Fullone-loop electroweak corrections to $e^+e^- \to \nu\bar{\nu}H$

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Motivation

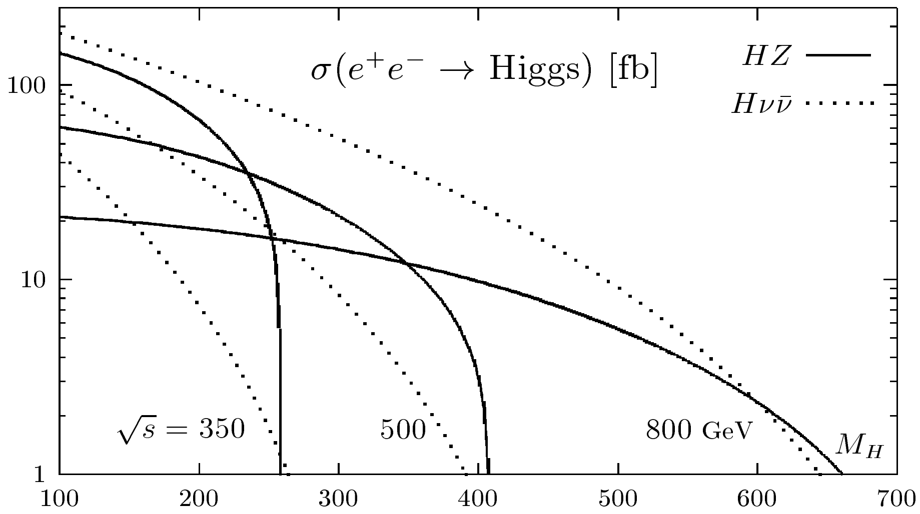
GRACE/1-loop

Checks

Results and comparisons

Higgs production in e^+e^-

Higgsstrahlung vs WW fusion



$e^+e^- \to \nu\bar{\nu}h$ dominant process for Higgs production at high energies

Status of one-loop corrections

- Complete $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to Zh$
  A. Denner, J. Kublbeck, R. Mertig, M. Bohm, Z.Phys. C56 (1992) 261.
  B. A. Kniehl Z. Phys. C55 (1992) 605.
- Radiative corrrections to $e^+e^- \to \nu\bar{\nu}h$ in region $\sqrt{s}, m_h \ll 2M_Z$
  B.A. Kniehl, Phys. Rep. 240 (1994) 211.

- Loop corrections to $h \to W^+W^-$ from fermion/sfermion loops in MSSM

  E. Eberl, W. Majerotto, V.C. Spanos, Phys. Lett. B538 (2002) 353

  T. Hahn, S.Heinemeyer, G. Weiglein, hep-ph/0211204.

- Full one-loop electroweak radiative corrections to $e^+e^- \to \nu\bar{\nu}h$

  G.B., Boudjema, Fujimoto, Ishikawa, Kaneko, Kato, Shimizu, hep-ph/0212261

  Denner, Dittmaier, Roth, Weber, hep-ph/0302198

GRACE/1-loop

- On-mass shell renormalization scheme (Kyoto group) Counterterms are generated automatically
- Symbolic manipulation code based on REDUCE or FORM writes for each diagram

$$T_i^{loop} T_j^{tree\dagger} \tag{1}$$

Symbolic manipulation performs: Dirac algebra, tensor manipulation in $n = 4 - 2\epsilon$, Feynman parametric integrals, shift momentum... Then Fortran code is generated.

- Two libraries for Loop integration: numerical integration, FF package
- Integration over phase space : BASES

System tested in SM at tree-level and at one-loop

- Internal checks: ultraviolet divergence, infrared finiteness...
- Gauge parameter independence
- Comparison with other calculations $2 \to 2$

$\boxed{e^+e^- \to \nu\bar{\nu}h \text{ at one-loop}}$

Number of diagrams

- **12** tree-level diagrams
- **1350** one-loop diagrams
  Among these **98** 5-point functions

Neglecting electron-scalar couplings

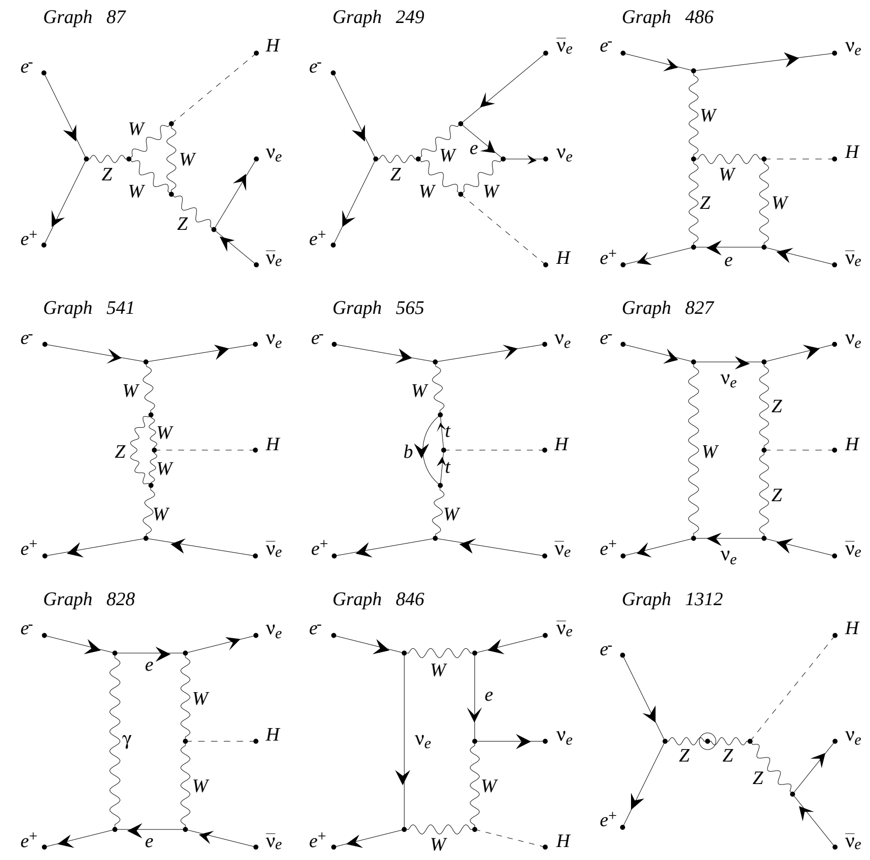
- **2** tree-level diagrams: fusion +higgstrahlung
- **249** one-loop diagrams
  Among these **5** 5-point functions

Keep full set of diagrams for checks at one point in phase space
Results for total cross-section based on 249 diagrams (keeping m_e to handle collinear singularities)

Define s-channel

$$\sigma_s = 3\sigma(e^+e^- \to \nu_\mu\bar{\nu}_\mu h)$$
$$\sigma_t = \sigma_{TOTAL} - \sigma_s$$

Selection of loop diagrams



produced by GRACEFIG

$\boxed{\text{Checks on calculation}}$

- Checks performed with
  1. One random phase space point
  2. Full set of diagrams
  3. Quadruple precision

- Ultraviolet Finiteness
  regularise infrared divergence by fictitious photon mass $\lambda = 10^{-15} GeV$
  extract term in $C_{UV} = \frac{1}{\epsilon} - \gamma_E + \log 4\pi$
  stability of result for different values of $C_{UV} \rightarrow$ 30 digits precision)
  repeat this test with non-zero non-linear gauge parameters

- Infrared finiteness
  Regulate infrared divergence in photon exchange diagram with fictitious photon mass λ
  Dependence on λ cancels with the one in soft bremstrahlung
  Soft photon is calculated analytically

$$d\sigma_{soft}(\lambda, E_\gamma, k_c) = d\sigma_0 \delta_{soft}(\lambda, E_\gamma, k_c) \tag{2}$$

  check independence on $\lambda \rightarrow$ 23 digits stability

- Gauge parameter dependence
  Use non-linear gauge fixing (here $C_{UV} = 0$)
  Non-trivial check for:
  Correct input file: Feynman diagrams, counterterms...
  Symbolic manipulation part
  Correctness of reduction formulae for tensor integral (including pentagon)

General Non-linear gauge fixing

$$\begin{aligned}\mathcal{L}_{GF} \;=\; & -\frac{1}{\xi_W}|(\partial_\mu \;-\; ie\tilde{\alpha}A_\mu \;-\; igc_W\tilde{\beta}Z_\mu)W^{\mu+} \\ & +\xi_W\frac{g}{2}(v+\tilde{\delta}H+i\tilde{\kappa}\chi_3)\chi^+|^2 \\ & -\frac{1}{2\xi_Z}(\partial.Z+\xi_Z\frac{g}{2c_W}(v+\tilde{\varepsilon}H)\chi_3)^2 \;-\; \frac{1}{2\xi_A}(\partial.A)^2\end{aligned}$$

Choose t'Hooft-Feynman gauge ($\xi_W = \xi_Z = \xi_A = 1$)

1. avoid large numerical cancellations
2. no need for higher order tensor structure in loop integrals

Vary parameters $\zeta = (\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\kappa}, \tilde{\epsilon})$
For each loop graph, g

$$\mathrm{d}\sigma_g = \mathrm{d}\sigma_g(\zeta) = \Re\left(\mathcal{T}_g^{loop}\cdot\mathcal{T}^{tree\;\dagger}\right)$$

Comprehensive checks of NLG parameters independence, for example for only one non-zero parameter, $d\sigma$ polynomial in ζ

$$\mathrm{d}\sigma_g = \mathrm{d}\sigma_g^{(0)} + \zeta\mathrm{d}\sigma_g^{(1)} + \zeta^2\mathrm{d}\sigma_g^{(2)} + \zeta^3\mathrm{d}\sigma_g^{(3)}$$

can reconstruct $d\sigma_g^{(0,1..4)}$ and verify that $d\sigma$ is independent of ζ

$$\mathrm{d}\sigma = \sum_g \mathrm{d}\sigma_g = \sum_g \mathrm{d}\sigma_g^{(0)}\;, \tag{3}$$

$$\mathrm{sum}_i = \frac{\sum_g \mathrm{d}\sigma_g^{(i)}}{\mathrm{Max}_g(|\mathrm{d}\sigma_g^{(i)}|)} = 0\quad,\quad i=1,2,3\;, \tag{4}$$

For this, set $\Gamma_Z = 0$ (choose point away from resonance).

$\boxed{\text{Independence on NLG parameters}}$

	# graphs	sum_3	sum_2	sum_1
$\tilde{\alpha}$	149	–	10^{-28}	10^{-30}
$\tilde{\beta}$	314	–	10^{-31}	10^{-23}
$\tilde{\delta}$	477 *(1059)*	10^{-20}	10^{-20}	10^{-26}
$\tilde{\kappa}$	122	–	10^{-23}	10^{-23}
$\tilde{\epsilon}$	128 *(132)*	–	10^{-21}	10^{-30}

Additional check on total cross-section

- Hard bremstrahlung : $e^+e^- \to \nu\bar{\nu}h\gamma$ computed automatically
  Integration over phase space done with BASES
  Check k_c independence, $k_c = .001 \to .1$, stability within precison of MonteCarlo (4 digits)

- Comparison with other calculations $e^+e^- \to Zh, h \to WW$ (excellent agreement)

$e^+e^- \to ZH$	GRACE-LOOP	Denner, et al. ZPhys. C56
$\sqrt{s} = 500\text{GeV}$ $M_H = 100\text{GeV}$	4.15239	4.1524
$\sqrt{s} = 500\text{GeV}$ $M_H = 300\text{GeV}$	6.90166	6.9017
$\sqrt{s} = 1000\text{GeV}$ $M_H = 100\text{GeV}$	-2.16561	-2.1656
$\sqrt{s} = 1000\text{GeV}$ $M_H = 300\text{GeV}$	-2.49949	-2.4995
$\sqrt{s} = 1000\text{GeV}$ $M_H = 800\text{GeV}$	26.10942	26.1094
$\sqrt{s} = 2000\text{GeV}$ $M_H = 100\text{GeV}$	-11.54131	-11.5414
$\sqrt{s} = 2000\text{GeV}$ $M_H = 300\text{GeV}$	-12.82256	-12.8226
$\sqrt{s} = 2000\text{GeV}$ $M_H = 800\text{GeV}$	11.24680	11.2468

Results

Input parameters:
$\alpha^{-1} = 137.0359895$
$M_Z = 91.1876\text{GeV}.$
$m_e = 0.510999\text{MeV}, m_\mu = 105.6584\text{MeV}, m_\tau = 1.777\text{GeV}$
$M_t = 174\text{GeV}$
$M_u = M_d = 58\text{MeV},\ M_s = 92\text{MeV}$
$M_c = 1.5\text{GeV},\ M_b = 4.7\text{GeV}$

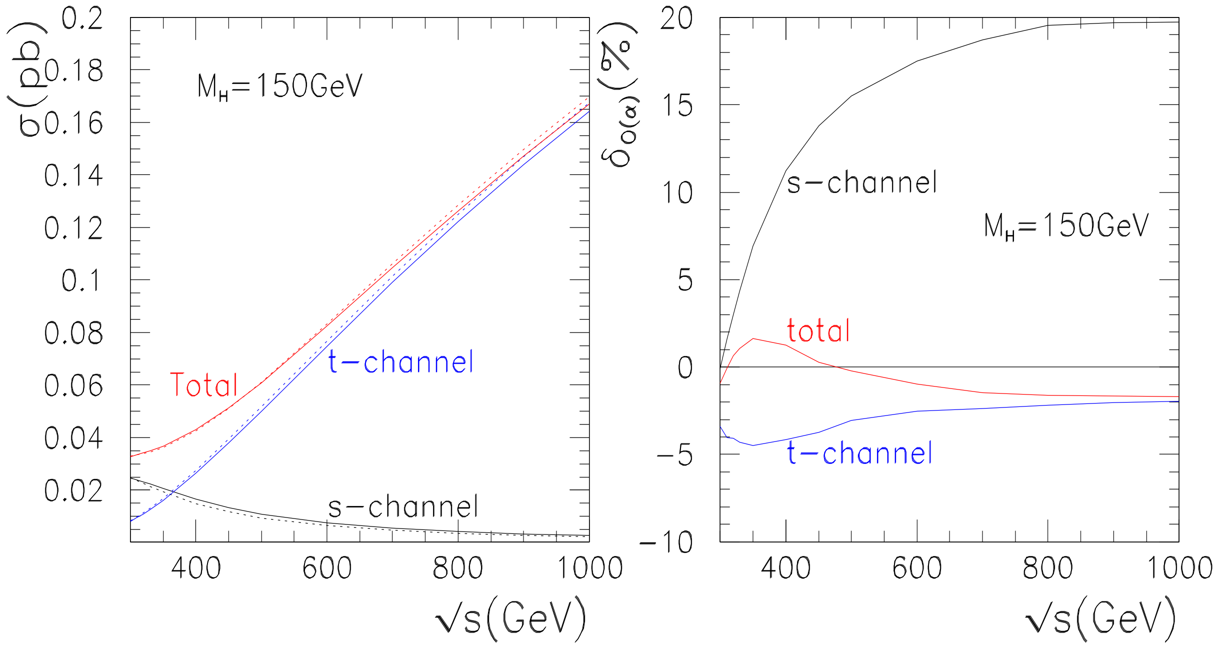
constant fixed Z-width, $\Gamma_Z = 2.4952\text{GeV}$
(required especially for Higgs-strahlung processes)

Our numerical value of M_W is derived through Δr even though on-shell renormalisation program uses M_W as input parameter,

$$M_W = M_W(M_H)$$

In Δr, we include NLO QCD corrections and two-loop Higgs effects. We take $\alpha_s(M_Z^2) = .118$, $G_\mu = 1.16639 \times 10^{-5}\text{GeV}^{-2})$
$\quad\rightarrow\quad \Delta\alpha(M_Z) = .059258$
$M_W = 80.3767\text{GeV}$ for $M_H = 150\text{GeV}$
$M_W = 80.3158\text{GeV}$ for $M_H = 350\text{GeV}.$

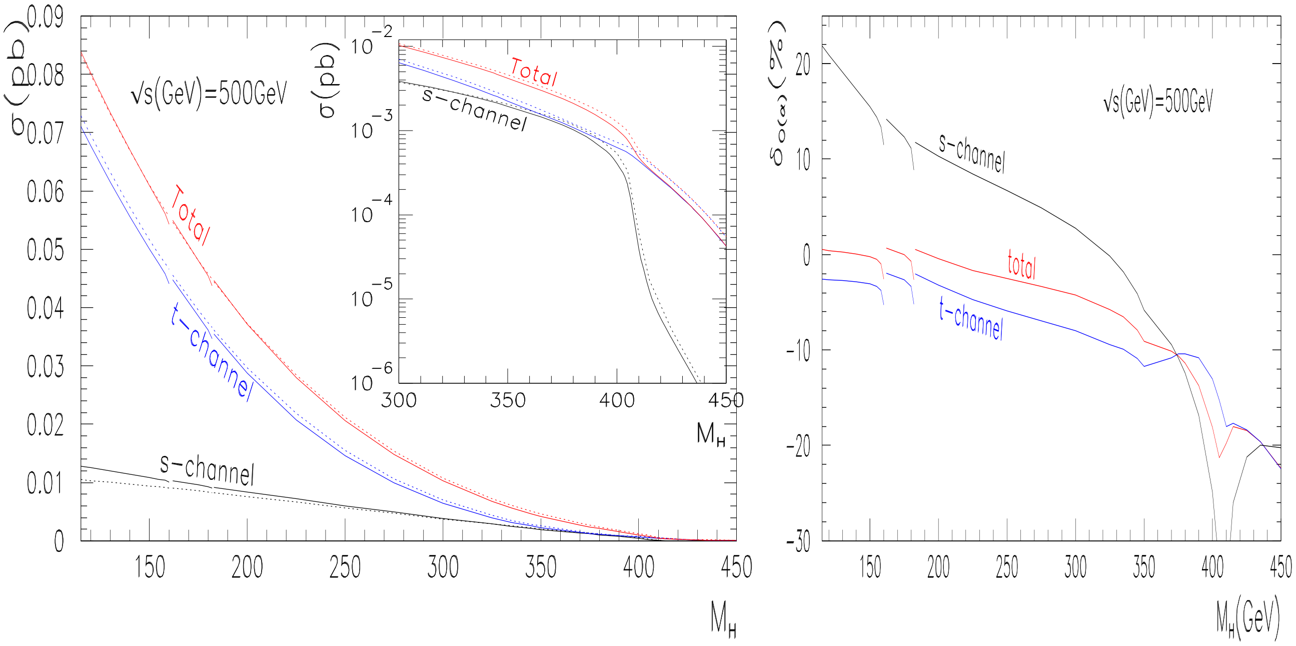
$$\boxed{M_H = 150\text{GeV}}$$



$$\delta = \frac{\sigma_{\mathcal{O}(\alpha)}}{\sigma_{tree}} - 1$$

- Recall s-channel $\sigma_s = 3\sigma_{\nu_\mu \bar{\nu}_\mu h}$ includes diagrams beyond $e^+e^- \to Zh$
- s-channel: large positive correction mostly virtual+real QED corrections
- Large corrections when Higgstrahlung contribution is small
- Total correction small, lowest around 500GeV, $\delta \approx .2\%$

$\boxed{\sqrt{s} = 500\text{GeV}}$



- spikes at $M_H = 2M_W, 2M_Z$
- Large QED corrections in s-channel

Genuine weak corrections

Subtract QED corrections

s-channel:
correction to $eeZ+$ soft bremstrahlung, universal soft photon factor.

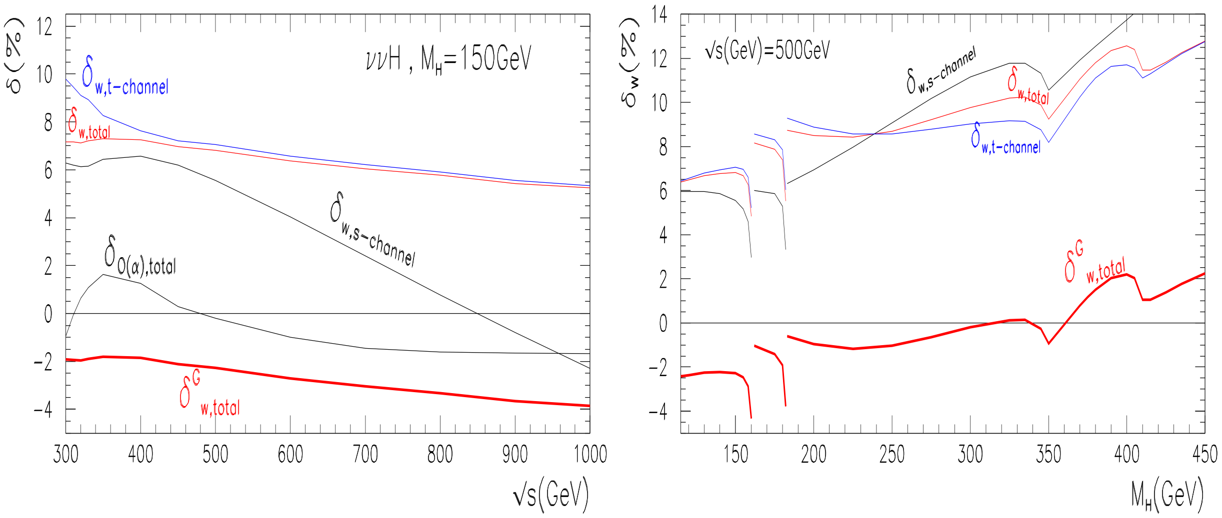
$$\delta_{V+S}^{QED} = \frac{2\alpha}{\pi}\left((L_e - 1)\ln\frac{k_c}{E_b} + \frac{3}{4}L_e + \frac{\pi^2}{6} - 1\right)\ ,\ L_e = \ln(s/m_e^2)$$

E_b the beam energy

Subtracting this contribution from our k_c dependent (numerical) result $\rightarrow$ genuine weak correction
$\rightarrow \delta_{W,s-channel}$

t-channel :
subtract δ_{V+S}^{QED} from full $\mathcal{O}(\alpha)$ since leading (infrared+collinear) contributions are given by universal factor
$\rightarrow \delta_{W,t-channel}$

Large QED corrections require higher order treatment



Weak corrections in term of G_μ

Write tree-level in term of G_μ $\rightarrow$ subtract universal weak corrections (fermionic contribution) to two-point function

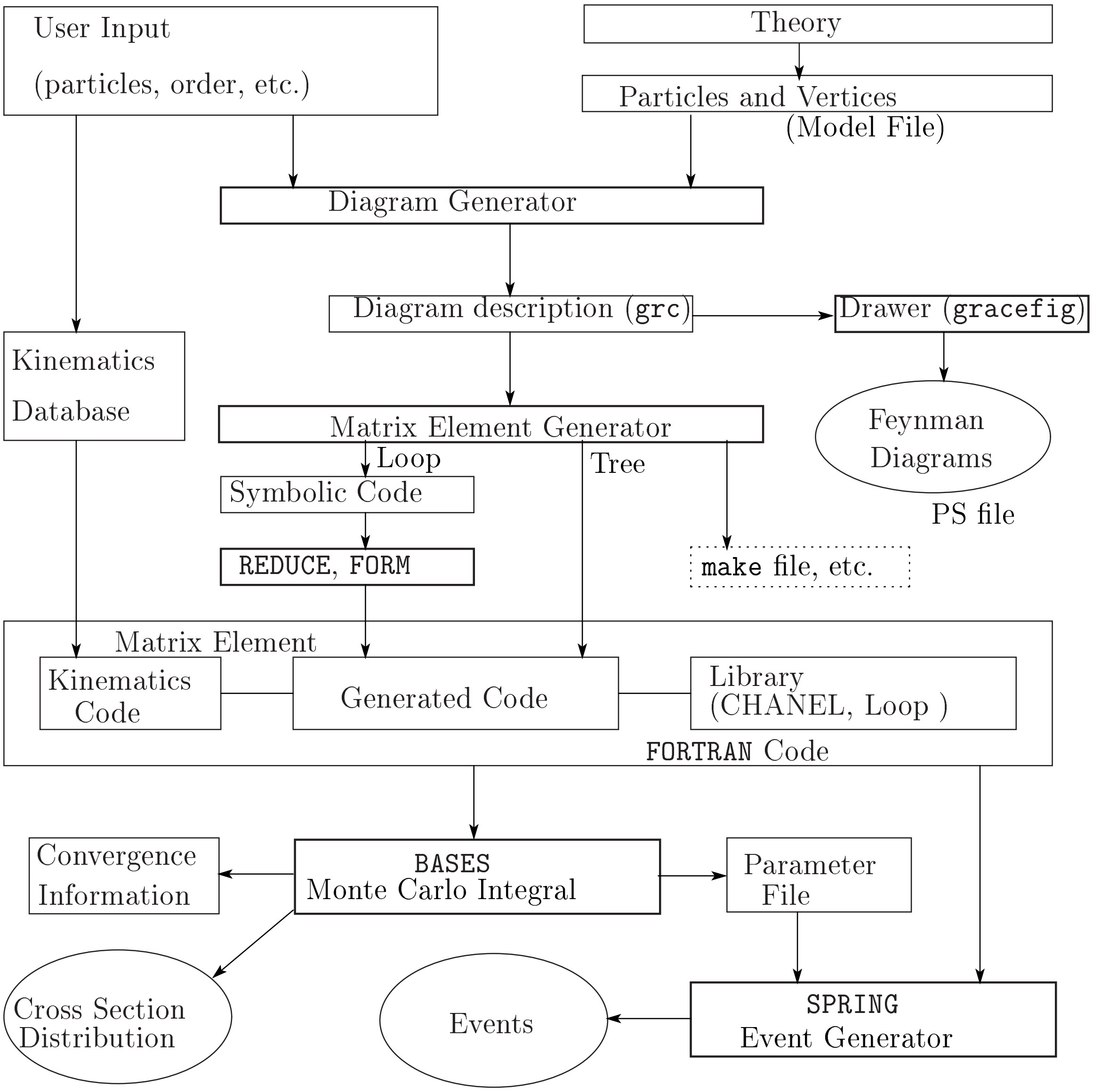
Define non-universal weak correction

$$\delta_W^G = \delta_W - 3\Delta r$$

- Note $\Delta r \approx 3\%$
- $\delta_W^G \approx -(2-4)\%$, given the precision in e^+e^- collider must be taken into account
- $\delta^G_{W,s-channel} \approx -12\%$ at 1TeV , similar to $e^+e^- \rightarrow Zh$

Conclusions

- Full $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to \nu\bar{\nu}h$ with Grace/Loop
- Consistency of results checked with NLG fixing (full 5 parameters)
- Result in agreement with Denner et al. (next)
- For $M_H = 150\text{GeV}$ modest $\delta_{\mathcal{O}(\alpha)} = \pm 2\%$ (very small at 500GeV)
- Genuine weak corrections for $M_H = 150\text{GeV}$, range 5-7%
- For heavier Higgs, corrections can be larger, $\delta_{\mathcal{O}(\alpha)} = \pm 10\%$, $M_H = 350\text{GeV}$
- Need to include multiple photon emission


User Input
(particles, order, etc.)
Theory
Particles and Vertices
(Model File)
Diagram Generator
Diagram description (grc)
Drawer (gracefig)
Kinematics
Database
Matrix Element Generator
Feynman
Diagrams
PS file
Loop
Tree
Symbolic Code
REDUCE, FORM
make file, etc.
Matrix Element
Kinematics
Code
Generated Code
Library
(CHANEL, Loop)
FORTRAN Code
Convergence
Information
BASES
Monte Carlo Integral
Parameter
File
Cross Section
Distribution
Events
SPRING
Event Generator