Two-loop electroweak angular-dependent logarithms at high-energies

Ansgar Denner

Paul Scherrer Institut, Villigen, Switzerland

in collaboration with *M. Melles and S. Pozzorini*

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- Leading electroweak logarithms in the TeV range: origin and size
- Universality of one-loop logarithms
- Resummation prescriptions
- Calculation of two-loop leading and angular-dependent next-to-leading logarithms

Leading EW corrections at TeV energies

$E \gg M \sim M_{\mathrm{W}} \sim M_{\mathrm{Z}}$: EW corrections are enhanced by large logarithms $\ln(s/M^2)$ and grow with energy

at L-loop level

- leading (double) logarithms (LL)
  $\alpha^L \ln^{2L} \frac{s}{M^2}$
- next-to-leading logarithms (NLL)
  $\alpha^L \ln^{2L-1} \frac{s}{M^2}$
- next-to-next-to-leading logarithms (NNLL)
  $\alpha^L \ln^{2L-2} \frac{s}{M^2}$

typical size of one-loop double logarithms

$$\frac{\alpha}{4\pi s_{\mathrm{W}}^2} \ln^2 \frac{s}{M^2} = 6.6\% \ (18\%) \ @\, 1\,\mathrm{TeV} \ (5\,\mathrm{TeV})$$

typical size of one-loop single logarithms

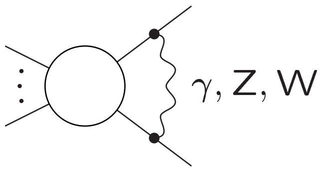
$$\frac{\alpha}{4\pi s_{\mathrm{W}}^2} \ln \frac{s}{M^2} = 1.3\% \ (2.1\%) \ @\, 1\,\mathrm{TeV} \ (5\,\mathrm{TeV})$$

Origin of leading EW logarithms

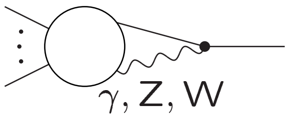
- parameter renormalization at scale $M^2 \ll s$
  $\Rightarrow$ running of electroweak couplings from M
  to $\sqrt{s}$

  single logarithms $\alpha \ln \frac{s}{M^2}$

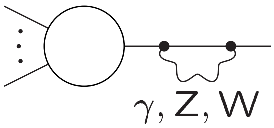
- mass singularities in virtual corrections
  related to external lines
  - soft and collinear virtual gauge bosons

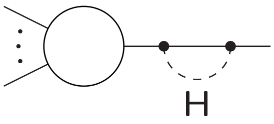
    

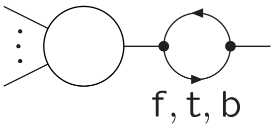
    double logarithms $\alpha \ln^2 \frac{s}{M^2}$
  - collinear or soft virtual gauge bosons,
    wave-function renormalization

    single logarithms

large logarithms are of universal origin

- infrared logarithms $\Leftrightarrow$ external particles
  of the process

- ultraviolet logarithms $\Leftrightarrow$ renormalization
  of parameters

High-energy approximation

Consider

- high-energy limit: $(p_k + p_l)^2 = r_{kl} \gg M^2$
  $M \sim M_{\mathrm{W}} \sim M_{\mathrm{Z}}$
  $\Rightarrow$ all scattering angles are finite

  mass-suppressed terms ($\propto M/\sqrt{|r_{kl}|}$) are neglected

- multi-scale problem
  $M \gg m_{f\neq \mathrm{t}} \gg \lambda = M_\gamma$
  $M_{\mathrm{H}} \sim m_{\mathrm{t}} \sim M$

- on-shell matrix elements $\mathcal{M}^{i_1 \dots i_n}$
  that are not mass-suppressed

- general electroweak processes
  with arbitrary polarized external particles
  $f_{\mathrm{L}}, f_{\mathrm{R}}, \quad \gamma, \mathrm{Z_T}, \mathrm{W_T}, \quad \mathrm{Z_L}, \mathrm{W_L}, \quad \mathrm{H}$

  External longitudinal gauge bosons have to be replaced by corresponding Goldstone bosons

Universality of EW logarithms at one loop

[Denner, Pozzorini '00 '01]

results depend only on external-leg properties and running couplings

$$\mathcal{M}_1 = \underbrace{\mathcal{M}_0}_{g\to g(s)} \left(1 + \underbrace{\delta^1_{\mathrm{EW}}}_{\text{soft,coll.}}\right), \qquad \delta^1_{\mathrm{EW}} = \frac{\alpha}{4\pi} \sum_{\mathbf{legs}\, k} \delta^1_{\mathrm{EW}}(k)$$

EW logarithms can be split in

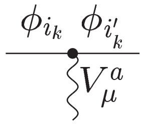
- symmetric electroweak part resulting from EW theory with photon of mass M

$$\begin{aligned}\delta_{\mathrm{sew}}(k) = \delta^1_{\mathrm{EW}}(k)\Big|_{\lambda=M} &= -\tfrac{1}{2}C^{\mathrm{ew}}(k)\ln^2\tfrac{s}{M^2} + \gamma^{\mathrm{ew}}(k)\ln\tfrac{s}{M^2} \\ &\quad + \sum_{l\neq k}\sum_{a=\gamma,\mathrm{Z},\mathrm{W}^\pm} I^a(k)I^{\bar{a}}(l)\ln\tfrac{|r_{kl}|}{s}\ln\tfrac{s}{M^2}\end{aligned}$$

- and subtracted electromagnetic part resulting from mass gap between M and λ

$$\begin{aligned}\delta_{\mathrm{sem}}(k) = \delta^1_{\mathrm{EW}}(k) - \delta^1_{\mathrm{EW}}(k)\Big|_{\lambda=M} &= -\tfrac{1}{2}Q^2(k)\Big[2\ln\tfrac{s}{m_k^2}\ln\tfrac{M^2}{\lambda^2} - \ln^2\tfrac{M^2}{m_k^2} \\ &\quad -2\ln\tfrac{M^2}{\lambda^2} - \ln\tfrac{M^2}{m_k^2}\Big] + \sum_{l\neq k} Q(k)Q(l)\ln\tfrac{|r_{kl}|}{s}\ln\tfrac{M^2}{\lambda^2}\end{aligned}$$

Universality of EW logarithms at one loop II

gauge coupling matrices $I^a_{i'_k i_k}$ 

electroweak Casimir operator

$$C^{\mathrm{ew}} = \frac{1}{c_{\mathrm{W}}^2}\left(\frac{Y}{2}\right)^2 + \frac{1}{s_{\mathrm{W}}^2} C^{\mathrm{SU(2)}}$$

anomalous dimensions for fermions

$$\gamma^{\mathrm{ew}}(k) = \begin{cases} \frac{3}{2} C^{\mathrm{ew}}(k) & \text{light fermions} \\ \frac{3}{2} C^{\mathrm{ew}}(k) - \frac{m_{\mathrm{t}}^2}{8 s_{\mathrm{W}}^2 M_{\mathrm{W}}^2} & \mathrm{t_L, b_L} \\ \frac{3}{2} C^{\mathrm{ew}}(k) - \frac{m_{\mathrm{t}}^2}{4 s_{\mathrm{W}}^2 M_{\mathrm{W}}^2} & \mathrm{t_R} \end{cases}$$

anomalous dimensions for bosons

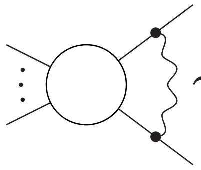
$$\gamma^{\mathrm{ew}}(k) = \begin{cases} \frac{1}{2} b^{\mathrm{ew}} & \mathrm{W}^\pm_{\mathrm{T}} \\ \frac{1}{2} b^{\mathrm{ew}} + \frac{1}{2} b^{\mathrm{ew}}_{AZ} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} & \mathrm{Z_T}, \gamma_{\mathrm{T}} \\ 2 C^{\mathrm{ew}}(k) - \frac{3 m_{\mathrm{t}}^2}{4 s_{\mathrm{W}}^2 M_{\mathrm{W}}^2} & \mathrm{H}, \mathrm{W}^\pm_{\mathrm{L}}, \mathrm{Z_L} \end{cases}$$

with the coefficients of the one-loop β-functions

$$b^{\mathrm{ew}}_{ab} = \frac{11}{3} C^{\mathrm{ew}}_{ab} - \frac{1}{3} \mathrm{Tr}\left\{ I^a(\phi) I^b(\phi) \right\} - \frac{2}{3} \sum_{f,i} N_{\mathrm{C}}^f \sum_{\lambda} \mathrm{Tr}\left\{ I^a(f^\lambda) I^b(f^\lambda) \right\}$$

One-loop angular-dependent leading logarithms

- result only from soft-collinear gauge bosons (eikonal approximation)



- result from different scales in leading logarithms

$$\ln^2 \frac{|r_{kl}|}{M^2} \approx \underbrace{\ln^2 \frac{s}{M^2}}_{\text{leading}} + \underbrace{2 \ln \frac{s}{M^2} \ln \frac{|r_{kl}|}{s}}_{\text{subleading angular-dependent}}$$

- are next-to-leading

- involve sum over pairs of external legs

$$\frac{\alpha}{4\pi} \sum_k \sum_{l \neq k} \sum_{a=\gamma,\mathrm{Z},\mathrm{W}^\pm} I^a(k) I^{\bar{a}}(l) \ln \frac{|r_{kl}|}{s} \ln \frac{s}{M^2}$$

$\Rightarrow$ no simple factorization!

- involve non-diagonal couplings $I^a_{i'_k i_k}$

$\Rightarrow$ depend on $\mathrm{SU}(2) \times \mathrm{U}(1)$ rotated Born matrix elements $\mathcal{M}_0^{i_1 \ldots i'_k \ldots i'_l \ldots i_n}$ (e.g. $\mathrm{e} \to \nu_\mathrm{e}$)

- in higher orders:

$$\alpha^L \ln^N \frac{s}{M^2} \ln^K \frac{|r_{kl}|}{s}$$

$$2L \geq N + K, \; N \geq 1, \; K \geq 1$$

- numerically important

Typical size of log. corr. at $\sqrt{s} \simeq 1\,\mathrm{TeV}$

for $2 \to 2$ processes

1-loop:

$$\left(\frac{\delta\sigma_{1,\mathrm{sew}}}{\sigma_0}\right)_{\mathrm{LL}} \simeq -\frac{\alpha}{\pi s_{\mathrm{W}}^2}\ln^2\frac{s}{M^2} \simeq -26\%$$
$$\left(\frac{\delta\sigma_{1,\mathrm{sew}}}{\sigma_0}\right)_{\mathrm{NLL}} \simeq +\frac{3\alpha}{\pi s_{\mathrm{W}}^2}\ln\frac{s}{M^2} \simeq +16\%$$

- contributions of $\mathcal{O}(10\%)$
- large cancellations between LL and NLL
- angular-dependent corrections have similar size as angular-independent corrections

2-loop (assuming exponentiation):

$$\left(\frac{\delta\sigma_{1,\mathrm{sew}}}{\sigma_0}\right)_{\mathrm{LL}} \simeq +\frac{\alpha^2}{2\pi^2 s_{\mathrm{W}}^4}\ln^4\frac{s}{M^2} \simeq 3.5\%$$
$$\left(\frac{\delta\sigma_{1,\mathrm{sew}}}{\sigma_0}\right)_{\mathrm{NLL}} \simeq -\frac{3\alpha^2}{\pi^2 s_{\mathrm{W}}^4}\ln^3\frac{s}{M^2} \simeq -4.1\%$$

- contributions of $\mathcal{O}(1\%)$
- NLL $\gtrsim$ LL
- large cancellations between NLL and LL

EW logarithms at two loops

Popular assumption:

- in high-energy limit EW theory is in symmetric phase

$\Rightarrow$ results of QED and QCD can be extended to electroweak theory

- evolution equations: $\mathrm{SU}(3)_c \to \mathrm{SU}(2) \times \mathrm{U}(1)_Y$
- all-order resummation of logarithms

non-trivial effects from spontaneous symmetry breaking, in particular in the gauge sector

- mass gap: $M_\mathrm{W}, M_\mathrm{Z} \gg \lambda$
- mixing: $(\mathrm{W}^3, \mathrm{B}) \to (\gamma, \mathrm{Z})$
- longitudinal gauge bosons

$\Rightarrow$ need calculations within the EW theory

- consider all mass-singular 2-loop diagrams
- evaluate massive loop integrals in logarithmic approximation

Existing Results at 2 loops

LL exponentiation for arbitrary processes

- infrared evolution equation (IREE) with 2 regimes: $\mathrm{SU}(2) \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{em}}$

  [Fadin et.al. 2000]

- temporal gauge for massive gauge bosons: two-loop calculation

  [Beenakker and Werthenbach 2000,2002]

NLL exponentiation for arbitrary processes

- matching of $\mathrm{SU}(2) \times \mathrm{U}(1)_Y$ and $\mathrm{U}(1)_{\mathrm{em}}$ at the EW scale [Melles 2001,2002]

NNLL exponentiation for massless $f\bar{f} \to f'\bar{f}'$

- QCD evolution equation and matching

  [Kühn et.al. 2000,2001]

no proof for exponentiation in EW SM exists!

existing analyses based on properties for symmetric theories (e.g. evolution equations) and matching conditions
$\Rightarrow$ need to be checked by diagrammatic higher-order calculations

New 2-loop electroweak calculation

[Denner, Melles, Pozzorini '02 '03]

- LL from **soft-collinear** $\gamma, \mathrm{Z}, \mathrm{W}^{\pm}$

$$\alpha^2 \ln^4 \frac{|r_{kl}|}{M^2}, \qquad r_{kl} = (p_k + p_l)^2$$

- including **NLL** **angular dependence** from external-leg correlations

$$\ln^4 \frac{|r_{kl}|}{M^2} \to \ln^4 \frac{s}{M^2} + 4 \ln \frac{|r_{kl}|}{s} \ln^3 \frac{s}{M^2}$$

$\Rightarrow$ angular-dependent NLL approximation ($\mathrm{NLL_a}$)

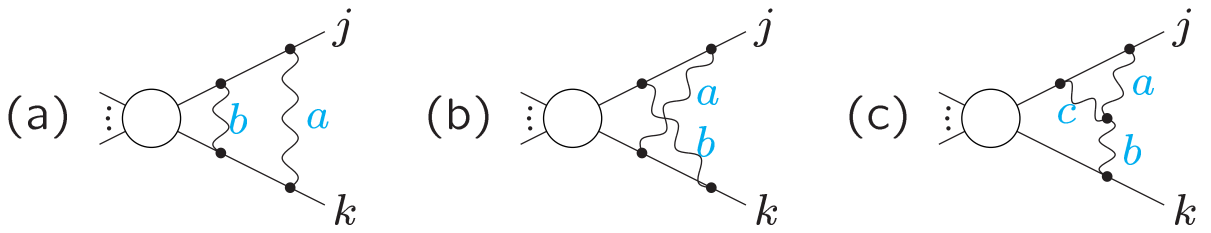
Outline of the calculation

- Feynman diagrams of order $\alpha^2 \ln^4 \frac{|r_{kl}|}{M^2}$
- eikonal approximation
- loop integrals to logarithmic accuracy
- sum of diagrams using global gauge invariance

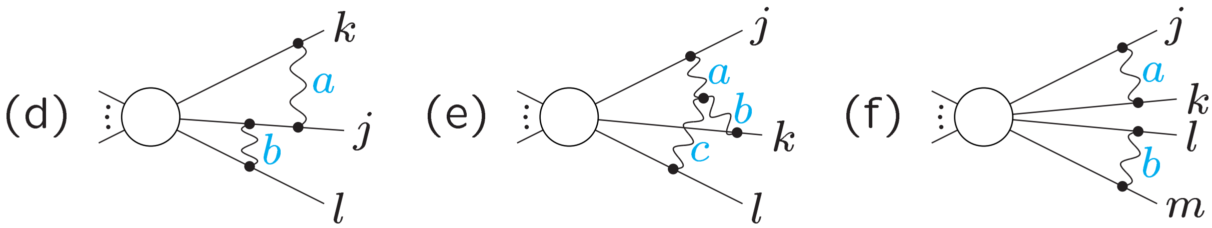
Feynman diagrams of order $\alpha^2 \ln^4(|r_{kl}|/M^2)$

't Hooft–Feynman gauge $\Rightarrow$ diagrams with soft-collinear $\gamma, \mathrm{Z}, \mathrm{W}^\pm$ coupling to

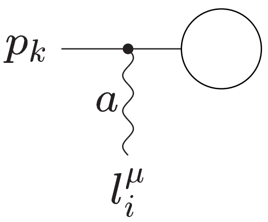
- 2 different external legs



- 3 and 4 different external legs



Eikonal approximation



$$\Rightarrow \mathcal{M}_0|_{l_i=0} \; \frac{-2ep_k^\mu I^a(k)}{\left((p_k-l_i)^2-m_k^2+\mathrm{i}\epsilon\right)} \, \frac{-\mathrm{i}}{\left(l_i^2-M_a^2+\mathrm{i}\epsilon\right)}$$

- universal factor $-2ep_k^\mu \, I^a(k)$ for eikonal vertices
- hard part of $\mathcal{M}$ in the soft limit $l_i = 0$

2-loop massive integrals in logarithmic app.

A) Sudakov method [Sudakov '54]

- Sudakov parametrization

$$l_1^\mu = x_1 p_i^\mu + y_1 p_j^\mu + l_{1,\perp}^\mu$$
$$l_2^\mu = x_2 p_k^\mu + y_2 p_l^\mu + l_{2,\perp}^\mu$$

- restrict integration region to the part where the integrand has 4 poles and approximate integrand in this region
- $l_\perp^\mu$ negligible $\Rightarrow$ simple integration
- soft-collinear singularities $\sim 1/(x_1 y_1 x_2 y_2)$

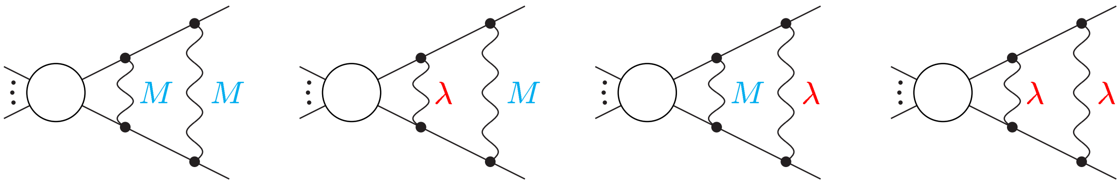
B) Sector-decomposition method

UV singularities [Hepp '66]
1-loop massive integrals [Denner and Roth '96]
3-loop massless integrals [Binoth and Heinrich '00]

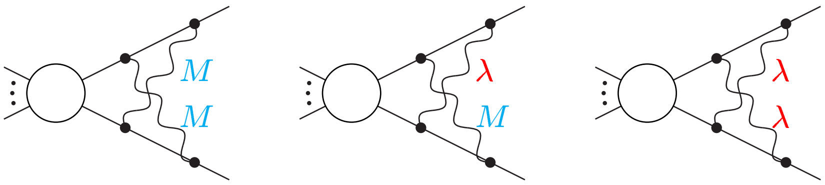
- Feynman parametrization
- factorization of generic mass singularities
- calculation of coefficient integrals

Results for loop integrals

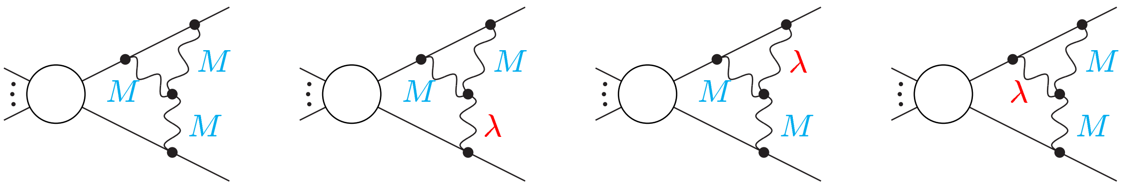
(a)



(b)



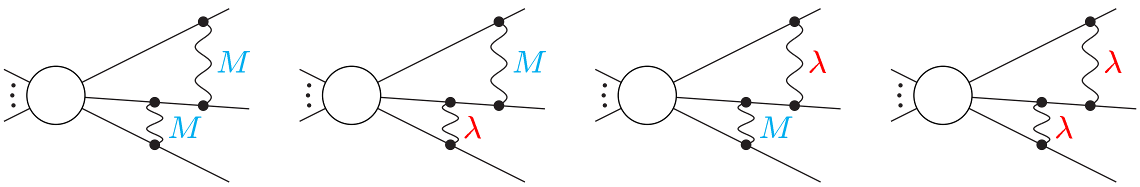
(c)



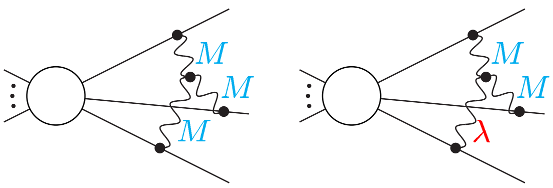
Diagrams (a–c) in agreement with form-factor calculation for **massless** external particles

[Hori, Kawamura and Kodaira 2000]

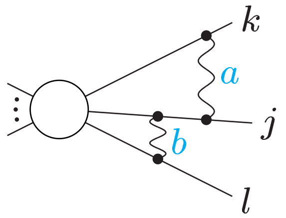
(d)



(e)



Sample diagram: 3-leg ladder



$$\overset{\text{eik}}{=} \left(\frac{\alpha}{4\pi}\right)^2 S_{3\mathrm{L}}(M_a, M_b; p_j, p_k, p_l) \times \mathcal{M}_0^{i_1,\ldots,i'_j,\ldots,i'_k,\ldots,i'_l,\ldots,i_n} I^a_{i'_k i_k} (I^b I^{\bar{a}})_{i'_j i_j} I^{\bar{b}}_{i'_l i_l},$$

integral

$$S_{3\mathrm{L}}(M_a, M_b; p_j, p_k, p_l) := -(4\pi)^4 \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4} \frac{1}{[q_1^2 - M_a^2][q_2^2 - M_b^2]}$$
$$\times \frac{16(p_j p_k)[(p_j - q_1)p_l]}{[(p_j - q_1)^2 - m_j^2][(p_j - q_1 - q_2)^2 - m_j^2][(p_k + q_1)^2 - m_k^2][(p_l + q_2)^2 - m_l^2]}$$

Sudakov representation

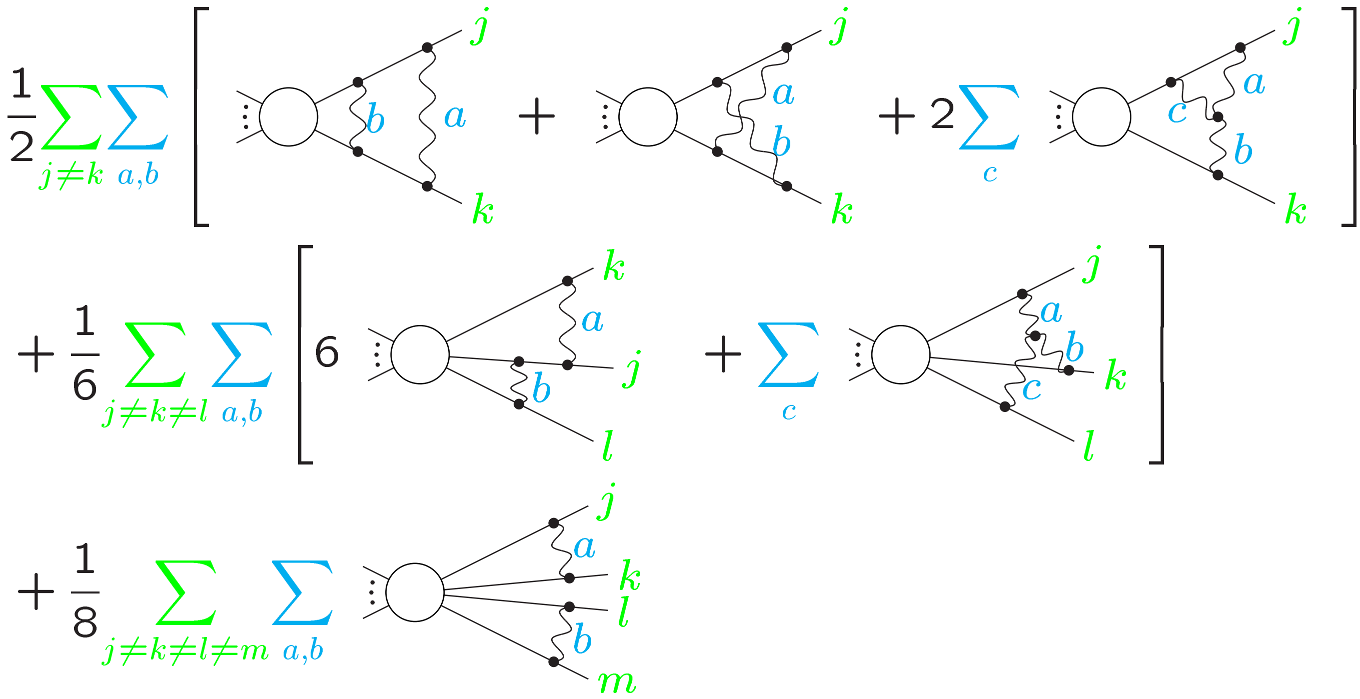
$$S_{3\mathrm{L}}(M_a, M_b; p_j, p_k, p_l) \overset{\text{Sud}}{=}$$
$$4 \int_0^1 \frac{\mathrm{d}x_1}{x_1} \int_0^1 \frac{\mathrm{d}y_1}{y_1} \int_0^1 \frac{\mathrm{d}x_2}{x_2} \int_0^1 \frac{\mathrm{d}y_2}{y_2}\, \theta\left(x_1 y_1 - \frac{M_a^2}{|2p_j p_k|}\right)$$
$$\times\, \theta\left(x_2 y_2 - \frac{M_b^2}{|2p_j p_l|}\right) \theta\left(x_2 - \frac{|p_j p_k|}{|p_j p_l|} x_1\right) \theta\left(x_1 - \frac{m_j^2}{|2p_j p_k|} y_1\right)$$
$$\times\, \theta\left(x_2 - \frac{m_j^2}{|2p_j p_l|} y_2\right) \theta\left(y_1 - \frac{m_k^2}{|2p_j p_k|} x_1\right) \theta\left(y_2 - \frac{m_l^2}{|2p_j p_l|} x_2\right)$$

result for $M_a = \lambda, M_b = M$

$$S_{3\mathrm{L}}(\lambda, M; p_j, p_k, p_l) \overset{\mathrm{NLL_a}}{=} \ln \frac{\lambda^2}{|2p_j p_k|} \ln \frac{m_j^2 m_k^2}{(2p_j p_k)^2} \ln^2 \frac{M^2}{|2p_j p_l|} - \frac{1}{6} \ln^4 \frac{M^2}{|2p_j p_k|}$$
$$- \frac{1}{2} \ln^2 \frac{M^2}{|2p_j p_l|} \left(\ln^2 \frac{m_j^2}{|2p_j p_k|} + \ln^2 \frac{m_k^2}{|2p_j p_k|} \right)$$
$$- \frac{2}{3} \ln^3 \frac{M^2}{|2p_j p_k|} \ln \frac{m_k^2}{|2p_j p_k|}$$

Sum of eikonal contributions

summing over external legs $j, k, l, m, = 1, \ldots, n$ and gauge bosons $\gamma, \mathrm{Z}, \mathrm{W}^{\pm}$



simplifying the sum with global gauge symmetry

$$\sum_{k=1}^{n} \mathcal{M}\, I^{a}(k) = \mathcal{O}\left(\frac{M}{\sqrt{s}}\right) \times \mathcal{M} \simeq 0$$

and relating the result to the 1-loop eikonal corrections we find ...

Result of two-loop calculation

- Two-loop result

$$\delta\mathcal{M}_2 \stackrel{\mathrm{NLL_a}}{=} \frac{1}{2}\mathcal{M}_0\left((\delta_{\mathrm{sew}})^2 + 2\delta_{\mathrm{sew}}\delta_{\mathrm{sem}} + (\delta_{\mathrm{sem}})^2\right)$$

- the symmetric electroweak corrections

$$\delta_{\mathrm{sew}} = \delta^1_{\mathrm{EW}}\Big|_{\lambda=M}$$

  and the subtracted electromagnetic corrections

$$\delta_{\mathrm{sem}} = \delta^1_{\mathrm{EW}} - \delta^1_{\mathrm{EW}}\Big|_{\lambda=M}$$

  do not commute in $\mathrm{NLL_a}$

$$[\delta_{\mathrm{sew}}, \delta_{\mathrm{sem}}] = \mathcal{O}\left[\ln\frac{|2p_kp_l|}{s}\ln^2\frac{s}{M^2}\ln\frac{M^2}{\lambda^2}\right]$$

- subtracted electromagnetic corrections factorize outside electroweak matrix element

  δ_{sem} involves only charges of external legs

Exponentiation

two-loop $\mathrm{NLL_a}$ result

- consistent with separate exponentiation of δ_{sew} and δ_{sem}

$$\mathcal{M}_{2,} = \mathcal{M}_0 \exp(\delta_{\mathrm{sew}}) \exp(\delta_{\mathrm{sem}})$$
$$\neq \mathcal{M}_0 \exp(\delta_{\mathrm{sem}} + \delta_{\mathrm{sew}})$$

- at LL level: confirms results from IREE [Fadin et al. '00]
  and Coulomb gauge [Beenakker, Werthenbach '00, '02]

- at angular-dependent NLL level:
  in agreement with the *matching prescriptions*
  for $f\bar{f} \to f'\bar{f}'$ [Kühn et al. '00, '01]
  and for arbitrary processes [Melles '02]

- confirms the physical picture that *"the electromagnetic corrections below the electroweak scale are only sensitive to the external-leg charges"*

Conclusion

In the TeV region electroweak logarithmic
corrections become potentially very important!
in signal and background calculations

electroweak logarithms are universal
related to external lines and
to renormalization of parameters

next-to-leading logarithms $\propto \alpha^L \ln^{2L-1}(s/M^2)$
numerically sizeable in TeV range

general results for one-loop logarithms known

higher-order contributions:

leading logarithms

- arguments for exponentiation have been given
- exponentiation confirmed at two-loop level

next-to-leading logarithms

- exponentiation formula has been proposed
- exponentiation checked for angular-dependent contributions at two-loop level

Still much to be done