Three-Loop Contributions to $\Delta\rho$ and Δr for large m_t

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I Introduction

central prediction of SM:

$$M_W = f(\underbrace{G_F,\ M_Z,\ \alpha}_{\text{Born}};\quad \underbrace{M_t,\ M_H,\ \ldots}_{\text{radiative corrections}}\quad)$$

[similarly for couplings of fermions: $\sin^2\theta_{\text{eff}}$]

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}(1+\Delta r)$$

Δr dominated by: $\Delta r = -\frac{c^2}{s^2}\Delta\rho + \Delta\alpha$

$$\Delta\rho \sim \left(G_F\, M_t^2\right) + \ldots$$

aim: experiment

δM_W [MeV]	$\delta \sin^2\theta_{\text{eff}} \times 10^{-5}$	
33	17	status
12 – 16	20	Tevatron (run II), LHC
7	1.3	TESLA (GigaZ)

requires theory uncertainty better than 5 MeV!

correlation with δM_t:

$$\delta M_t = 5 \text{ GeV} \Rightarrow \delta M_W = 32 \text{ MeV}$$
$$\Rightarrow \delta \sin^2\theta_{\text{eff}} = 16 \cdot 10^{-5}$$

Tevatron (run II) 2.7 $\to$ 1.4 GeV

LHC 1.0 GeV

LC 0.2 GeV

remaining uncertainties:

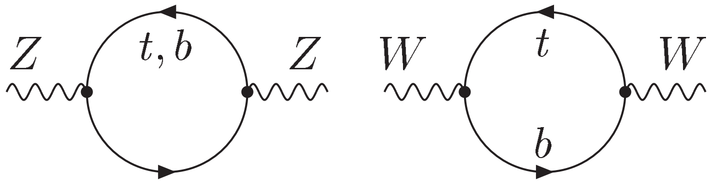
SNOWMASS (Baur et al)

	order	sector	estimate	size $(\times 10^5)$	M_W [MeV]	
2-loop	α^2	fermionic	$N(\alpha/4\pi\hat{s}^2)^2$	8.7	complete [1]	
	α^2	bosonic	$(\alpha/\pi\hat{s}^2)^2$	11.6	2.1	
3-loop	$\alpha\alpha_s^2$	top-bottom doublet	$N_C C_F C_A \alpha\alpha_s^2/4\pi^3\hat{s}^2$	4.7	complete [2]	
	$\alpha\alpha_s^2$	light doublets	$2\ N_C C_F C_A \alpha\alpha_s^2/4\pi^3\hat{s}^2$	9.4	complete	
	$\alpha^3 m_t^6$	heavy top	$5.3\ N_C^2(\alpha m_t^2/4\pi\hat{s}^2 M_W^2)^3$	7.0	4.1	complete now
	$\alpha^3 m_t^6$	heavy top	$3.3\ N_C(\alpha m_t^2/4\pi\hat{s}^2 M_W^2)^3$	1.5	0.9	
	$\alpha^2\alpha_s m_t^4$	heavy top	$3.9\ N_C C_F \alpha^2\alpha_s m_t^4/16\pi^3\hat{s}^4 M_W^4$	7.8	4.5	
4-loop	$\alpha\alpha_s^3 m_t^2$	heavy top	$N_C C_F C_A^2 \alpha\alpha_s^3 m_t^2/4\pi^4\hat{s}^2 M_W^2$	2.3	1.3	
		total			7	

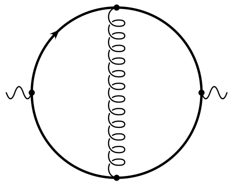
[1] (Freitas, Hollik, Walter, Weiglein)

[2] (Chetyrkin, JK, Steinhauser)

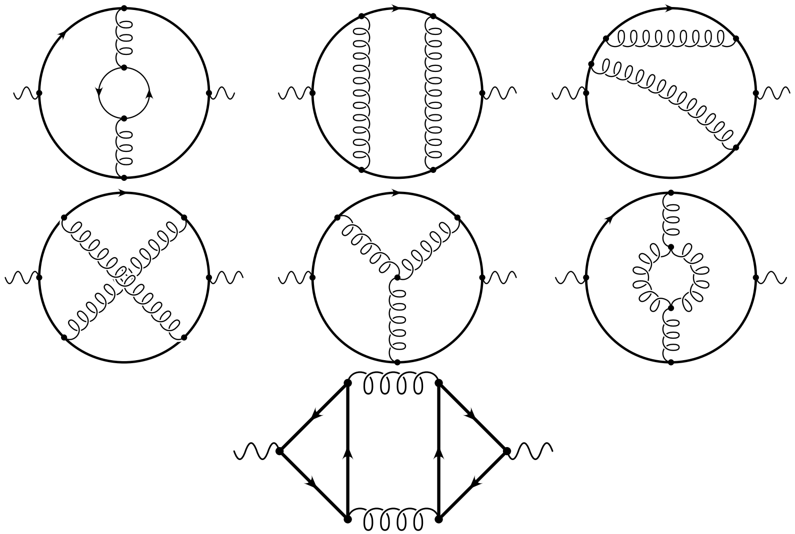
II Three-Loop QCD-Results



one-loop
(Veltman)



two-loop
(Djouadi; Kniehl, JK, Stuart)



three-loop
(Chetyrkin, JK, Steinhauser)

evaluation:

$$\Delta\rho = \frac{\Pi_{ZZ}(0)}{M_Z^2} - \frac{\Pi_{WW}(0)}{M_W^2}$$

$$\begin{aligned}
\Delta\bar{r}_{tb} &\equiv \Delta r_{tb} - \tilde{\Pi}_{\gamma\gamma}(0) + \mathrm{Re}\tilde{\Pi}_{\gamma\gamma}(M_Z^2) \\
&= \frac{c^2}{s^2}\left(\frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}\right) \\
&\quad + \frac{c^2}{s^2}\left(\Pi'_{WW}(0) - \Pi'_{ZZ}(0)|_t - \mathrm{Re}\frac{\Pi_{ZZ}(M_Z^2)|_b}{M_Z^2}\right) \\
&\qquad\qquad + \tilde{\Pi}_{\gamma\gamma}(0)|_t + \mathrm{Re}\tilde{\Pi}_{\gamma\gamma}(M_Z^2)|_b - \Pi'_{WW}(0) \\
&\quad + \frac{c^2}{s^2}\left(\Pi''_{WW}(0)\frac{M_W^2}{2} - \Pi''_{ZZ}(0)|_t\frac{M_Z^2}{2}\right) + \tilde{\Pi}'_{\gamma\gamma}(0)|_t M_Z^2 - \Pi''_{WW}(0)\frac{M_W^2}{2} \\
&\quad + \ldots
\end{aligned}$$

(Chetyrkin, JK, Steinhauser; Avdaev, Fleischer, Michailov, Tarasov)

tadpoles! expansion in $\frac{M_W^2}{M_t^2}$!

Result:

δM_W in MeV	α_s^0	α_s^1	α_s^2
M_t^2	611.9	-61.3	-10.9
const.	136.6	-6.0	-2.6
$1/M_t^2$	-9.0	-1.0	-0.2

III Weak Corrections: Technicalities

approximation: $M_t^2 \gg M_W^2$ $\Rightarrow$ scalar bosons only

status: two-loop $\left\{ \begin{array}{l} \text{Barbieri, Beccaria, Ciafaloni, Curci, Vicere} \\ \text{Fleischer, Jegerlehner, Tarasov} \end{array} \right.$

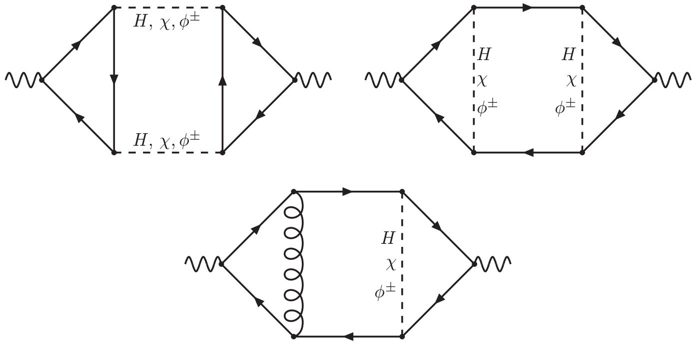
$$\Delta\rho = X_t^2 \, f(M_t/M_H) \quad \text{with} \quad X_t \equiv \frac{G_F \, M_t^2}{8\sqrt{2}\pi^2} = \frac{g^2_{\text{Yukawa}}}{16\pi^2}$$

status: three-loop

$M_H = 0$ (van der Bij, Chetyrkin, Faisst, Jikia, Seidensticker)

X_t^3 and $\alpha_s X_t^2$ tiny corrections

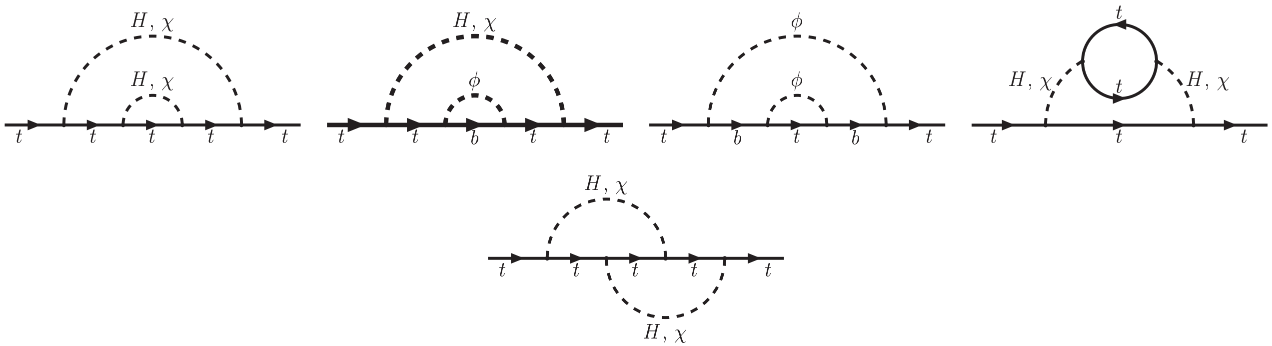
$M_H \neq 0$ (Faisst, JK, Seidensticker, Veretin) requires: 3-loop tadpoles $\left(\, \Pi_{WW}(0),\ \Pi_{ZZ}(0)\, \right)$



$X_t^3\, f(M_t/M_H)$

$\alpha_s X_t^2\, f(M_t/M_H)$

and two-loop on-shell diagrams: $M_t \iff m_t(\overline{\mathsf{MS}})$



in general two (!) mass scales, three loops

special cases:

$M_H = 0:$ $\quad\surd$

$M_H \gg M_t:$ hard mass expansion in $(M_t^2/M_H^2)^n$ mod. log

$M_H = M_t:$ one scale

M_H in neighbourhood of M_t:

Taylor expansion: $\delta = (M_H - M_t)/M_t$

$\Rightarrow$ reduction to one-scale two- or three-loop integrals

Algebraic programs

- generating diagrams: QGRAF (Nogueira)
- hard-mass expansion: EXP (Seidensticker)
- Taylor-expansion / algebraic evaluation of tadpoles:
  MATAD (Steinhauser)
- massless propagators:
  MINCER (Larin, Tkachov, Vermaseren)
- on-shell diagrams:
  TARCER (Mertig, Scharf),
  ONSHELL2 (Fleischer, Kalmykov)

IV Results

define: $X_t = \frac{G_F M_t^2}{8\sqrt{2}\pi^2} \approx 3.2 \cdot 10^{-3}$ and $\frac{\alpha_s}{\pi} \approx 3.5 \cdot 10^{-2}$

- $M_H = 0$

$$
\begin{aligned}
\Delta\rho^{(3\text{ Loop})} = X_t^3 n_c \Big(& 68 + 96\zeta(2)\log 2 + 6\zeta(2) - 612\zeta(3) \\
& + 729 S_2 + 324\zeta(4) - 72 B_4 + 36 D_3 \Big) \\
+ X_t^3 n_c^2 \Big(& -\frac{6572}{15} - \frac{4374}{5} S_2 + \frac{1472}{15}\zeta(2) + 440\zeta(3) \Big) \\
+ X_t^2 \frac{\alpha_s}{\pi} n_c C_F \Big(& \frac{185}{3} + \frac{729}{4} S_2 - 48\zeta(2)\log 2 \\
& - \frac{151}{6}\zeta(2) + 29\zeta(3) - 24\zeta(4) + 12 B_4 \Big)
\end{aligned}
$$

Numerically:

$$
\begin{aligned}
\Delta\rho^{(3\text{ Loop})} = \; & 3\,X_t - 2.2176\,X_t^2 + 249.74\,X_t^3 \\
& - 8.5797 X_t \frac{\alpha_s}{\pi} + 2.9394\,X_t^2 \frac{\alpha_s}{\pi}
\end{aligned}
$$

$\Leftarrow$ accidentally small

- $M_H \approx M_t$: Taylor expansion in $\delta = \frac{M_H - M_t}{M_t}$

$$
\begin{aligned}
\Delta\rho^{(\text{3 Loop})} = {} & X_t^3 \, (95.92 - 111.98\,\delta + 8.099\,\delta^2 + 9.36\,\delta^3 + \ldots) \\
& + X_t^2 \frac{\alpha_s}{\pi} \, (157.295 + 112.00\,\delta - 24.73\,\delta^2 + 7.39\,\delta^3 + \ldots)
\end{aligned}
$$

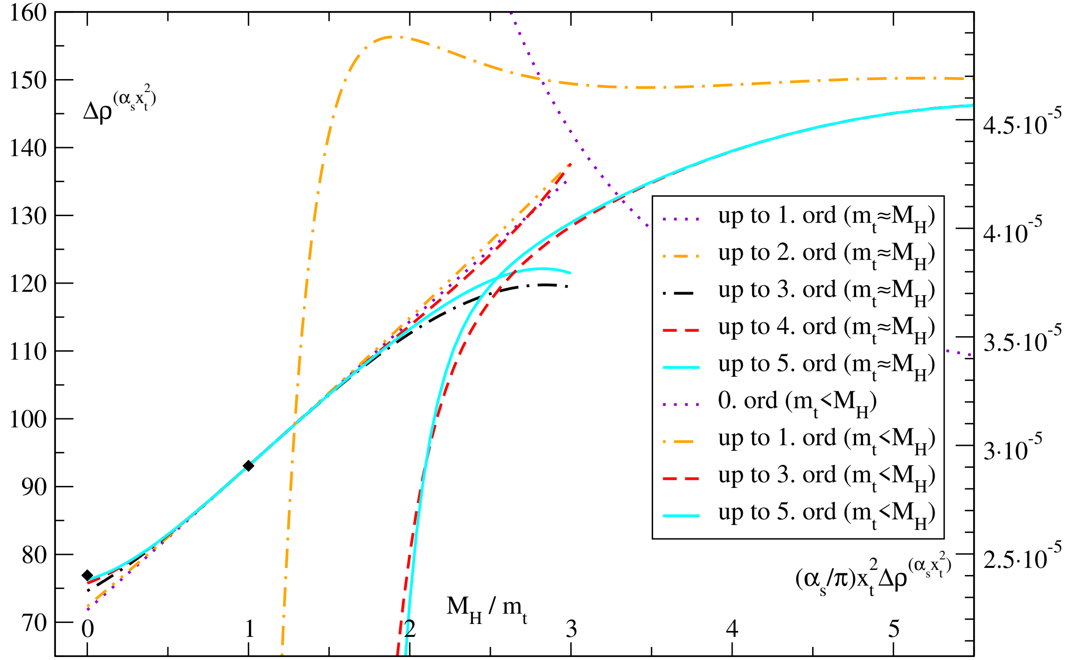
- $M_H \gg M_t$: asymptotic expansion in $Y = \frac{4M_t^2}{M_H^2}$

$$
\begin{aligned}
\Delta\rho^{(\text{3 Loop})} = X_t^3 \Big(& Y^{-1} (-3.17 - 83.25\log Y) \\
& + 1\,(-189.93 - 231.48\log Y - 142.06\log^2 Y + 2.75\log^3 Y) \\
& + Y\,(-332.34 + 77.71\log Y - 68.67\log^2 Y + 51.79\log^3 Y) \\
& + Y^2\,(227.55 - 510.55\log Y + 87.77\log^2 Y + 6.41\log^3 Y)\Big) \\
+ X_t^2 \frac{\alpha_s}{\pi} \Big(& 1\,(79.73 - 47.77\log Y + 42.07\log^2 Y + 9.00\log^3 Y) \\
& + Y\,(225.16 - 179.74\log Y + 70.22\log^2 Y - 19.22\log^3 Y) \\
& + Y^2\,(-76.07 + 25.33\log Y - 9.17\log^2 Y - 5.57\log^3 Y)\Big) + \ldots
\end{aligned}
$$

two steps:

- $\Delta\rho$ in $\overline{\text{MS}}$-scheme

example: $x_t^2\alpha_s$-term



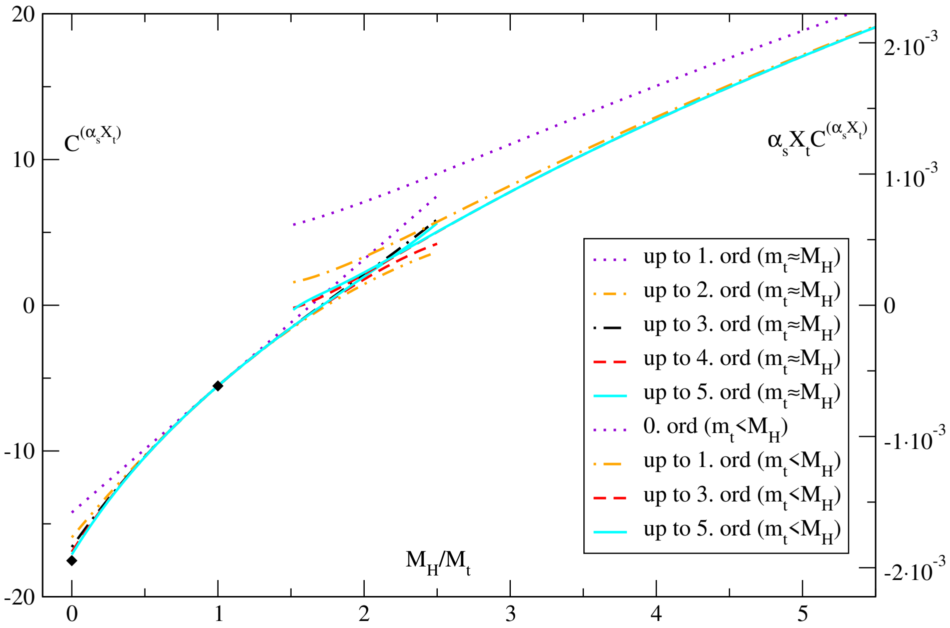
Contributions of order $\alpha_s x_t^2$ to $\Delta\rho$ in the $\overline{\text{MS}}$ definition of the top quark mass. The black squares indicate the points where the exact result is known.

Expansion around $M_H/M_t = 1$ approaches the result at $M_H = 0$.

The two expansions agree for $M_H/m_t \approx 2$.

- transition from $\overline{\text{MS}}$ to on-shell top quark mass

$$\frac{m_t}{M_t} = 1 + \frac{\alpha_s}{\pi} C^{(\alpha_s)} + X_t C^{(X_t)} + \left(\frac{\alpha_s}{\pi}\right)^2 C^{(\alpha_s^2)} + \frac{\alpha_s}{\pi} X_t C^{(\alpha_s X_t)} + X_t^2 C^{(X_t^2)} + \ldots$$



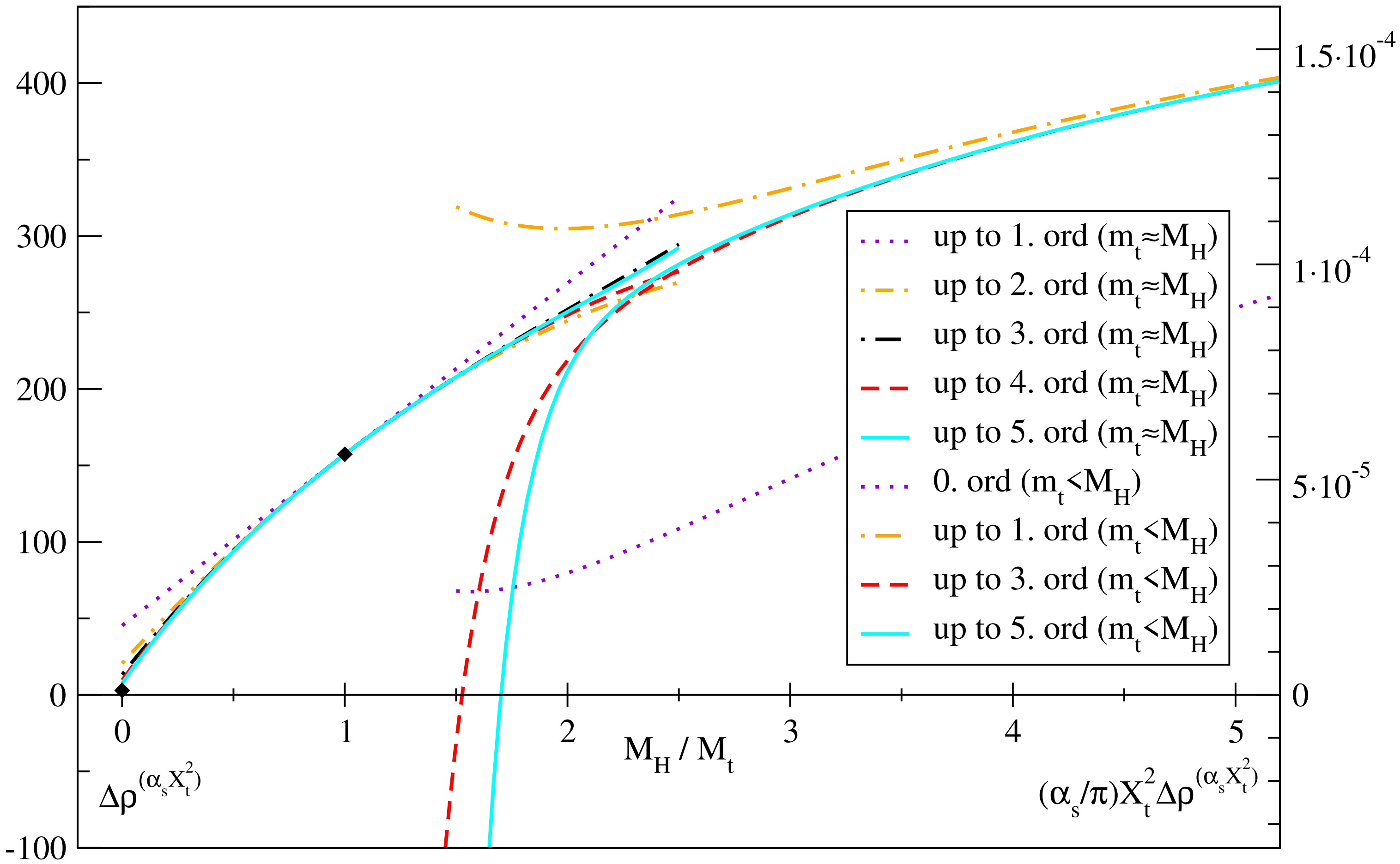
Contributions of order $\alpha_s x_t$ to the relation between the $\overline{\text{MS}}$ and on-shell top quark mass. The black squares indicate the points where the exact result is known.

smooth behaviour in overlapping regions

corresponds to a shift between M_t and m_t by nearly 200 MeV!

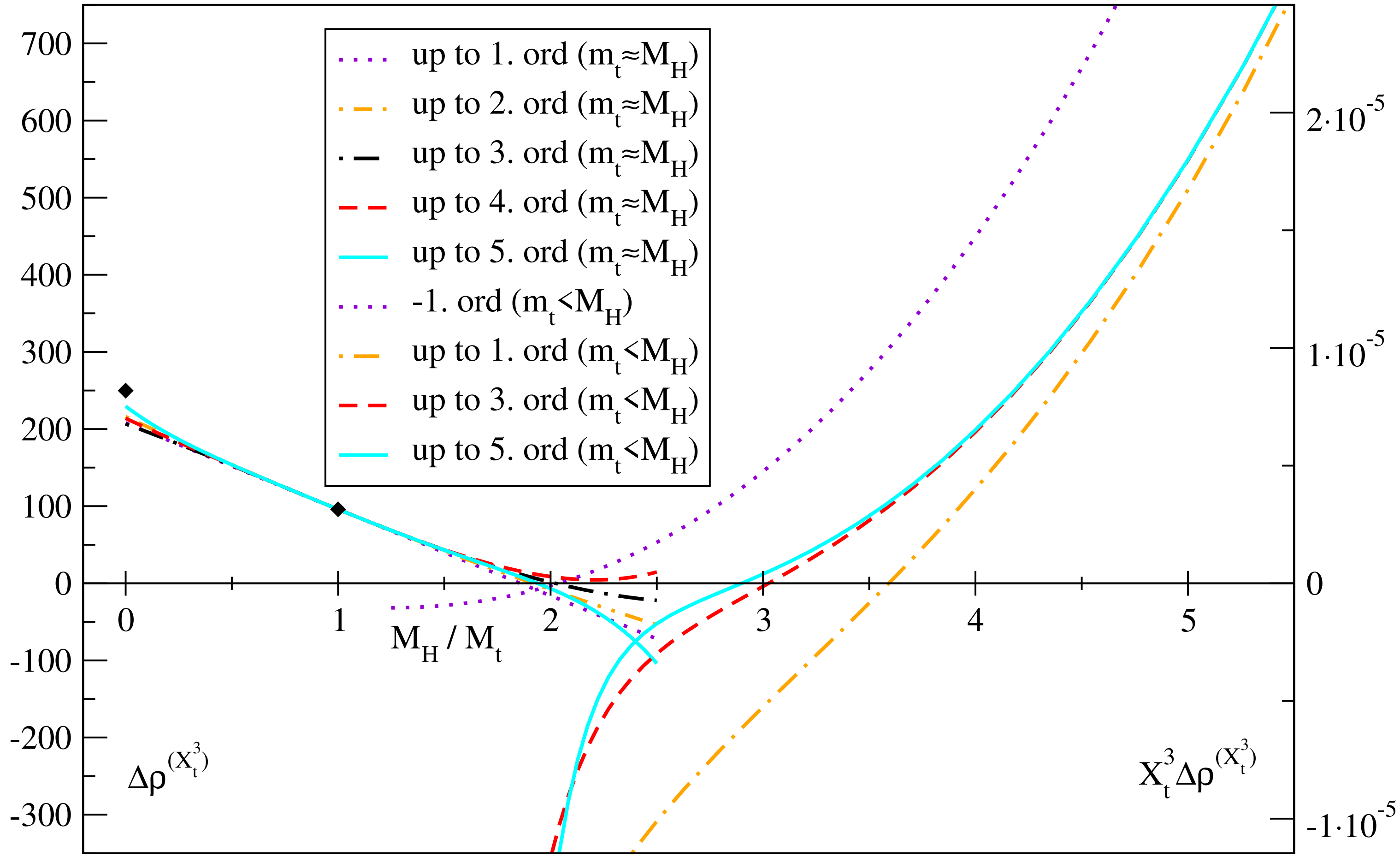
relevance for threshold measurement!

on-shell result

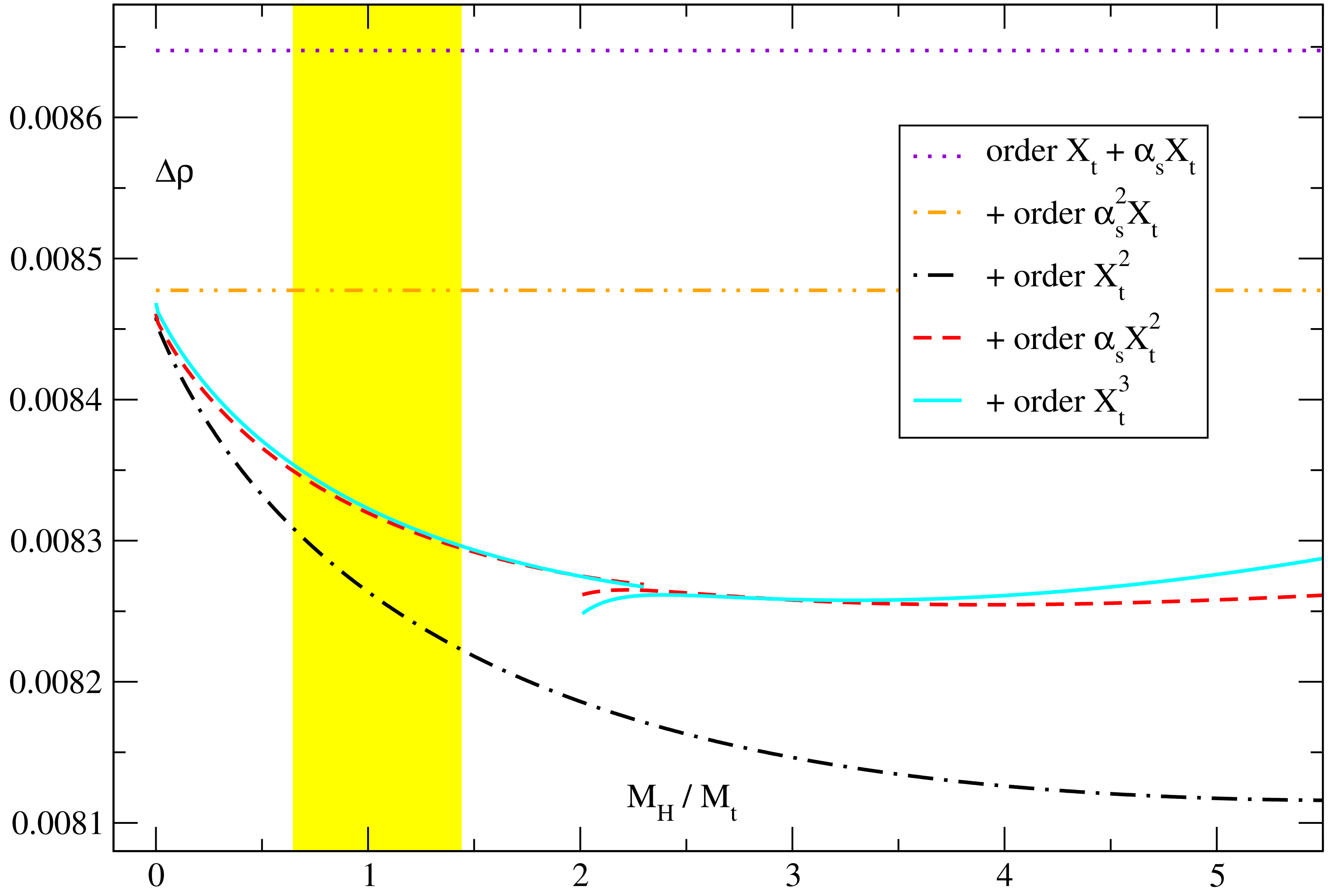


Contributions of order $\alpha_s X_t^2$ to $\Delta\rho$ in the on-shell definition of the top quark mass. The black squares indicate the points where the exact result is known.

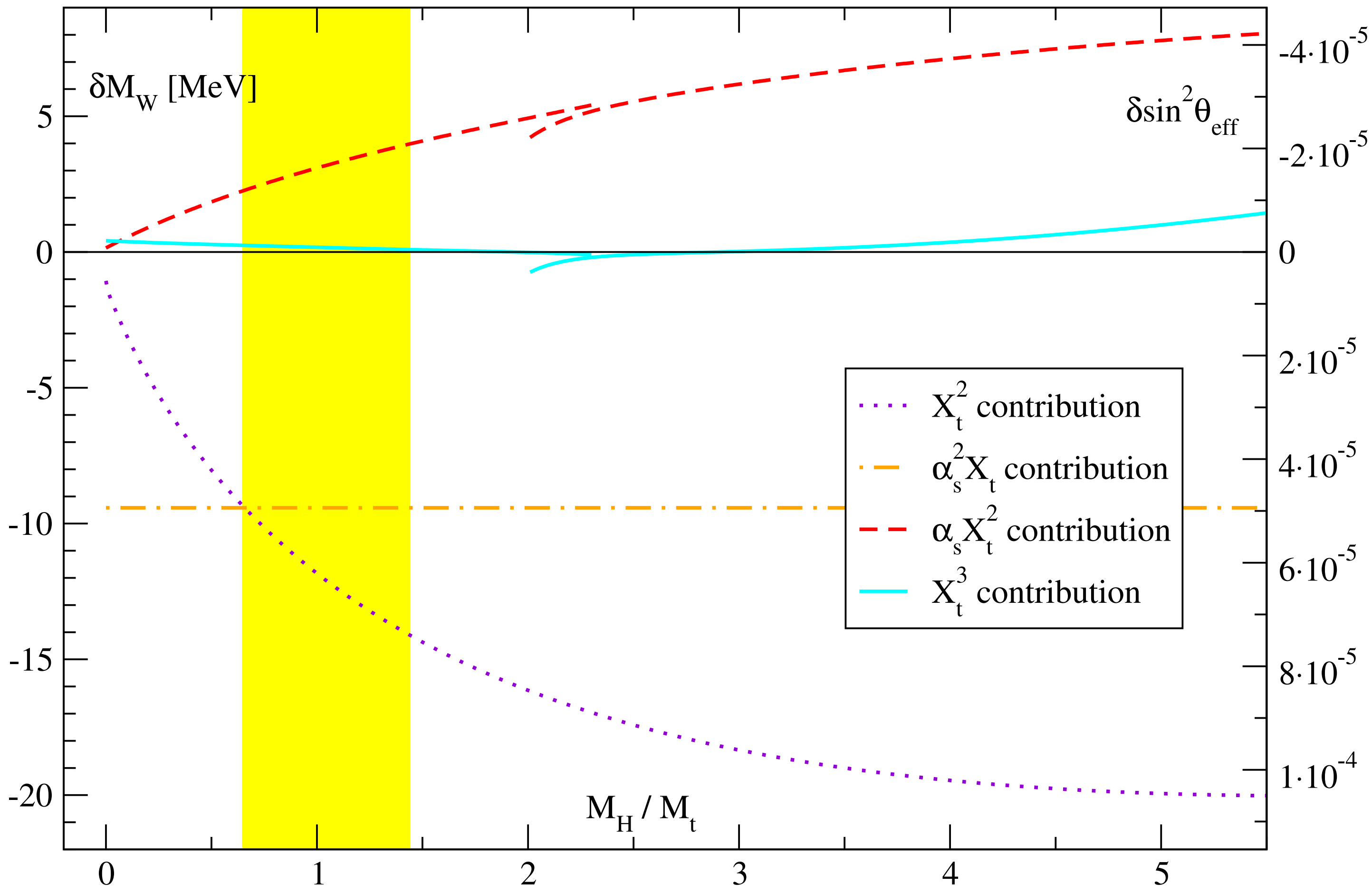
similarly for purely weak corrections



Contributions of order X_t^3 to $\Delta\rho$ in the on-shell definition of the top quark mass. The black squares indicate the points where the exact result is known.



Effect of several contributions to $\Delta\rho$ in the on-shell definition of the top quark mass.



δM_W between 2 and 4 MeV.

$\delta \sin^2 \theta_{\text{eff}}$ between 1 and $2 \cdot 10^{-5}$

V Summary

- dominant electroweak and mixed three-loop corrections are under control
- $\alpha_s X_t^2$-term is comparable to GigaZ precision
- for fixed M_W, M_t upward shift of M_H by 5 GeV
- for fixed M_W, M_H $\to$ shift of M_t by 500 MeV
- corrections comparable to non-enhanced 2-loop effects in Δr

$$\alpha_s X_t^2 \sim \text{fermionic terms}$$
$$X_t^3 \sim \text{bosonic terms}$$

- essential for LC, about a factor 2–4 below LHC