Final Report on Two Loop

Electroweak Corrections to Muon Lifetime

Małgorzata Awramik

University of Silesia, Katowice

Institute of Nuclear Physics, Cracow

in collaboration with M. Czakon

Outline:

1. Introduction
2. Matching
3. Methods and Software
4. Tests and Results
5. Conclusions

Motivation

- μ decay lifetime $\Rightarrow M_W^{th}$
  - input parameter
  - precision tests
  - M_H determination
- The experimental and theoretical precision are of the same order:
  - Experimental precision:

$M_W^{exp} = 80.423$	$\pm$	0.033 GeV	present
	$\pm$	0.015 GeV	LHC
	$\pm$	0.006 GeV	TESLA

  - Error on M_W^{th}
    - input parameters:

      top quark mass:

      $\delta M_W \sim$ 30 MeV (10MeV, 1MeV)

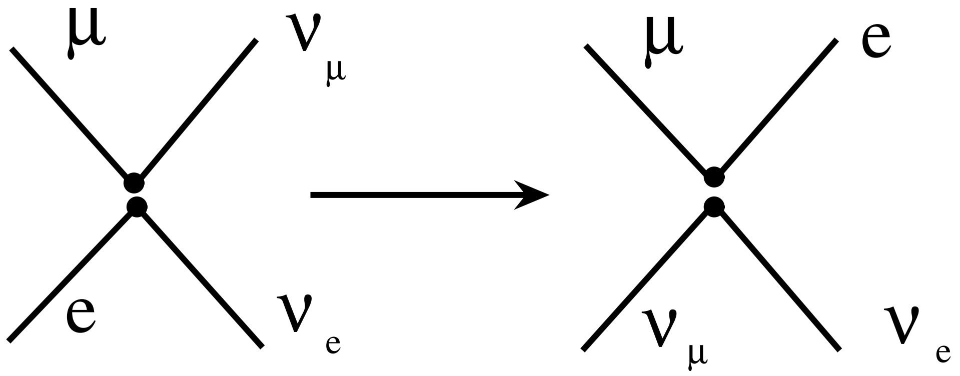
      running of α: $\delta M_W \sim$ 6 MeV
    - Perturbative expansion $\delta M_W <$ 6 MeV

Muon Decay

- Dynamics of the muon decay described in the Fermi Model (effects of the light scales)

$$\frac{1}{\tau_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}(1+\Delta q)$$

Fierz rearrangement:



- Point interaction corrected in the SM (effects of the heavy particles)

$$G_\mu = \frac{\alpha\pi}{2 s_W^2 M_W^2}(1+\Delta r)$$

Previous results

- One-loop corrections $\mathcal{O}(\alpha)$

  A. Sirlin '80

  W.J. Marciano, A. Sirlin '80

- QCD corrections $\mathcal{O}(\alpha\alpha_S)$, $\mathcal{O}(\alpha\alpha_S^2)$

  A. Djouadi '88, F. Halzen and B.A. Kniehl '91

  L. Avdeev, J. Fleischer, S. Mikhailov, O. V. Tarasov '94

  K. Chetyrkin, J. Kühn, M. Steinhauser '95

- Two-loop electroweak Fermionic $\mathcal{O}(\alpha^2)$

  - expansions in m_t and M_H

    J. v.d Bij, F. Hoogeveen '87 ($\mathcal{O}(G_\mu^2\, m_t^4)$ for $M_H = 0$)

    J. Fleischer, O.V. Tarasov, F. Jegerlehner '95

    G. Degrassi, P. Gambino, A. Vicini '96 ($\mathcal{O}(G_\mu^2\, \frac{m_t^2}{M_W^2})$)

    G. Degrassi, P. Gambino, A. Sirlin '97

  - exact

    A. Freitas, W. Hollik, W. Walter, G. Weiglein '00

    $\Uparrow$ disagreement

    M.Awramik, M.Czakon '03

- Two-loop electroweak Bosonic $\mathcal{O}(\alpha^2)$
  - Higgs boson mass dependence

    A. Freitas, W. Hollik, W. Walter, G. Weiglein '00

    $\Uparrow$ disagreement
  - exact M.Awramik,M.Czakon '02

    $\Updownarrow$ agreement

    in the range of applicability
  - expansions in M_H^2 and $\sin\theta_W^2$

    A.Onishchenko, O.Veretin '02
- Toward three-loop fermionic corrections
  - Top quark contributions to $\Delta\rho$

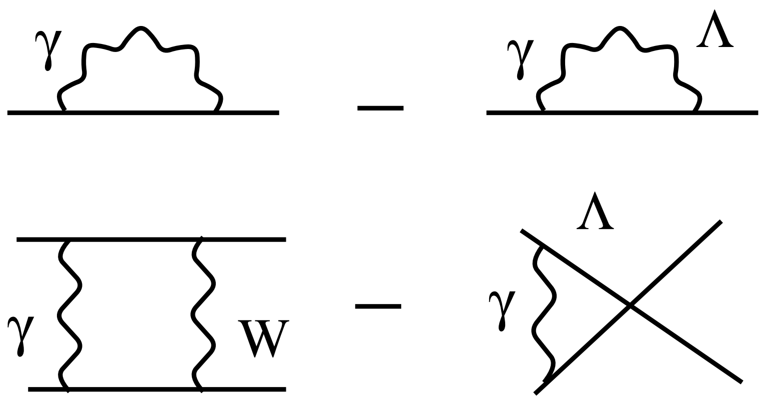
    M. Faisst, J.H. Kuhn, T. Seidensticker, O. Veretin '03

2. MATCHING

- Originally, factorization obtained by playing tricks diagram by diagram (Sirlin '80), *e.g.*
  - for external fermion lines write photon propagator as

$$\frac{1}{k^2} = \frac{1}{k^2 - M_W^2} + \frac{1}{k^2}\frac{-M_W^2}{k^2 - M_W^2}$$

    and keep first term.
  - throw away box diagram with photon exchange
- amounts to subtraction of the respective Fermi model contribution in Pauli-Villars regularization



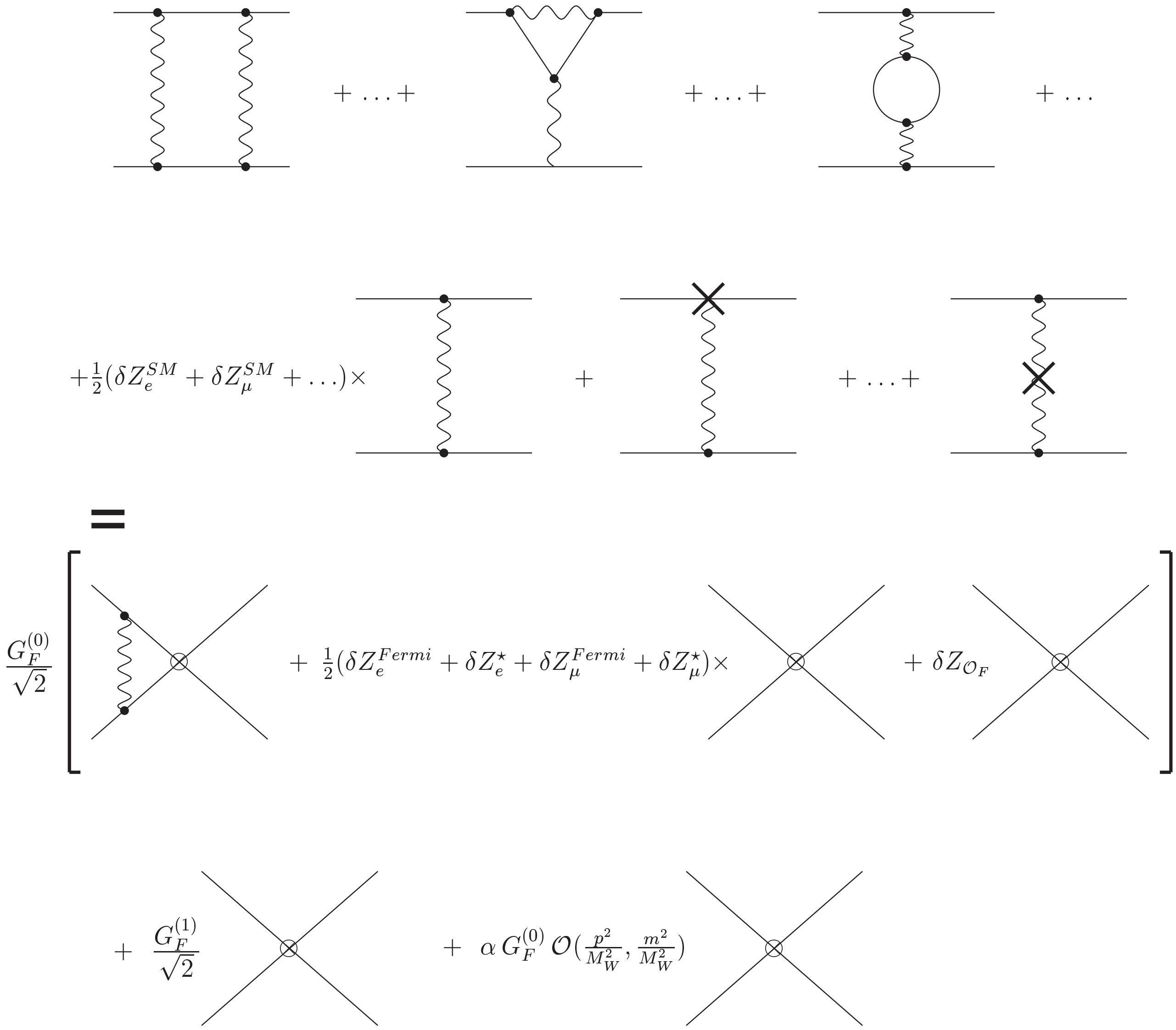
- Analogous methods used for previous two loop calculations

General method

For the renormalized amputated Green functions:

$$\mathcal{G}^{\mathrm{SM}} = \mathcal{G}^{\mathrm{Fermi}} + \mathcal{O}(\frac{1}{M_W^4})$$

e.g. at first order :

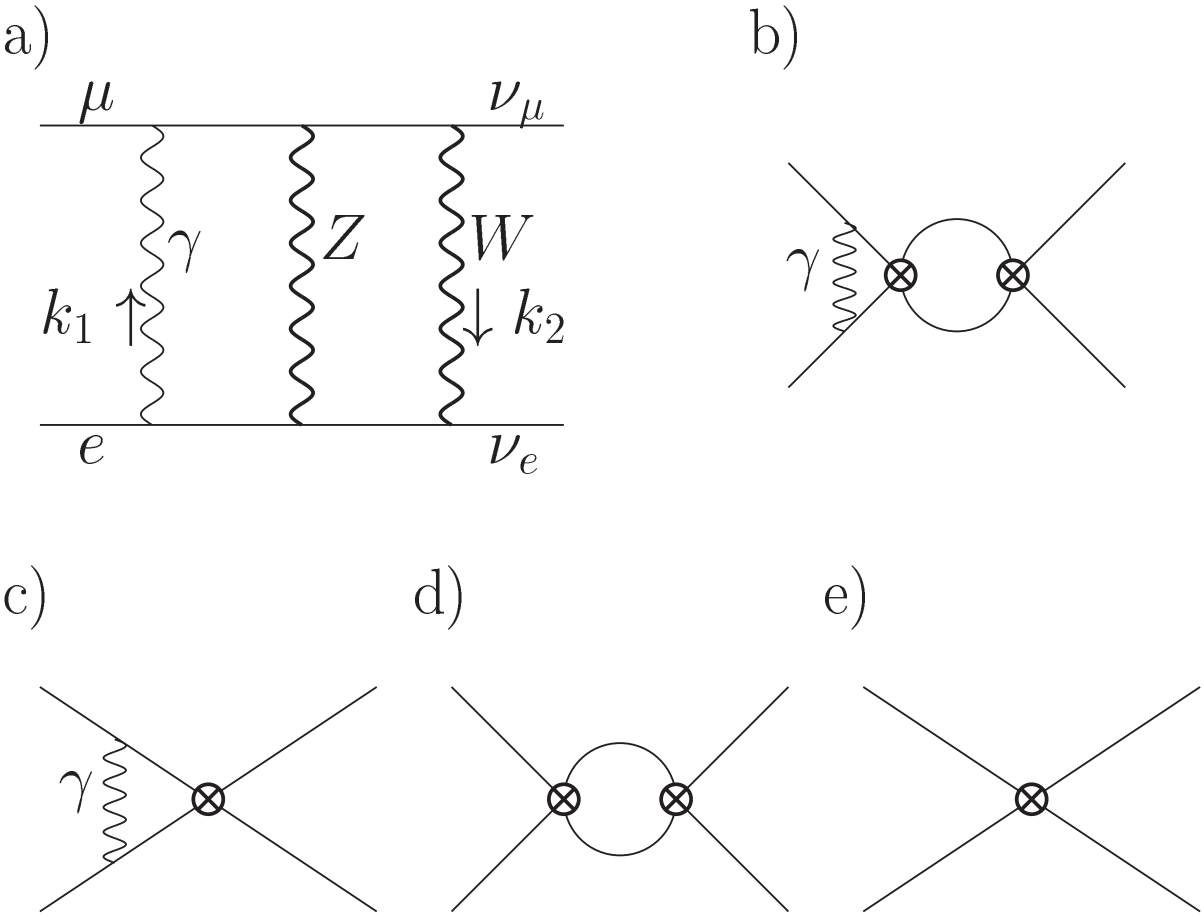


- Alternative point of view:

  The Factorization Theorem

  Based on the large mass expansion

$$\mathcal{F} \cong \sum_{\mathcal{H}\subseteq\mathcal{F}} \mathcal{S} \cdot \mathrm{T}(\mathcal{H})$$

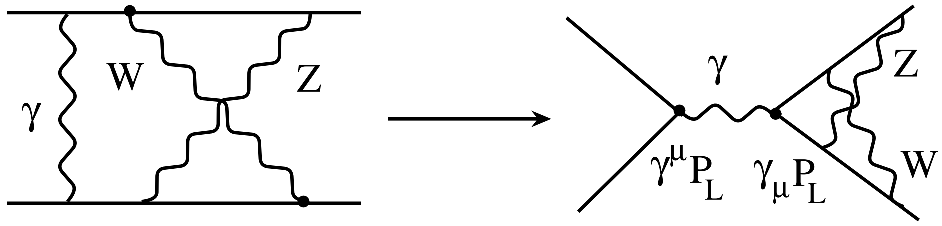


Problem With γ Matrices

Projections of chains of gamma matrices onto the Born level operator:

$$\Gamma_1 \otimes \Gamma_2 \to A\, \gamma_\mu \mathrm{P_L} \otimes \gamma^\mu \mathrm{P_L}$$

Define $\Gamma_1 \otimes \Gamma_2$ by Fierz transformation:



In practice, use a projector

$$A = -\frac{1}{2\,d\,(d-2)}\,\mathrm{Tr}\left(\Gamma_1\,\gamma_\mu\,\mathrm{P_R}\,\Gamma_2\,\gamma^\mu\,\mathrm{P_R}\right)$$

$\longrightarrow$ explicitly respects the Fierz Symmetry

$\longrightarrow$ no problems with γ_5

$\longrightarrow$ the result is finite and gauge independent

$\longrightarrow$ in massive case, IR finite, checked to be correct!

Calculation

- RENORMALIZATION
  - special treatment of tadpoles $\longrightarrow$ 1PI diagrams and gauge invariant mass counter-terms
  - on-shell and $\overline{\mathrm{MS}}$ renormalization schemes
- ALGEBRAIC REDUCTION
  - propagator two loop diagrams

    $\longrightarrow$ G. Weiglein, R. Scharf, M. Böhm, '94
  - vacuum diagrams $\longrightarrow$ Integration by Parts

    $\longrightarrow$ K.G. Chetyrkin, F.V. Tkachov '81

    $\longrightarrow$ A.I. Davydychev, J.B. Tausk, '93
- NUMERICAL EVALUATION
  - propagator master integrals

    $\longrightarrow$ S. Bauberger and M. Böhm, '95 (**S2LSE**)
  - vacuum master integral

    $\longrightarrow$ A.I. Davydychev, J.B. Tausk, '93

- SOFTWARE
  - diagram generation - **DiaGen**
    - $\rightarrow$ diagrams with counterterm insertions
    - $\rightarrow$ draws topologies
    - $\rightarrow$ output suitable for further evaluation
    - $\rightarrow$ very fast
  - algebraic evaluation routines - **FORM**
  - numerical evaluation - **S2LSE**
    - $\rightarrow$ one dimensional integral representation
    - $\rightarrow$ quadrupole precision added
  - fully automatized system
    - $\rightarrow$ **AWK** - intercommunications
    - $\rightarrow$ Makefiles - overall calculation control
    - $\rightarrow$ shell scripts - testing utilities (expanders)
    - $\rightarrow$ execution time - 1.5 h on 1.4GHz PC

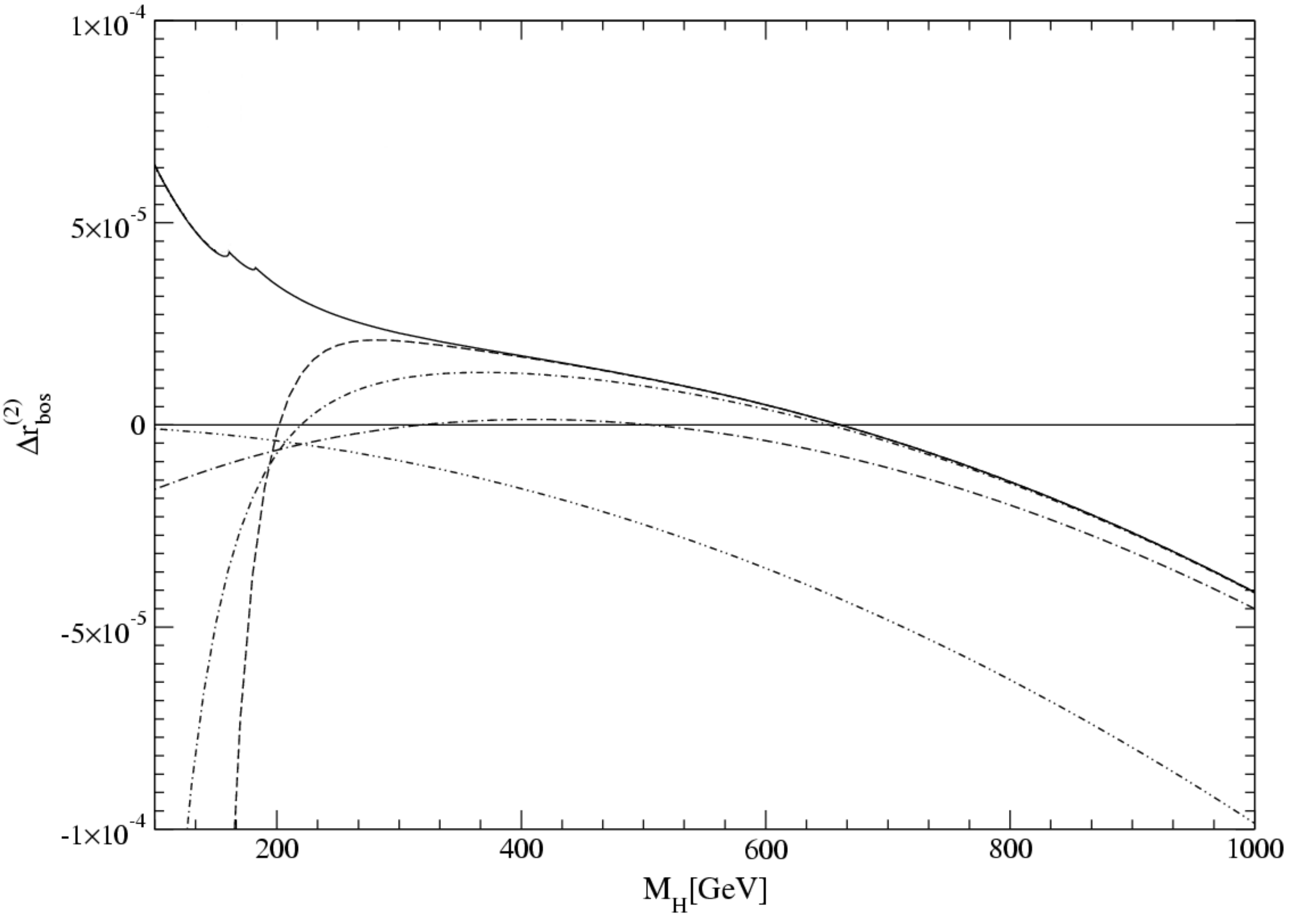
Tests and Results

- Lagrangian - BRS symmetry
- propagators - Slavnov-Taylor identities.
- $\delta M^2_{W,Z}, \; \delta \sin^2_W, \; \delta e, \; \Delta r$

  - independence of gauge parameters
- Δr - UV and IR finiteness
- $\Delta r_{\rm bos}$ - calculation performed in two schemes, $\overline{\rm MS}$ and on-shell; after translation - analytic results agree.
- Matching procedure:

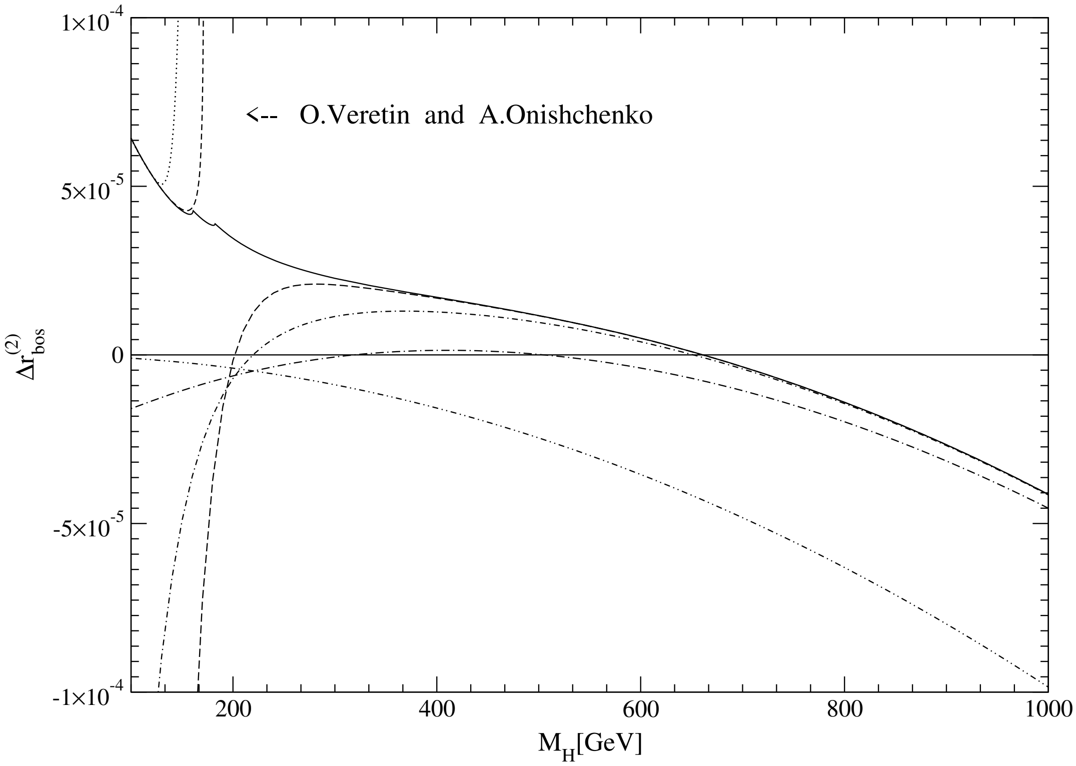
  matching equation at second order with IR divergences regulated with massive fermions:

- Tests of reduction and numerics:
  - expansions of every W and Z boson propagator (at least 6 digits the same)
  - expansions of every master integral
  - large Higgs boson mass expansion of $\Delta r^{(2)}_{\mathrm{bos}}$:



- Comparisons with independent calculations:

$\rightarrow\ \Delta r^{(2)}_{\mathrm{bos}}$ with A. Onishchenko, O. Veretin

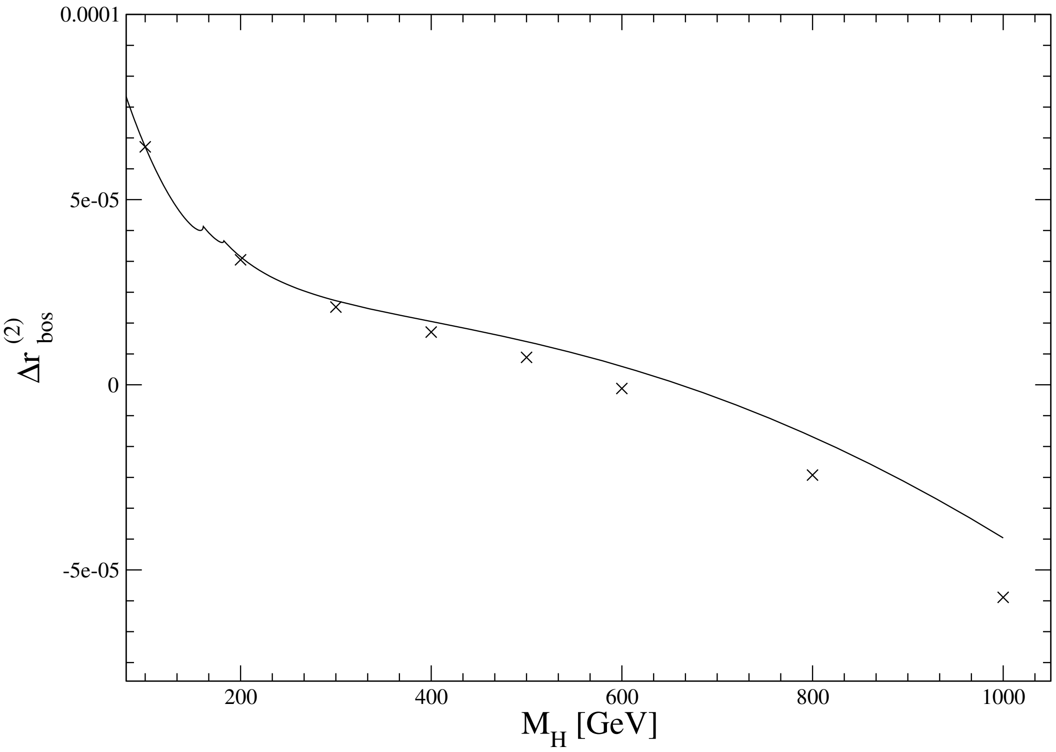


$\Rightarrow$ methods and software were independent, except matching procedure (therefore tested)

$\Rightarrow$ our result fully confirmed for small and large values of the Higgs boson mass

$\longrightarrow$ Higgs boson mass dependence of $\Delta r^{(2)}_{\rm bos}$ with A. Freitas, W. Hollik, W. Walter, G. Weiglein

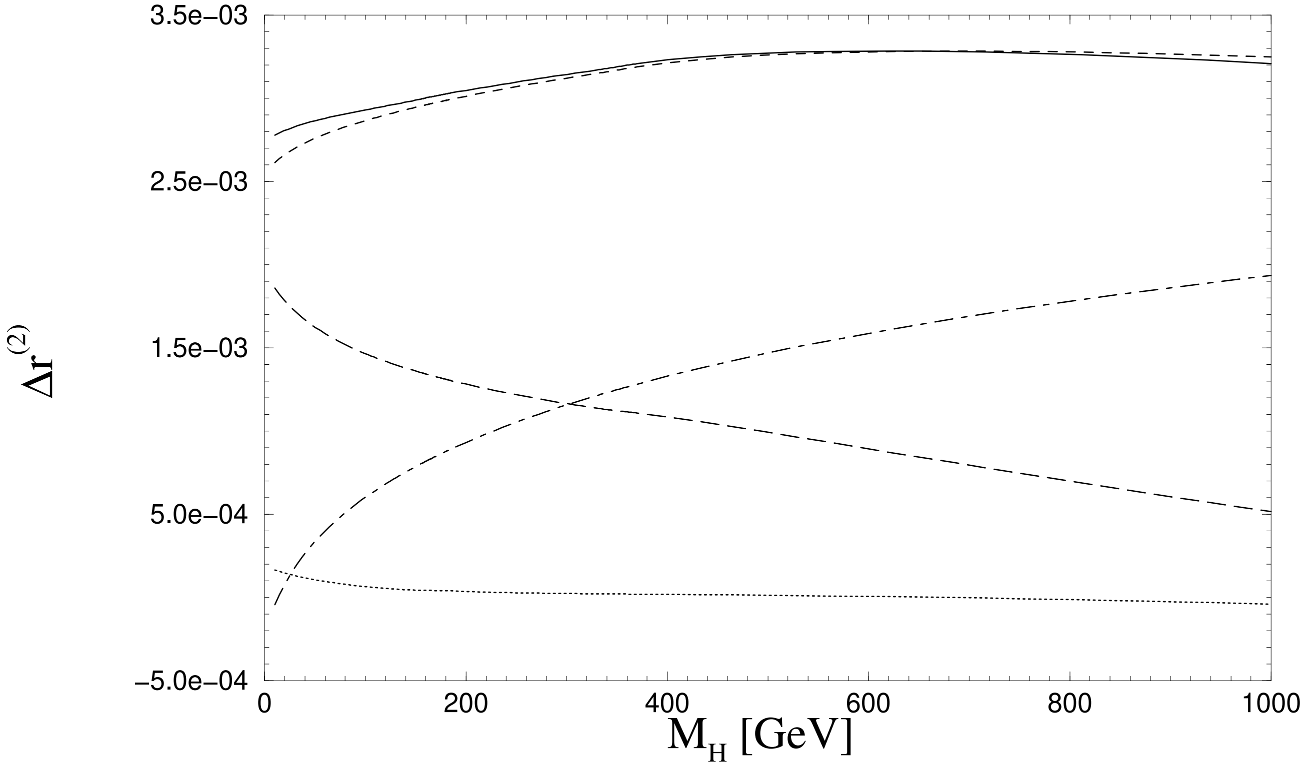
$$\Delta r^{(2)}_{\rm bos}(M_H) = \Delta r^{(2)}_{\rm bos,sub}(M_H, M_H^0) + \Delta r^{(2)}_{\rm bos}(M_H^0)$$



$\Rightarrow$ methods and software were different, except core of S2LSE

$\Rightarrow$ origin of disagreement identified, our result fully confirmed

- Full electroweak contributions to $\Delta r^{(2)}$:



- M_W prediction against the fitting formula of

  A. Freitas, W. Hollik, W. Walter, G. Weiglein

	$\Delta r^{(\alpha)} + \Delta r^{(\alpha^2)}_{\rm ferm}$	$+ \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)}$		$+\Delta r^{(2)}_{\rm bos}$	
M_H [GeV]	$M_W[FHWW]$ [GeV]	M_W [GeV]	ΔM_W [MeV]	M_W [GeV]	ΔM_W [MeV]
100	80.3771	80.3758	-1.3	80.3747	-2.4
200	80.3338	80.3326	-1.2	80.3321	-1.7
600	80.2521	80.2509	-1.2	80.2508	-1.3
1000	80.2135	80.2122	-1.3	80.2129	-0.6

- M_W prediction after inclusion of the result of

  M. Faisst, J.H. Kuhn, T. Seidensticker, O. Veretin

		$-c_w^2/s_w^2\Delta\rho_{\rm t}^{(\alpha^3)}$		$-c_w^2/s_w^2\Delta\rho_{\rm t}^{(\alpha^2\alpha_s)}$	
M_H [GeV]	M_W [GeV]	M_W [GeV]	ΔM_W [MeV]	M_W [GeV]	ΔM_W [MeV]
100	80.3747	80.375	0.3	80.3771	2.4
200	80.3321	80.3322	0.1	80.3358	3.7
600	80.2508	80.2506	-0.2	–	–
1000	80.2129	80.2142	1.3	–	–

$m_t = 174.3$ GeV,

$\Delta\alpha(M_Z) = 0.05923$,

$\alpha_s(M_Z) = 0.119$

Conclusions

- full two loop result for Δr available
  - $\Delta r^{(2)}_{\rm bos}$
    - matching procedure tested
    - confirmed by independent calculation
  - $\Delta r^{(2)}_{\rm ferm}$
    - agrees with the corrected result of

      A. Freitas, W. Hollik, W. Walter, G. Weiglein
- new approach to calculate Δr, based on effective theories. In principle, works to arbitrary order.
- new large computer system - can be used for other projects ($\sin^2\theta^{\rm lep\,(2)}_{\rm eff}$ under way)
- theoretical error on M_W reduced
- new fitting formula under way