Precision of m_h Predictions in the MSSM

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1. Introduction

Higgs sector of the MSSM:

Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}$$

Higgs potential:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2(\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) + \frac{g'^2 + g^2}{8}(H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2}|H_1 \bar{H}_2|^2$$

Physical states: $h^0, H^0, A^0, H^\pm$

Goldstones: $G^0, G^\pm$

Input parameters:

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$$

$\tilde{t}$ sector of the MSSM:

(scalar partner of the top quark)

Mass matrix for $\tilde{t}_L, \tilde{t}_R$:

$$M^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{1t} & m_t X_t \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{2t} \end{pmatrix}$$

$$\Downarrow \leftarrow \text{Diagonalization, } \theta_{\tilde{t}}$$

$$\begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

$X_t = A_t - \mu \cot\beta$; large mixing possible

$\Rightarrow$ Physical parameters: $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$

Soft SUSY-breaking parameters: $M_{\tilde{t}_L}$, $M_{\tilde{t}_R}$, A_t

Similar expressions for $\tilde{b}, \tilde{\tau}, \ldots$

Yukawa couplings: $\frac{e\, y_b}{2 s_w \cos\beta}$, $\frac{e\, y_\tau}{2 s_w \cos\beta}$, ...

$\Rightarrow$ non-negligible contributions for large $\tan\beta$

Higgs sector tree-level predictions:

$$m^2_{H,h} = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4 M_Z^2 M_A^2 \cos^2 2\beta}\right]$$

$$\Rightarrow m_h \leq M_Z \ , \qquad M_A \approx m_H \approx m_{H^\pm}$$

Huge radiative corrections:

$$\Delta m_h^2 \sim \frac{m_t^4}{M_W^2 \sin^2\beta} \log\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right)$$

$$\approx \frac{m_t^4}{M_W^2 \sin^2\beta} \log\left(\frac{(M_{\rm SUSY}^2 + m_t^2)^2 - m_t^2 X_t^2}{m_t^4}\right)$$

Experimental error:

Prospective accuracies:

$$\Delta m_h^{\rm exp} \approx 200 \text{ MeV} \quad \text{(LHC)}$$
$$\Delta m_h^{\rm exp} \approx 50 \text{ MeV} \quad \text{(TESLA)}$$

Q: Are theoretical uncertainties under control?
A: No!

Q': Will they be under control?
A': Maybe ...

Relevant theoretical uncertainties:

– Parametric uncertainties from m_t

$$\Delta m_h^{\delta m_t} \approx \delta m_t$$

(numerical coincidence)

– Parametric uncertainties from other SM input

→ M_W, α_s, m_b, ...

– Parametric uncertainties from SUSY input

→ $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$,
$m_{\tilde{b}_1}$, $m_{\tilde{b}_2}$, $\theta_{\tilde{b}}$, ...
M_A, $\tan\beta$, $M_{1,2,3}$, ...

⇒ use m_h to constrain SUSY parameters
(any analysis strongly model dependent!)

– Intrinsic uncertainties from unknown higher-order corrections

→ missing 2-loop corrections
missing corrections beyond 2-loop

2. Parametric uncertainties

[S.H., S. Kraml, W. Porod, G. Weiglein '03]

m_t:

future: $\delta m_t^{\text{LC}} \approx 100$ MeV $\Rightarrow \Delta m_h^{m_t} \approx 100$ MeV

M_W:

today: $\delta M_W = 34$ MeV $\Rightarrow \Delta m_h^{M_W} \approx 100$ MeV

future: $\delta M_W^{\text{GigaZ}} \approx 7$ MeV negligible

α_s:

today: $\delta\alpha_s(M_Z) \approx 0.002 \Rightarrow \Delta m_h^{\alpha_s} \approx 0.3$ GeV

future: $\delta\alpha_s(M_Z) \lesssim 0.001 \Rightarrow \Delta m_h^{\alpha_s} \approx 0.1\text{–}0.2$ GeV

m_b:

today: $\delta m_b(m_b) \approx 100$ MeV

$\Rightarrow \Delta m_h^{m_b} \approx 0.1$ GeV (small $\tan\beta/\mu$)

$\Rightarrow \Delta m_h^{m_b} \approx 1$ GeV (large $\tan\beta/\mu$)

future: $\delta m_b(m_b) \approx 50$ MeV

$\Rightarrow \Delta m_h^{m_b} \approx 0.05$ GeV (small $\tan\beta/\mu$)

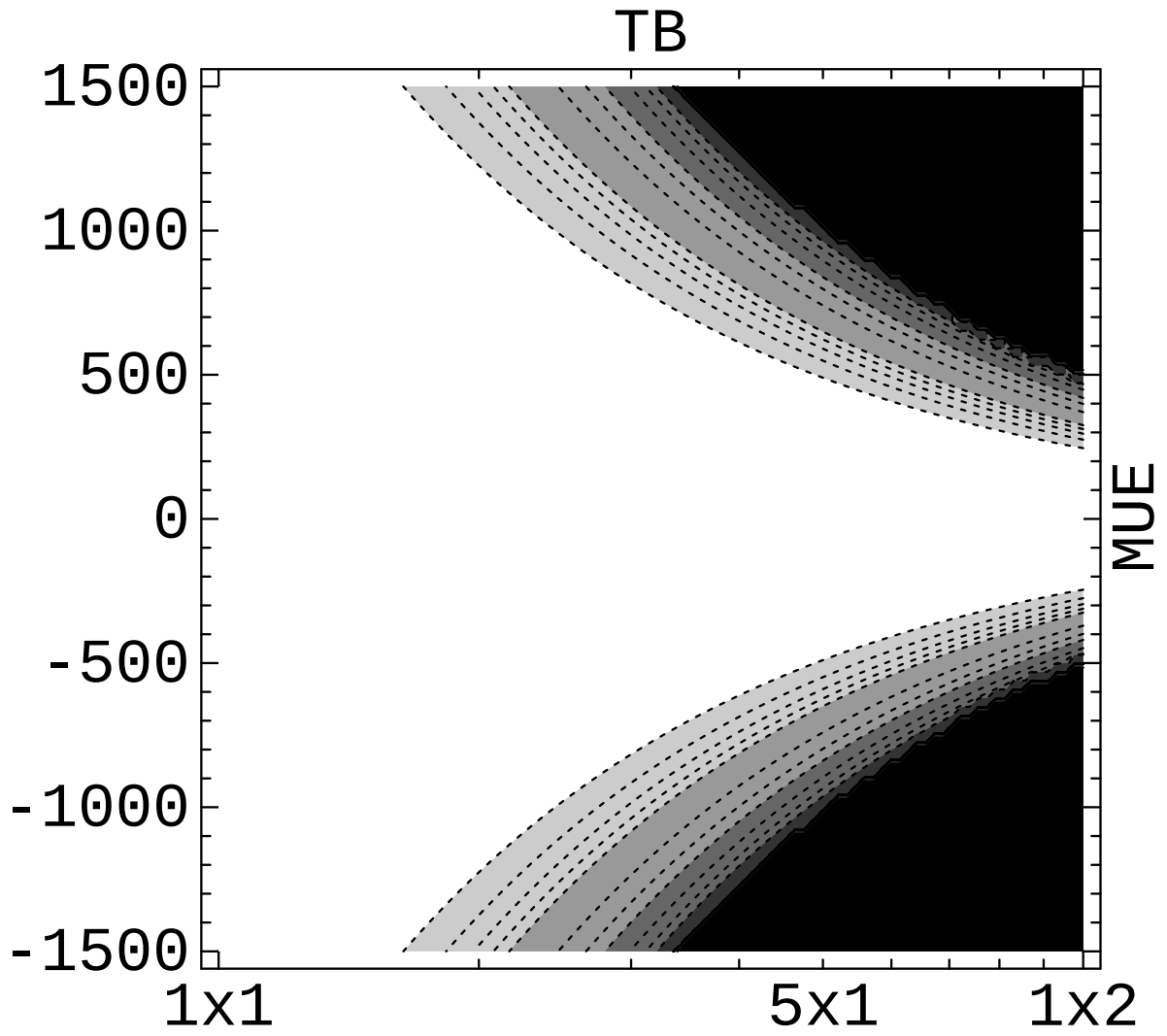
$\Rightarrow \Delta m_h^{m_b} \approx 0.5$ GeV (large $\tan\beta/\mu$) $\rightarrow$ T

$\Rightarrow m_b$ could be limiting factor!

$\Rightarrow$ parametric uncertainties hard to beat

Relevance of $b/\tilde{b}$ contributions (1-loop):

$M_{\rm SUSY} = M_A = 500$ GeV, $X_t = 0$



$\Rightarrow$ region of relevance exists

(but "small")

3. Higher-order (intrinsic) uncertainties

Recent higher-order corrections:

– New renormalization ($\overline{\rm MS}$/OS)
for 1-loop result $\Rightarrow \Delta m_h \approx 1-2$ GeV
[*M. Frank, S.H., W. Hollik, G. Weiglein '02*]

– subleading (non-)log $\mathcal{O}\left(\alpha_t^2\right)$ terms
$\Rightarrow \Delta m_h \lesssim 1-4$ GeV
[*A. Brignole, G. Degrassi, P. Slavich, F. Zwirner '01*]
[*J. Espinosa, R. Zhang '01*]

– "Δm_b" corrections for Higgs masses
$\Rightarrow$ leading $\mathcal{O}\left(\alpha_b \alpha_s\right)$ terms included
$\Rightarrow \Delta m_h$ for large $\tan\beta$
[*M. Carena, D. Garcia, U. Nierste, C. Wagner '00*]

– Remaining $\mathcal{O}\left(\alpha_b \alpha_s\right)$ corrections
$\Rightarrow \Delta m_h \lesssim 0-3$ GeV for very large μ, $\tan\beta$
[*A. Brignole, G. Degrassi, P. Slavich, F. Zwirner '02*]

All included in: *FeynHiggs*2.0

[*M. Frank, T. Hahn, S.H., W. Hollik, G. Weiglein '03*]
www.feynhiggs.de

– Evaluation of full 2-loop EP result
(no comprehensive analysis available)
[*S. Martin '02*]

Remaining higher-order uncertainties:

[*G. Degrassi, S.H., W. Hollik, P. Slavich, G. Weiglein '02*]

2-loop momentum independent:

$\mathcal{O}(\alpha_t\alpha_b)$: $\Delta m_h \lesssim 1-2$ GeV
(like $\mathcal{O}(\alpha_b\alpha_s)$, $\mu/\tan\beta$ large!)
(leading effects already included in *FeynHiggs*)

$\mathcal{O}\left(\alpha_b^2\right)$: $\Delta m_h \lesssim 1$ GeV
($\lesssim \mathcal{O}(\alpha_b\alpha_s)$,$\mu/\tan\beta$ large!)

remaining 2-loop, $q^2 = 0$: $\Delta m_h \lesssim 1.5$ GeV

- subleading $\mathcal{O}(\alpha_t\alpha_s)$
- subleading $\mathcal{O}\left(\alpha_t^2\right)$
- $\mathcal{O}\left(\alpha_\tau^2\right)$
- 2-loop gaugino contributions $\rightarrow$ T

2-loop momentum dependent:

Formally of $\mathcal{O}\left(\alpha_s\alpha_t m_t^2 m_h^2/M_W^2\right)$
(i.e. like the "remaining 2-loop" corrections)

1-loop: $\Delta m_h \lesssim 2$ GeV

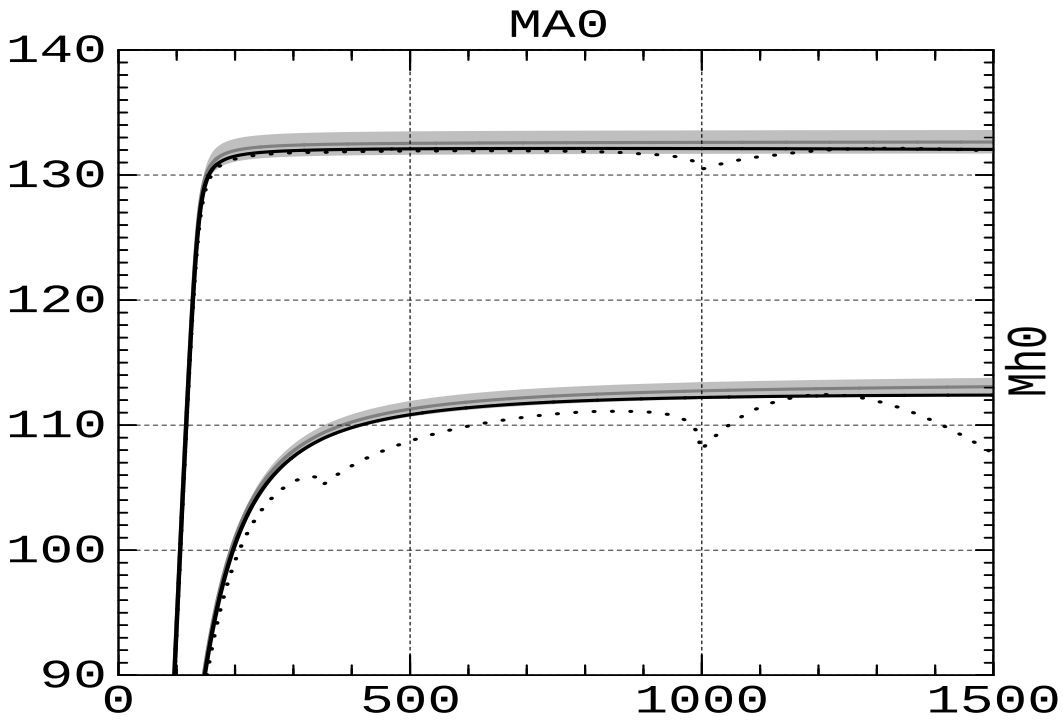
2-loop: $\Delta m_h \lesssim 1$ GeV

Variation of $\overline{\text{MS}}$ renormalization constant:

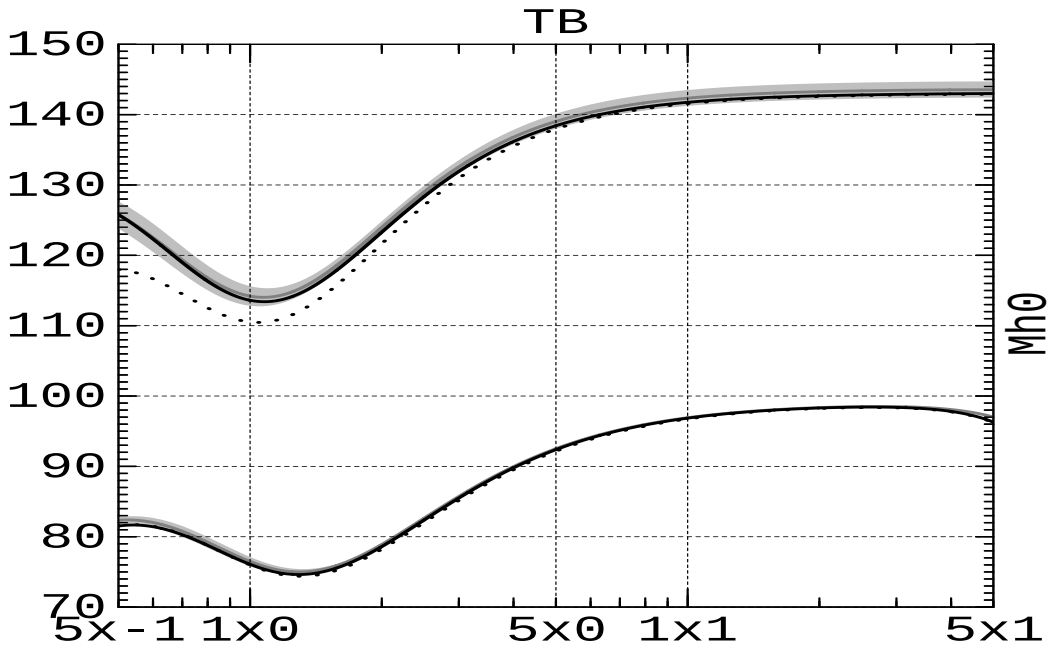
$\mu = 0.5 m_t \ldots 2 m_t$

[*M. Frank, S.H., W. Hollik, G. Weiglein '02*]

$M_{\text{SUSY}} = 500$ GeV, $X_t = 2 M_{\text{SUSY}}$, $\tan\beta = 2, 20$:



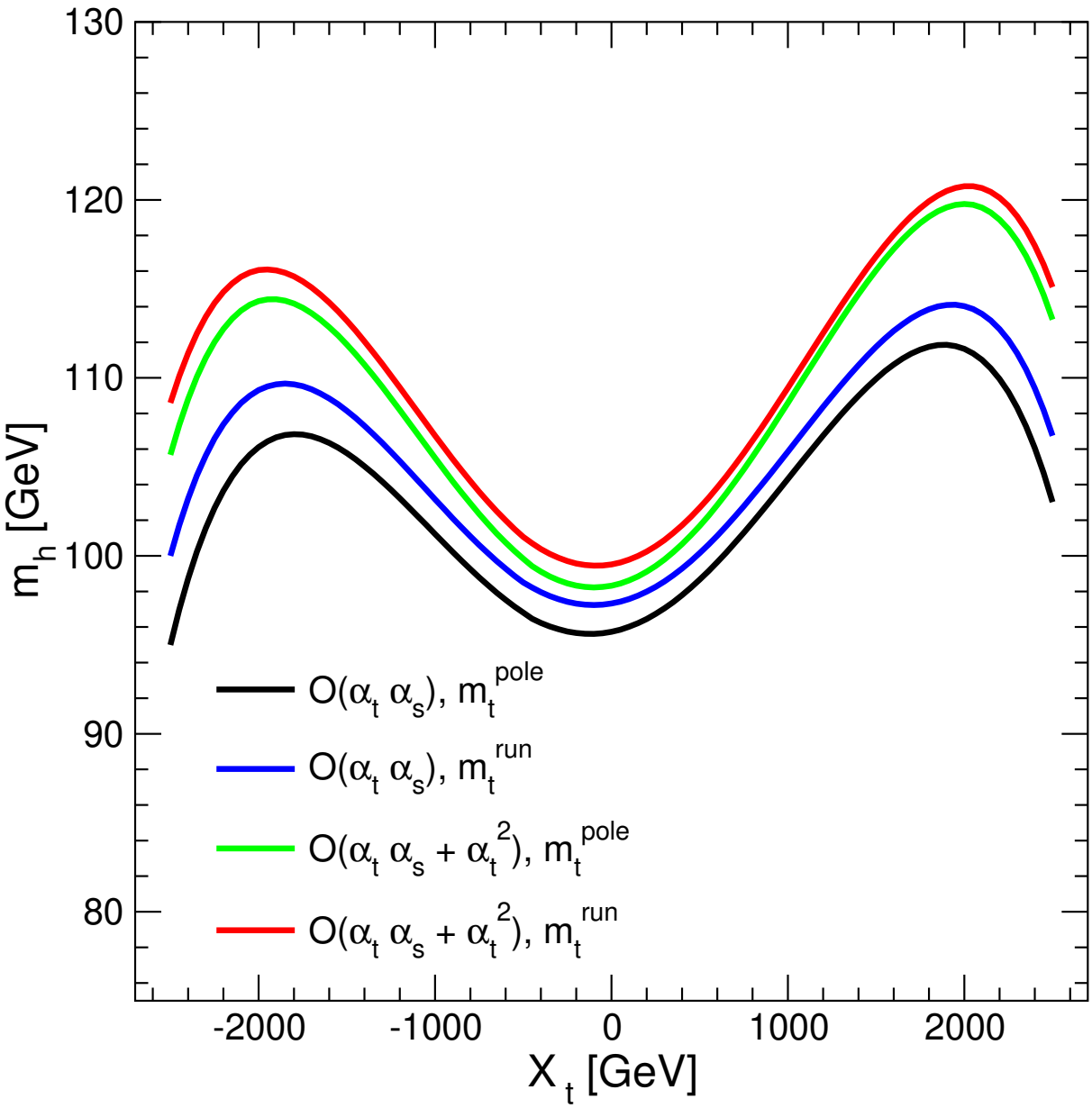
$M_{\text{SUSY}} = 1000$ GeV, $X_t = 2 M_{\text{SUSY}}$, $\tan\beta = 10$, $M_A = 100, 500$ GeV:



3-loop, 4-loop, . . . :

1) changing renormalization of m_t at 2-loop: [*G. Degrassi, S.H., W. Hollik, P. Slavich, G. Weiglein '02*]

$M_A = M_{\rm SUSY} = 1$ TeV, $\tan\beta = 3$:



$\Rightarrow$ $\Delta m_h \lesssim 1.5$ GeV from leading 3-loop corrections

3-loop, 4-loop, . . . :

2) explicit formula for simplified case:

[*G. Degrassi, S.H., W. Hollik, P. Slavich, G. Weiglein '02*]

$$\Delta m_h^2 = \frac{3\alpha_s}{\pi^3} \log^3\left(\frac{M_{\rm SUSY}^2}{m_t^2}\right)\left[\frac{23}{6}\alpha_s^2 - \frac{5}{4}\alpha_s\alpha_t - \frac{33}{64}\alpha_t^2\right]$$

$(M_A = m_{\tilde{g}} = m_{\tilde{t}_1} = m_{\tilde{t}_2} \equiv M_{\rm SUSY},\ \tan\beta \to \infty)$

$\Rightarrow \Delta m_h \lesssim 1.5$ GeV

3) iterative numerical solution of RGEs:

[*A. Hoang '97*]

$\Rightarrow \Delta m_h \lesssim 1.5$ GeV

full intrinsic error:

(from unknown higher-order corr.)

today: $\Delta m_h^{\rm intr} \approx 3$ GeV

[*G. Degrassi, S.H., W. Hollik, P. Slavich, G. Weiglein '02*]

needed for future: $\Delta m_h^{\rm intr} \lesssim 0.5 - 0.1$ GeV

$\Rightarrow$ full 2-loop
$\Rightarrow$ full/leading+subleading 3-loop
$\Rightarrow$ leading 4-loop . . . (future is hard to tell)

4. Conclusinos

- $\Delta m_h^{\rm exp,LC} \approx 50$ MeV sets the mark

- theory error: parametric $\oplus$ intrinsic

- parametric error:

  today: dominated by m_t
  future: possibly dominated by m_b
  (depending on parameter space)
  $\delta m_h^{m_t} \approx \delta m_h^{\alpha_s} \approx 100$ MeV

  needed for future:

  $\Rightarrow \delta m_t \lesssim 100$ MeV
  $\Rightarrow \delta m_b \lesssim 50$ MeV
  $\Rightarrow \delta \alpha_s \lesssim 0.001$

- intrinsic error:

  missing 2-loop: $\Delta m_h \lesssim 2$ GeV
  missing 3-loop, 4-loop, . . . : $\Delta m_h \lesssim 1.5$ GeV

  needed for future: $\Delta m_h^{\rm intr} \lesssim 0.5 - 0.1$ GeV

  $\Rightarrow$ full 2-loop
  $\Rightarrow$ full/leading+subleading 3-loop
  $\Rightarrow$ leading 4-loop . . .