

— all parameters on–shell except

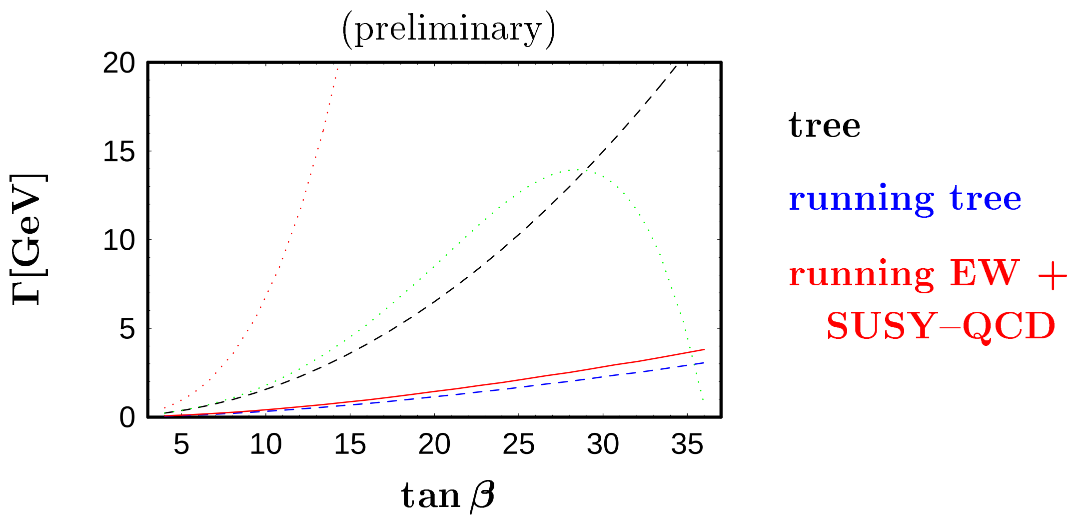
— running A_b

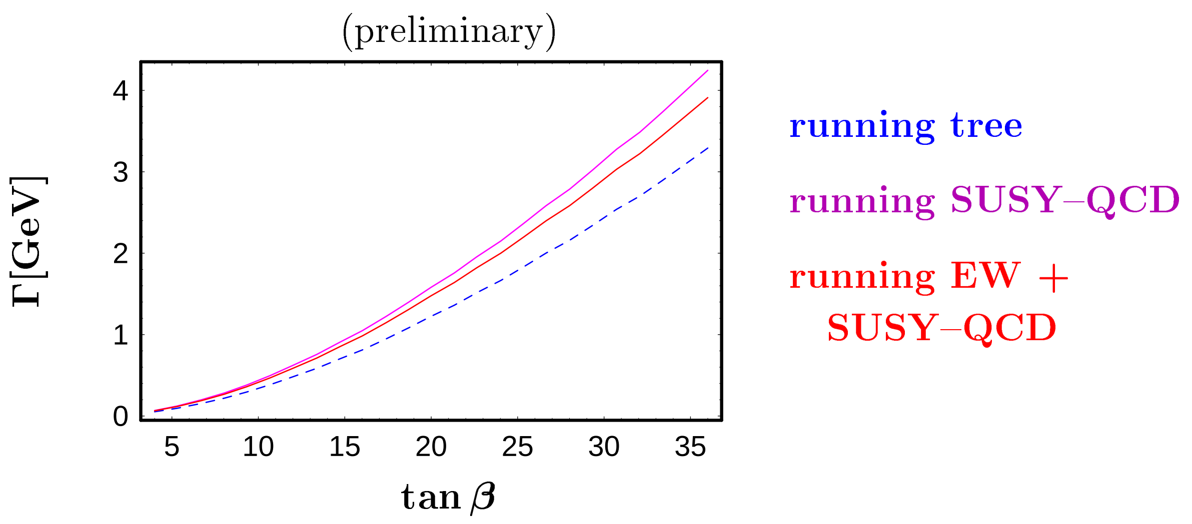
$\rightarrow$ perform iteration procedure to obtain running m_b and on–shell $m_{\tilde{b}_{1,2}}$ and $\theta_{\tilde{b}}$

$A^0 \to \tilde{b}_1 \bar{\tilde{b}}_2$

$\{M_{\tilde{Q}_1}, M_{\tilde{Q}}, A_t, A_b, A_\tau\} = \{1500, 300, 150, -700, -500\}$ GeV

$\{m_{A^0}, M, \mu\} = \{800, 120, 260\}$ GeV





Conclusions

- **Complete one–loop corrections to**
  $A^0 \to \tilde{f}_1\, \bar{\tilde{f}}_2$ **in pure on–shell scheme**

- **Numerical examples for decay into stops**
  PT not valid for decay into sbottoms

- **Improvement: redefine tree–level in terms of**
  $\overline{\text{DR}}$ **running values for** m_b **and** A_b

- **Electroweak corrections are not negligible**
  compared to SUSY–QCD corrections