Collider Phenomenology +

Model Building with NCQFT

- Background + Overview
- Testing NCQED at LC
  Problems with NC SM construction
- Summary / Conclusions

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PRD64, 075012 /2001
and
hep-ph/0112003

* See recent review by Douglas + Nekrasov hep-th/0106048

TGRizzo
12/7/01

Moller Scattering....

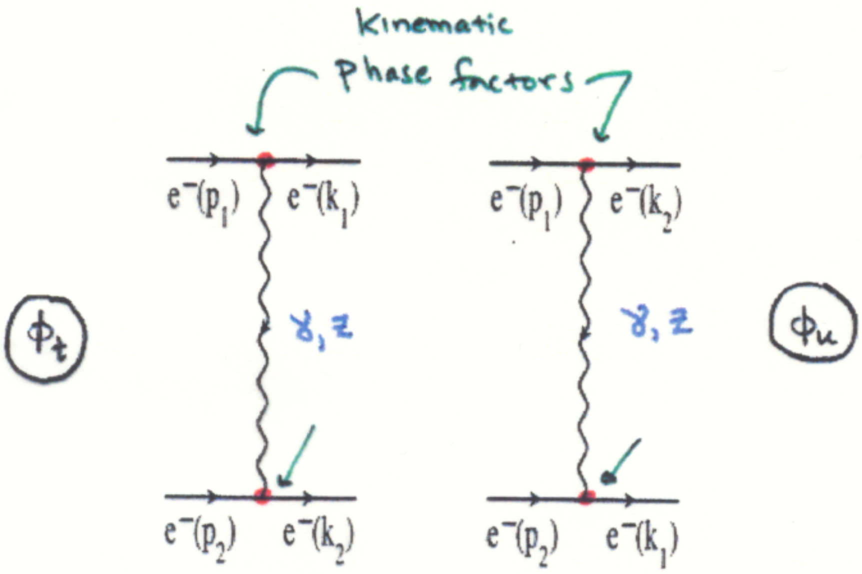


Figure 2: Feynman graphs contributing to Moller scattering with the exchanged particle corresponding to a photon and Z-boson.

We Assume the $Ze\bar{e}$ vertex is modified like the $\gamma e\bar{e}$ vertex *

Warning!! No complete NC-SM yet exists...

(more later)

* Qualitative results are independent of this assumption

Bhabha scattering

$e^+e^- \to e^+e^-$

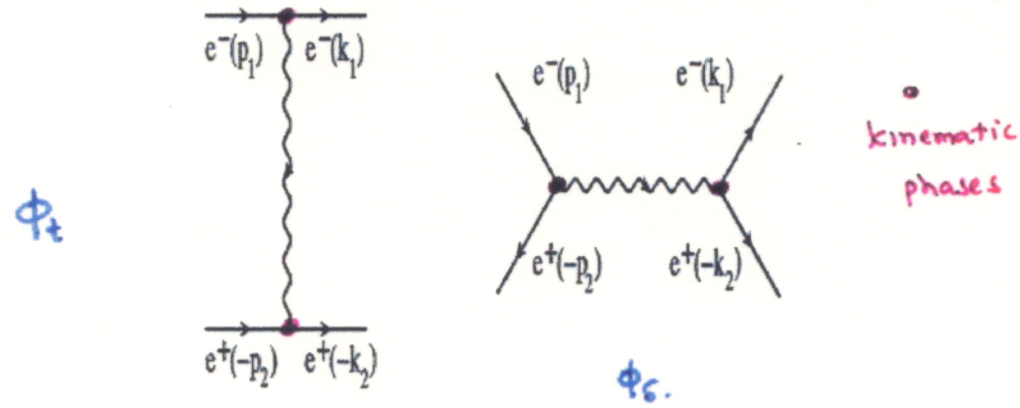
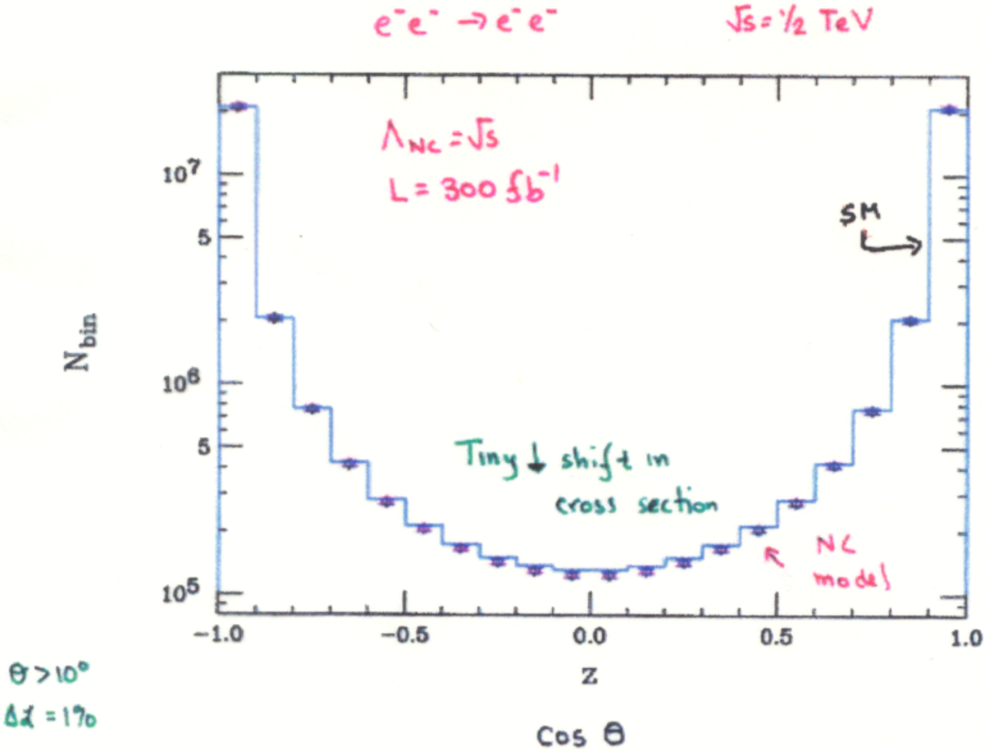
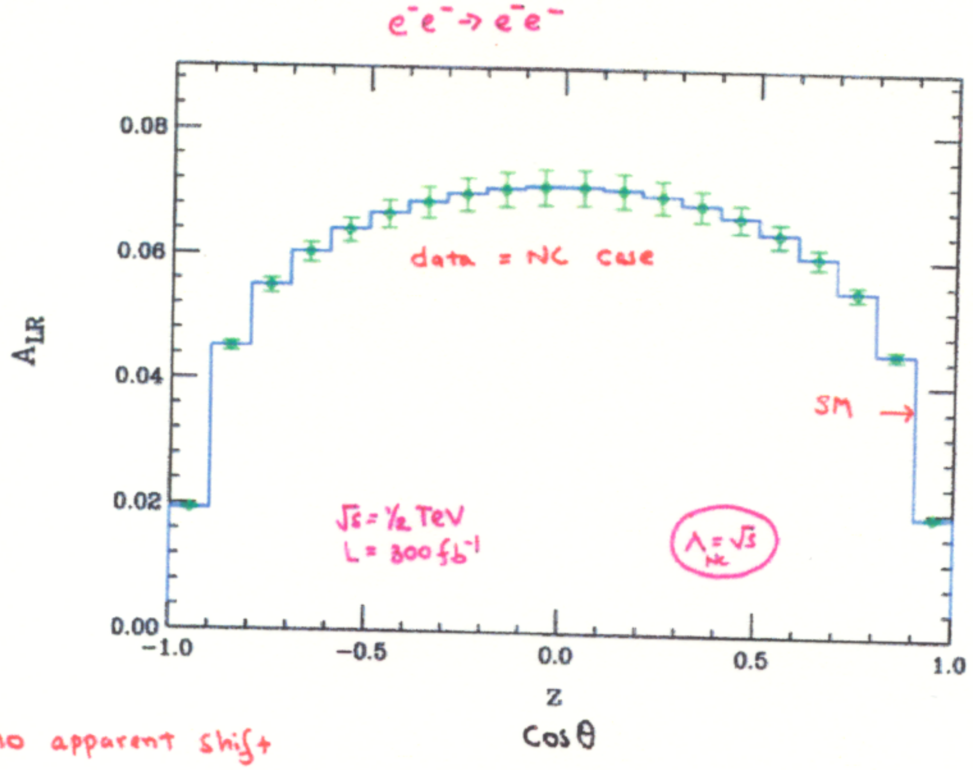


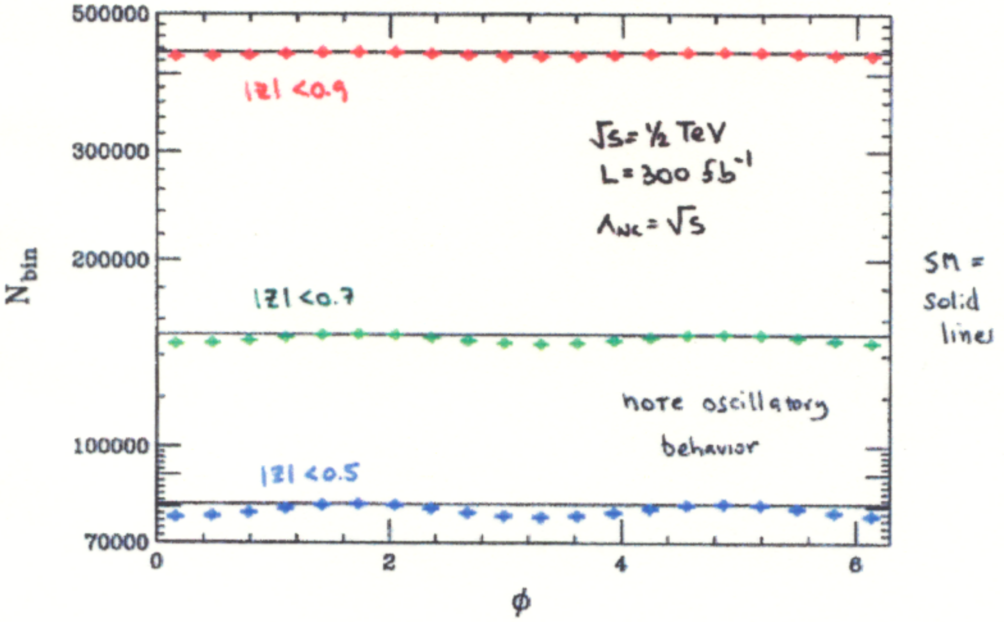
Figure 7: Feynman graphs contributing to Bhabha scattering with the exchanged particle corresponding to a photon and Z-boson.

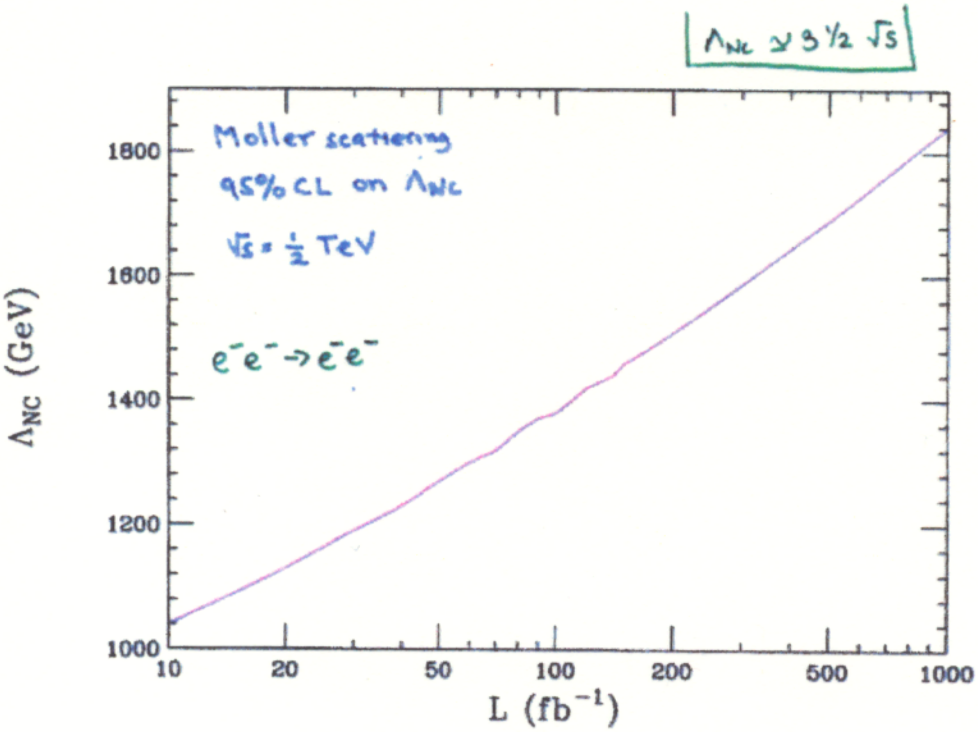

e⁻e⁻ → e⁻e⁻
√s = ½ TeV
Λ_NC = √s
L = 300 fb⁻¹
SM
N_bin
Tiny ↓ shift in cross section
NC model
z
θ > 10°
Δℒ = 1%
Cos θ



no apparent shift

$e^-e^- \to e^-e^-$ ϕ dependence





$\Lambda_{NC} \simeq 3\frac{1}{2}\sqrt{s}$
Moller scattering
95% CL on Λ_{NC}
$\sqrt{s} = \frac{1}{2}$ TeV
$e^-e^- \to e^-e^-$
1800
1600
1400
1200
1000
Λ_{NC} (GeV)
10
20
50
100
200
500
1000
L (fb^{-1})

Summary of 95 % CL Search Reach

Process	Structure Probed	Bound on Λ_{NC}
$e^+e^- \to \gamma\gamma$	Space-Time	740 – 840 GeV
Moller Scattering	Space-Space	1700 GeV
Bhabha Scattering	Space-Time	1050 GeV
$\gamma\gamma \to \gamma\gamma$	Space-Time	700 – 800 GeV
	Space-Space	500 Gev

$\sqrt{s} = 500$ GeV, $\mathcal{L} = 500\ fb^{-1}$

$\gamma\gamma \to e^+e^-$	Space-Time	450-540 GeV
$\gamma e \to \gamma e$	Space-Time Space-Space	925 - 1300 GeV

Godfrey + Doncheski

Moller scattering:

$$\frac{d\sigma}{dz\,d\phi} = \frac{\alpha^2}{4s}\left[(e_{ij}+f_{ij})(P_{ij}^{uu}+P_{ij}^{tt}+2P_{ij}^{ut}\cos\Delta_{Moller}) + (e_{ij}-f_{ij})(\frac{t^2}{s^2}P_{ij}^{uu}+\frac{u^2}{s^2}P_{ij}^{tt})\right]$$

Coupling constants

$$P_{ij}^{qr} = s^2\frac{(q-m_i^2)(r-m_j^2)+\Gamma_i\Gamma_j m_i m_j}{[(q-m_i^2)^2+(\Gamma_i m_i)^2][(r-m_j^2)^2+(\Gamma_j m_j)^2]},$$

Prop. factors

$$\Rightarrow \Delta_{Moller} = \phi_u - \phi_t = \frac{-\sqrt{ut}}{\Lambda_{NC}^2}[c_{12}c_\phi - c_{31}s_\phi],$$

Space - Space
NC only

Bhabha scattering

$$\frac{d\sigma}{dz\,d\phi} = \frac{\alpha^2}{2s}\left[(e_{ij}+f_{ij})(P_{ij}^{ss}+P_{ij}^{tt}+2P_{ij}^{st}\frac{t^2}{s^2}\cos\Delta_{Bhabha}) + (e_{ij}-f_{ij})(P_{ij}^{ss}\frac{t^2}{s^2}+P_{ij}^{tt})\right]$$

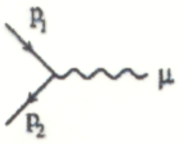
(I) (II)

$$\Delta_{Bhabha} = \phi_s - \phi_t = \frac{-1}{\Lambda_{NC}^2}[c_{01}t + \sqrt{ut}(c_{02}c_\phi + c_{03}s_\phi)].$$

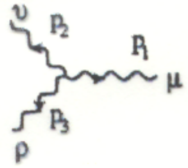
Space - Time
NC only

$\Delta_{Moller,\,Bhabha}$ are **not** Lorentz Invariants

Feynman rules for NCQED

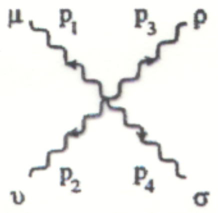


$$= ig\gamma^{\mu}\exp(ip_1\theta p_2/2)$$



$$= 2g\sin(p_1\theta p_2/2)\,[(p_1 - p_2)^{\rho}g^{\mu\upsilon} + (p_2 - p_3)^{\mu}g^{\upsilon\rho} + (p_3 - p_1)^{\upsilon}g^{\mu\rho}]$$

3 point!



$$= 4ig^2[(g^{\mu\sigma}g^{\upsilon\rho} - g^{\mu\rho}g^{\upsilon\sigma})\sin(p_1\theta p_2/2)\sin(p_3\theta p_4/2) + (g^{\mu\rho}g^{\upsilon\sigma} - g^{\mu\upsilon}g^{\rho\sigma})\sin(p_3\theta p_1/2)\sin(p_2\theta p_4/2) + (g^{\mu\upsilon}g^{\rho\sigma} - g^{\mu\sigma}g^{\upsilon\rho})\sin(p_1\theta p_4/2)\sin(p_2\theta p_3/2)]$$

4 point!

Figure 1: Feynman rules of NCQED.

$[t^A, t^B] = i f_{ABC}\, t^C$

$\{t^A, t^B\} = d_{ABC}\, t^C$

$A = 1, \dots, N^2$

$\left(t^0 = \frac{1}{\sqrt{2N}} \mathbb{1}_N\right)$

Armoni

5 Appendix A - Feynman rules for the non-commutative $U(N)$ Yang-Mills theory

The non-commutative Yang-Mills action including gauge fixing and ghosts takes the following form

$$S = \int d^4x \,\mathrm{tr}\, \left(-\frac{1}{2}F^{\mu\nu} \star F_{\mu\nu} + \xi(\partial^\mu A_\mu)^2 - \bar{c} \star \partial^\mu D_\mu c + \partial^\mu D_\mu c \star \bar{c}\right) \quad (36)$$

We use the Feynman-'t Hooft gauge $\xi = 1$.

$\tilde{p}q = \frac{1}{2} p^\mu \theta_{\mu\nu} q^\nu$

note there are non-zero for U(1)

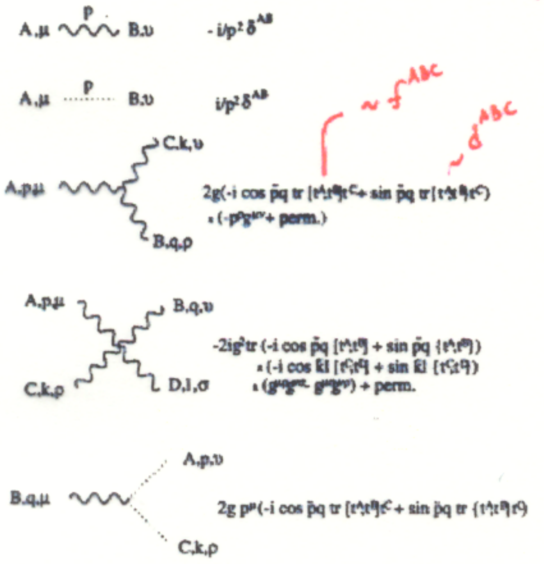


Figure 4: Feynman rules. Wavy lines and dotted lines denote gluons and ghosts, respectively. Capital letters and small letters denote $U(N)$ indices and momenta.

NC Theories arise from strings :

- Background n-form gauge fields 'polarize / magnetize' D-branes such that string end-points no longer commute

- We identify string end-points with SM fiel[ds]

<u>thus</u>

$$[\hat{x}_\mu, \hat{x}_\nu] = i\,\theta_{\mu\nu}$$

{ $\theta_{\mu\nu}$ is constant, real and anti-symmetric with dim = -2 }

$$\Rightarrow \begin{cases} \theta_{0i} \to \hat{c}_E/\Lambda_E^2 \neq 0 & \text{space-time NC} \\ \theta_{ij} \to \hat{c}_B/\Lambda_B^2 \neq 0 & \text{space-space NC} \end{cases}$$

$\hat{c}_{E,B}$ are unit vectors $\Lambda_{E,B}$ are NC scales $\gtrsim$ TeV (?)

- These are fixed "vectors" frame independent (preferred directions) $\Rightarrow$ Lorentz Inv. Violation

Lorentz Inv will be violated as we approach the $\Lambda_{E,B}$ scales !! $\left\{ \begin{array}{l} [p_\mu, p_\nu] = 0 \\ [x_i, p_j] = i\hbar\,\delta_{ij} \end{array} \right\}$

We CAN construct FT's on NC spaces

- They will conserve CPT and
- Be Unitary if $\theta_{\mu\nu}\theta^{\mu\nu} \geq 0$

$\Rightarrow$ $\boxed{\Lambda_B \leq \Lambda_E}$ {necessary but not sufficient!} {Seiberg et / Gomis et / Chaichian et.}

How?

- Seiberg-Witten Map

e.g.

$$\begin{cases} \hat{F}_{\mu\nu}(\hat{x}_\lambda) = F_{\mu\nu}(x_\lambda) + \mathcal{F}_{\mu\nu}(\theta, x_\lambda, A_\sigma, F_{\mu\nu}) \\ \hat{A}_\sigma(\hat{x}_\lambda) = A_\sigma(x_\lambda) + \mathcal{R}_\sigma(\theta, x_\lambda, A_\sigma, F_{\mu\nu}) \end{cases}$$

$\hat{F}, \hat{A}$: fields on the NC space

F, A: fields on ordinary R^4

$\mathcal{F}, \mathcal{R}$: Power series expansions in $\underline{\underline{\theta}}$

$\Rightarrow$ ∞ set of higher-dim operators - will work for any gauge group - difficult to employ

- Weyl-Moyal Correspondence

Introduce Fourier transform pair:

$$\hat{A}(\hat{x}) \sim \int d\alpha \;\; a(\alpha) \exp(i\alpha\hat{x})$$

$$a(\alpha) \sim \int d\dot{x} \;\; A(\dot{x}) \exp(-i\alpha x)$$

for any field A

Then

$$\hat{A}(\hat{x})\,\hat{B}(\hat{x}) \equiv A(x) * B(x)$$

$$= A(x)\, e^{\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu}\, B(x)$$

$$\approx A\cdot B + \frac{i}{2}\theta^{\mu\nu}\,\partial_\mu A\,\partial_\nu B + \ldots$$

Moyal or star product

and

$$[A, B]_{MB} = A * B - B * A$$

Moyal bracket

Replace products with $*$ products and commutators with MB's

→ equivalent NC FT on our (usual) R^4 spac[e]

⇒ Interesting Results

- New 'Uncertainty' relation $[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}$

  → $\Delta x_\mu \Delta x_\nu > \frac{1}{2}|\theta_{\mu\nu}|$ UV-IR Connectio[n]

- Only $U(n)$ groups closed under MB (Matsubara)

  → SM requires

  $U(3) \times U(2) \times U(1)$ [at least]

  ∴ at least 2 new gauge bosons !! (more later!)

- NC extension preserves renormalizability

  - U(N) is 1-loop renormalizable [Bonora + Salizzoni]
  - SSB U(1) is too [Petriello]
    {U(2) [Liao]}
  - ϕ^4 is at least 2-loop renorm.

- Gauge inv. requires only 1, F, $\bar{F}$ or Adj reps for U(N) and **$Q = 0, \pm 1$** for U(1)

  ∴ NCQED has <u>no</u> room for <u>quarks</u>

- Gauge inv. requires 3-point + 4-point coupling *even* for the GB's of 'Abelian' U(1) theories

- Vertices pick up momentum-dependent phase

  e.g.,

$$\int d^4\hat{x}\ \bar{\Psi}(\hat{x})\gamma_\mu\Psi(\hat{x})\ldots \quad \rightarrow \quad e^{-ip_f\hat{x}}\, e^{ip_i\hat{x}} = e^{i(p_i-p_f)\hat{x}}\, e^{-\frac{1}{2}p_f^\mu p_i^\nu[\hat{x}_\mu,\hat{x}_\nu]}$$

$$\bar{\Psi}(\hat{x}) \leftarrow \bar{\chi}(p_f)\, e^{-ip_f\hat{x}} \qquad \Psi(\hat{x}) \leftarrow e^{ip_i\hat{x}}\,\chi(p_i)$$

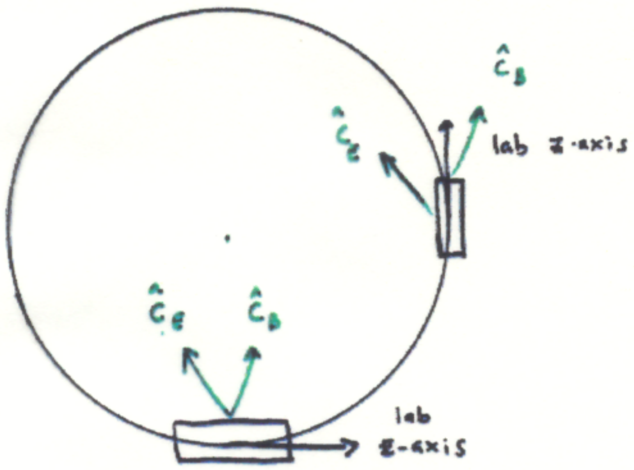
$$= e^{i(p_i-p_f)x}\, e^{i/2\,\theta_{\mu\nu}p_f^\mu p_i^\nu}$$

so  $ie\gamma_\mu \rightarrow ie\gamma_\mu \exp\left[\frac{i}{2}\theta^{\mu\nu}p_\mu^i p_\nu^f\right]$

… A kinematic phase growing with energy

Experiment vs Experiment : e.g. LEP

- We define the lab z-axis as the direction e^- goes in



- Two Experiments will see $\hat{c}_{E,B}$ point in different directions ... unless they convert to a common frame...

Even more:

- Even for a given experiment, $\hat{c}_i$ will vary with sidereal time due to Earth's rotation + motion around the sun.

→ Convert to astronomical co-ord system { "time stamp" data

$2 \to 2$ kinematics

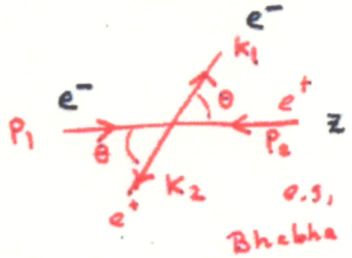
cm = lab frame

$$p_1^\mu = \frac{\sqrt{s}}{2}(1,-1,0,0) \qquad p_2^\mu = \frac{\sqrt{s}}{2}(1,1,0,0)$$

$$k_1^\mu = \frac{\sqrt{s}}{2}(1,-c_\theta,-s_\theta c_\phi,-s_\theta s_\phi) \qquad k_2^\mu = \frac{\sqrt{s}}{2}(1,c_\theta,s_\theta c_\phi,s_\theta s_\phi)\,.$$

(t, z, x, y)

$$\begin{cases} \hat{c}_{0i} = (\hat{c}_E/\Lambda_E^2)_i \\ \hat{c}_{ij} = (\hat{c}_B/\Lambda_B^2)_k \end{cases}$$



e.g., Bhabha Scattering

$$p_1 \cdot \Theta \cdot p_2 = \frac{s}{2}\hat{c}_{01}$$

$$k_1 \cdot \Theta \cdot k_2 = \frac{s}{2}[\hat{c}_{01}c_\theta + \hat{c}_{02}s_\theta \underline{c_\phi} + \hat{c}_{03}s_\theta \underline{s_\phi}]$$

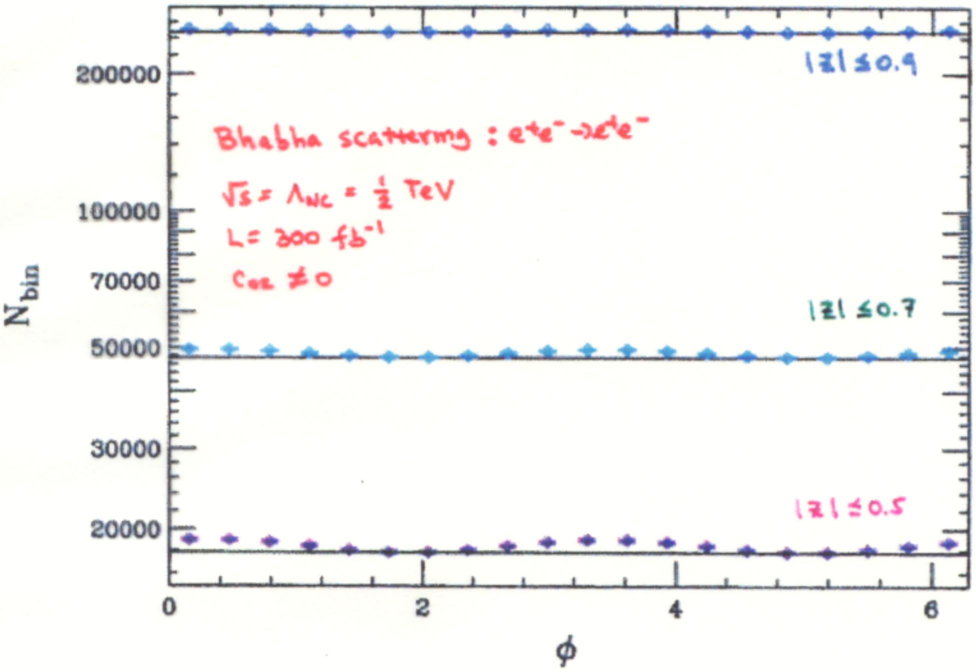
$$p_1 \cdot \Theta \cdot k_1 = \frac{s}{4}[\hat{c}_{01}(1-c_\theta) + (\hat{c}_{12} - \hat{c}_{02})s_\theta \underline{c_\phi} - (\hat{c}_{03} + \hat{c}_{31})s_\theta \underline{s_\phi}]$$

$$p_1 \cdot \Theta \cdot k_2 = \frac{s}{4}[\hat{c}_{01}(1+c_\theta) - (\hat{c}_{12} - c_{02})s_\theta \underline{c_\phi} + (\hat{c}_{03} + \hat{c}_{31})s_\theta \underline{s_\phi}]$$

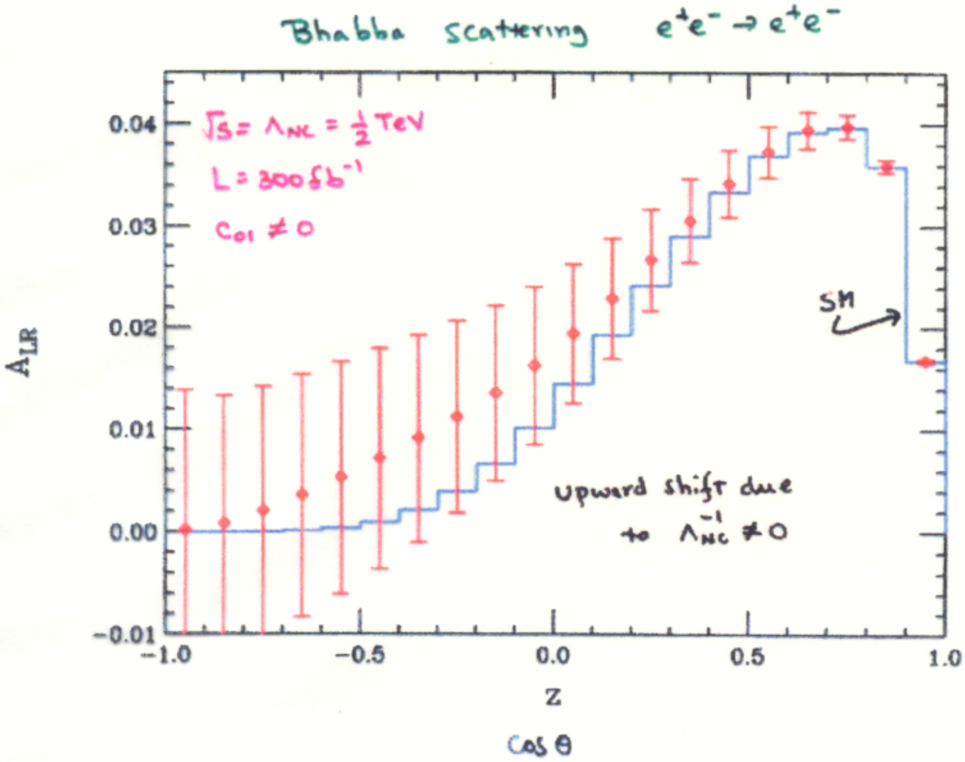
$$p_2 \cdot \Theta \cdot k_1 = \frac{s}{4}[-\hat{c}_{01}(1+c_\theta) - (\hat{c}_{12} + \hat{c}_{02})s_\theta \underline{c_\phi} - (\hat{c}_{03} - \hat{c}_{31})s_\theta \underline{s_\phi}]$$

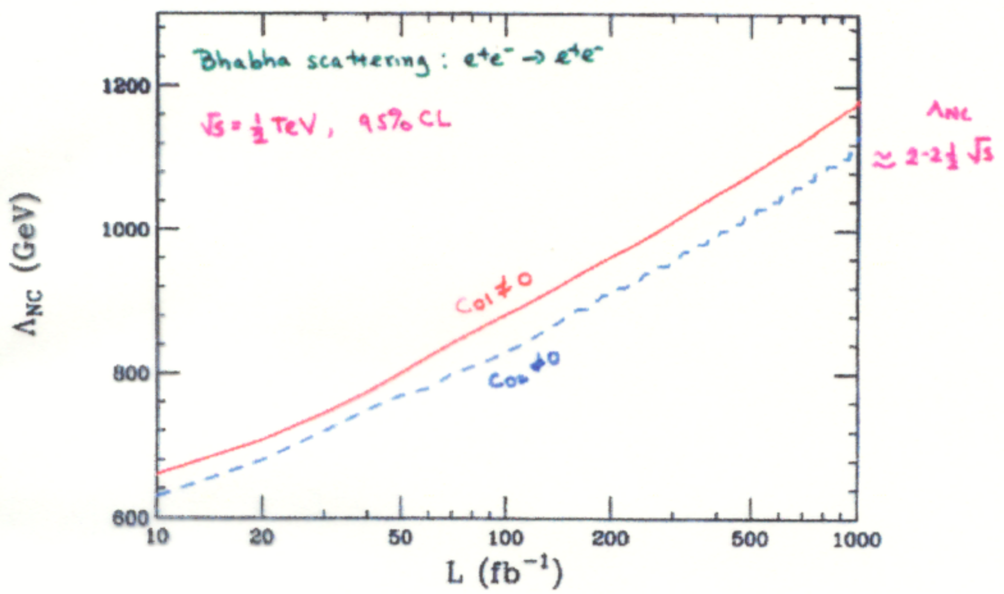
$$p_2 \cdot \Theta \cdot k_2 = \frac{s}{4}[-\hat{c}_{01}(1-c_\theta) + (\hat{c}_{12} + \hat{c}_{02})s_\theta \underline{c_\phi} + (\hat{c}_{03} - \hat{c}_{31})s_\theta \underline{s_\phi}]\,.$$

** Note: $\boxed{C_{23}}$ does not appear as initial beams along axis 1 (the z-axis)

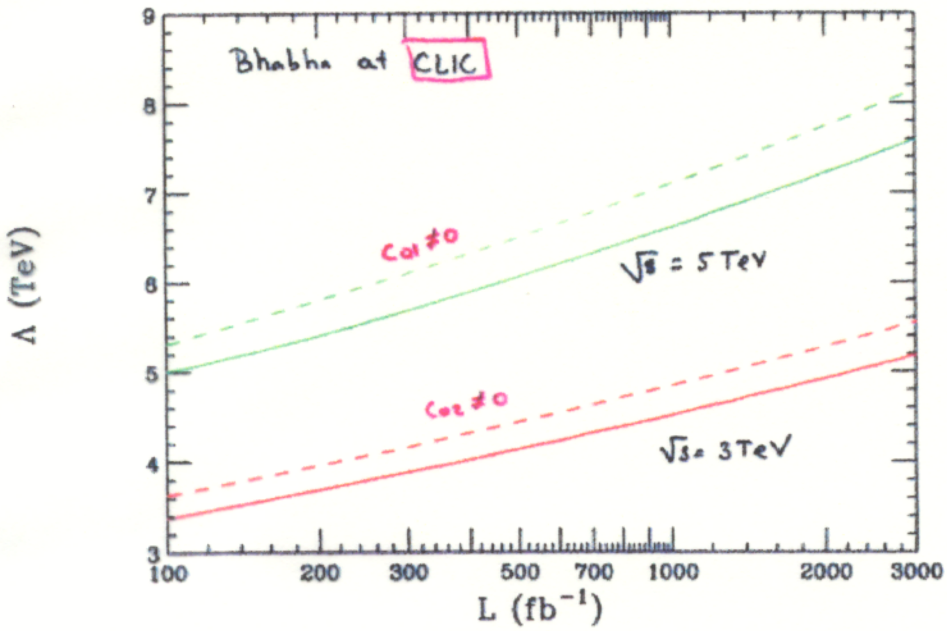


Bhabha scattering : e+e- → e+e-
√s = Λ_NC = 1/2 TeV
L = 300 fb⁻¹
|z| ≤ 0.9
|z| ≤ 0.7
|z| ≤ 0.5
N_bin
φ
200000
100000
70000
50000
30000
20000
0
2
4
6


Bhabba scattering $e^+e^- \to e^+e^-$
$\sqrt{s} = \Lambda_{NC} = \frac{1}{2}$ TeV
$L = 300\, fb^{-1}$
$c_{01} \neq 0$
SM
upward shift due to $\Lambda_{NC}^{-1} \neq 0$
A_{LR}
0.04
0.03
0.02
0.01
0.00
-0.01
-1.0
-0.5
0.0
0.5
1.0
z
cos θ

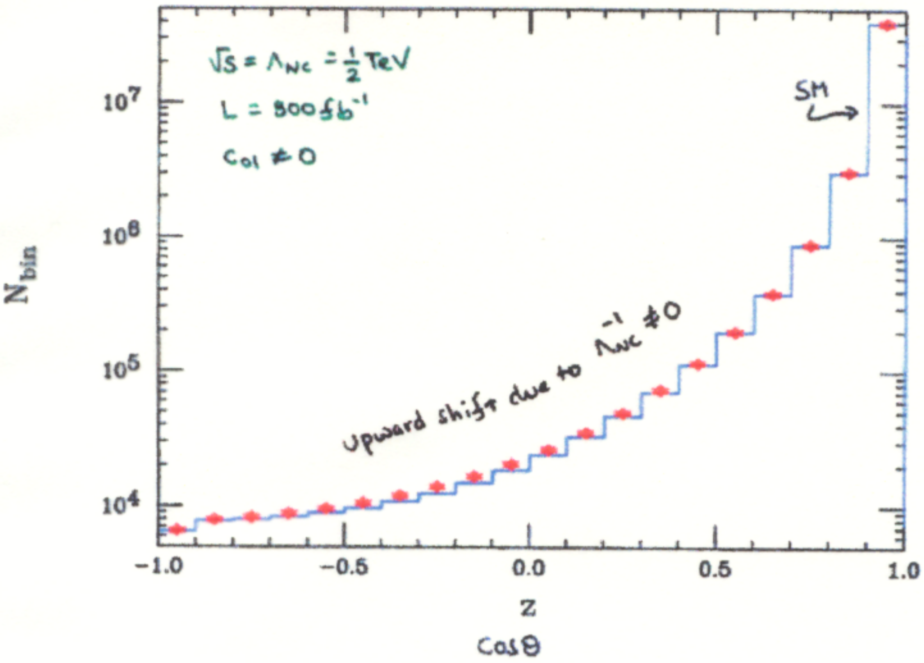


Bhabha scattering : $e^+e^- \to e^+e^-$
$\sqrt{s} = \frac{1}{2}$ TeV , 95% CL
Λ_{NC} (GeV)
L (fb^{-1})
1200
1000
800
600
10
20
50
100
200
500
1000
$c_{01} \neq 0$
$c_{0i} \neq 0$
Λ_{NC}
$\gtrsim 2\text{-}2\frac{1}{2}\sqrt{s}$



Bhabha at CLIC
$C_{01}\neq 0$
$\sqrt{s}$ = 5 TeV
$C_{02}\neq 0$
$\sqrt{s}$ = 3 TeV
Λ (TeV)
L (fb^{-1})

Bhabha scattering $e^+e^- \to e^+e^-$



Pair Annihilation

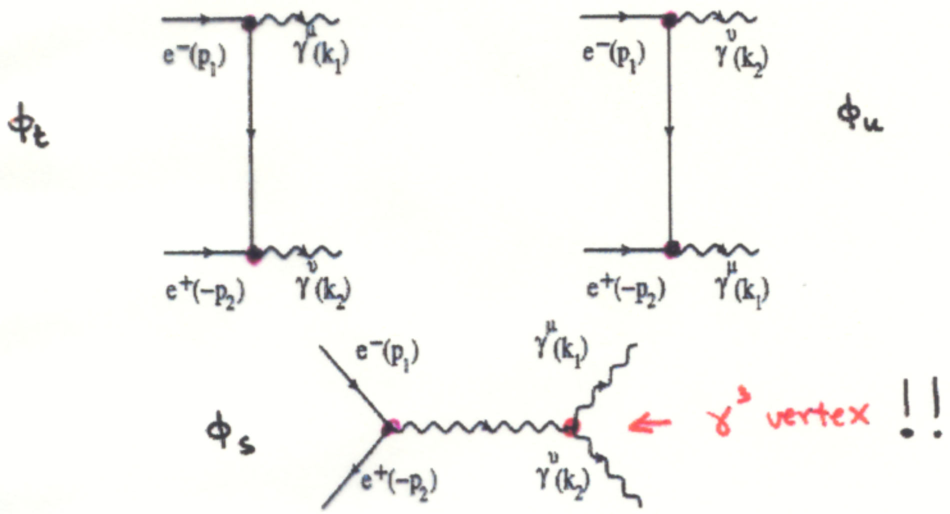


Figure 14: The three tree level contributions to $e^+e^- \to \gamma\gamma$ in NCQED.

$$\frac{d\sigma}{dz\,d\phi} = \frac{\alpha^2}{4s}\left[\frac{u}{t} + \frac{t}{u} - 4\,\frac{t^2+u^2}{s^2}\sin^2(\underbrace{\frac{1}{2}k_1 \wedge k_2}_{\Delta_{PA}})\right],$$

time-space NC

$$\Delta_{PA} \equiv \frac{1}{2}k_1 \wedge k_2 = \frac{-s}{2\Lambda_{NC_E}^2}\left[c_{01}c_\theta + c_{02}s_\theta c_\phi + c_{03}s_\theta s_\phi\right].$$

not Lorentz invariant

$\sum_i |c_{0i}|^2 \equiv 1$

$c_{01} = \cos\alpha$

$c_{02} = \sin\alpha\cos\beta$

$c_{03} = \sin\alpha\sin\beta$

Pair Annihilation

Bin integrated distributions : $e^+e^- \to \gamma\gamma$

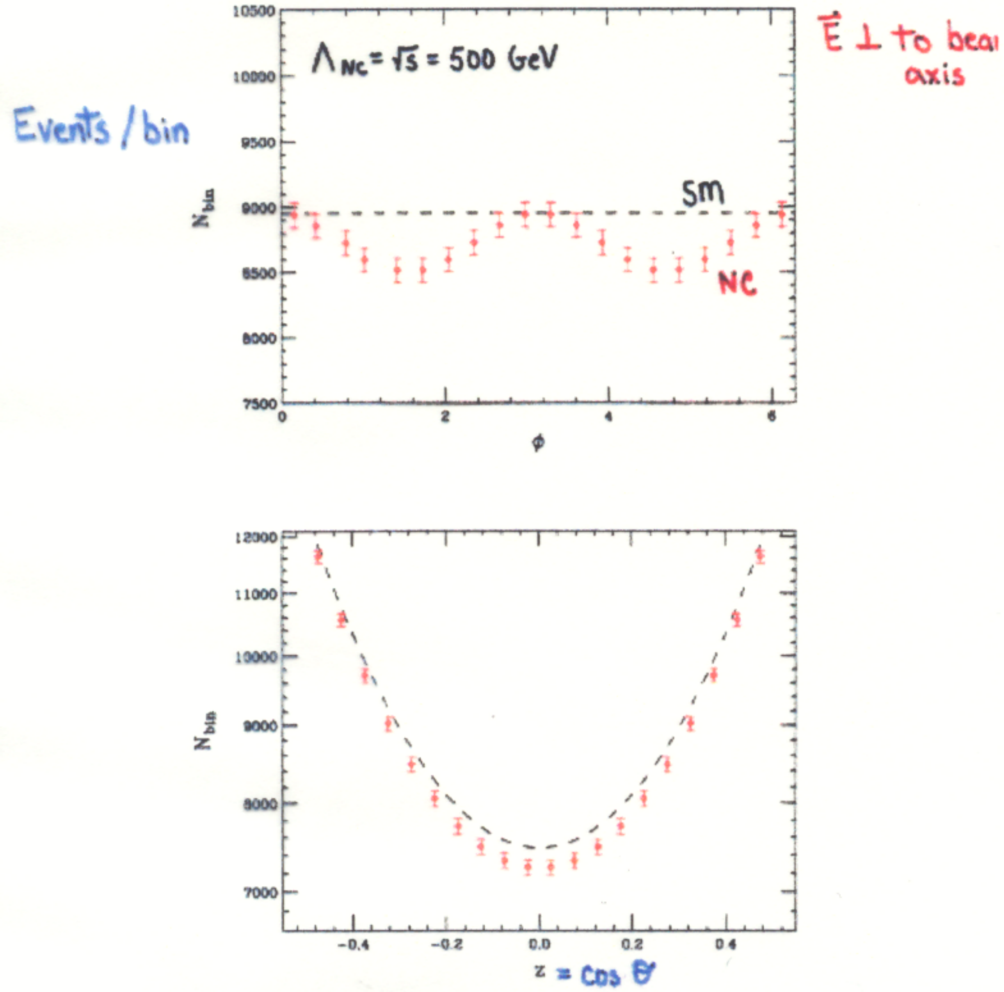


Figure 15: ϕ dependence (top) and θ dependence (bottom) of the $e^+e^- \to \gamma\gamma$ cross section for the case $\alpha = \pi/2$. We take $\Lambda_{NC} = \sqrt{s} = 500$ GeV, and assume a luminosity of 500 fb^{-1}. In the top panel a cut of $|z| < 0.5$ has been employed. The dashed line corresponds to the SM expectations and the 'data' points represent the NCQED results.

Bin integrated distributions: $e^+e^- \to \gamma\gamma$

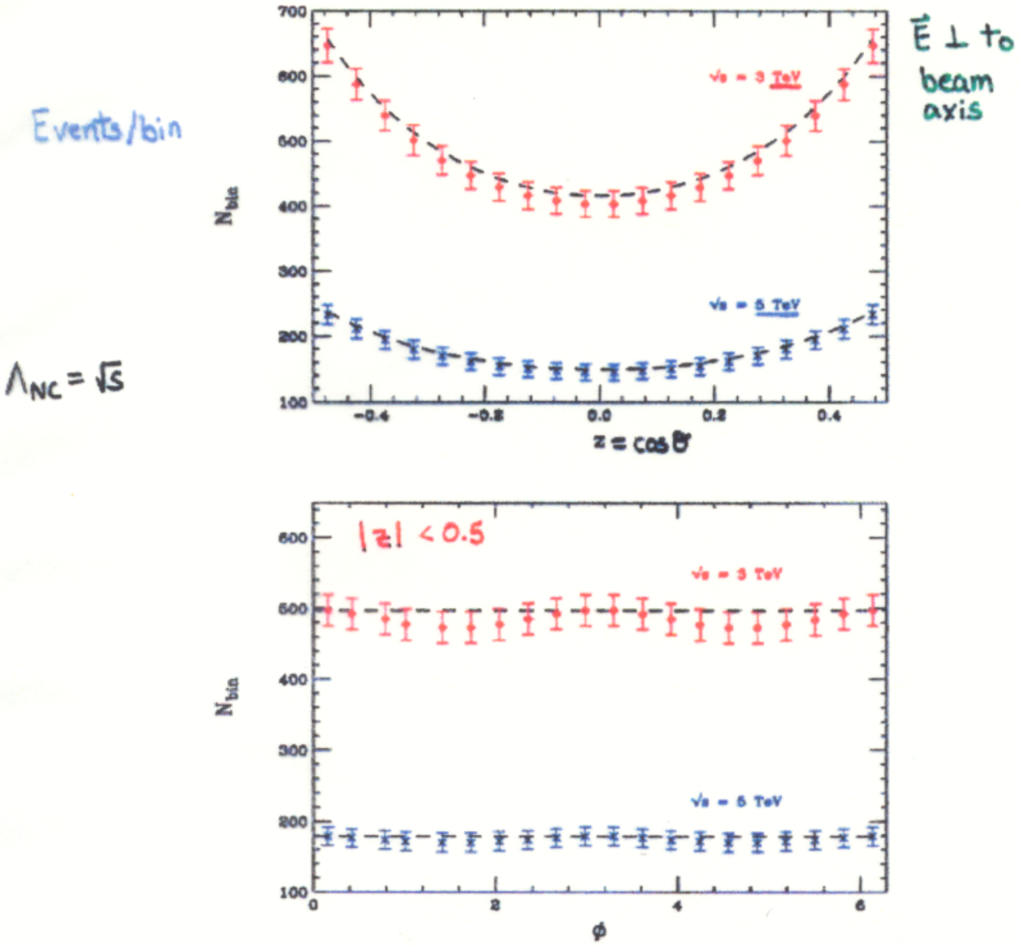
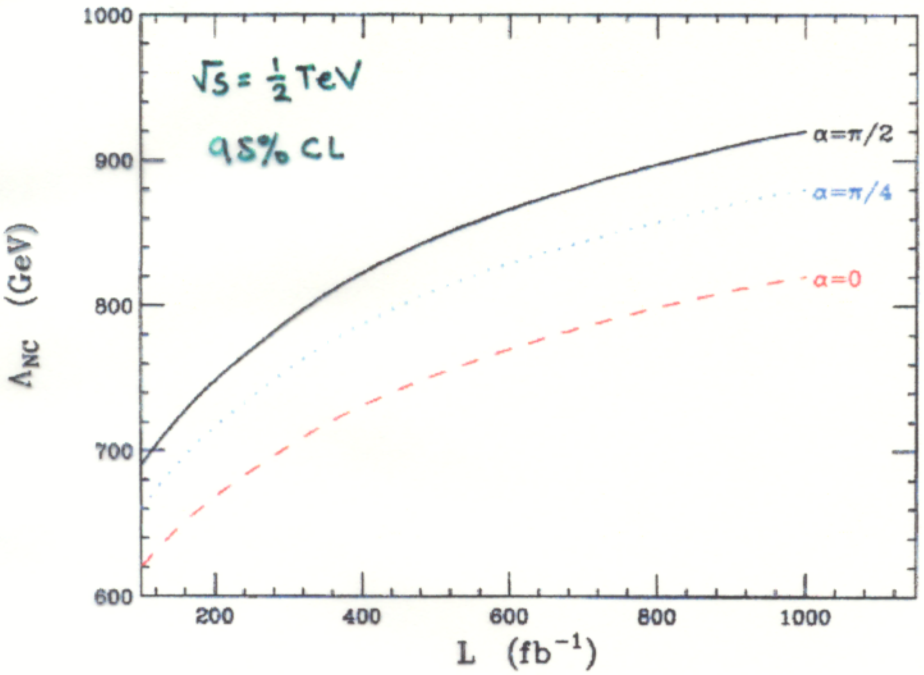


Figure 1: θ dependence (top) and ϕ dependence (bottom) of the $e^+e^- \to \gamma\gamma$ cross section for the case $\alpha = \pi/2$. We take $\Lambda_{NC} = \sqrt{s}$, and assume a luminosity of 1000 fb^{-1}. In the bottom panel a cut of $|z| < 0.5$ has been employed. The dashed lines correspond to the SM expectations, the data points represent the NCQED results, and the bars correspond to the associated statistical errors.

$e^+e^- \to 2\gamma$



reach is relatively poor ...

$\Lambda_{NC} \sim 2\sqrt{s}$

95% CL Search Reach for Λ_{NC}

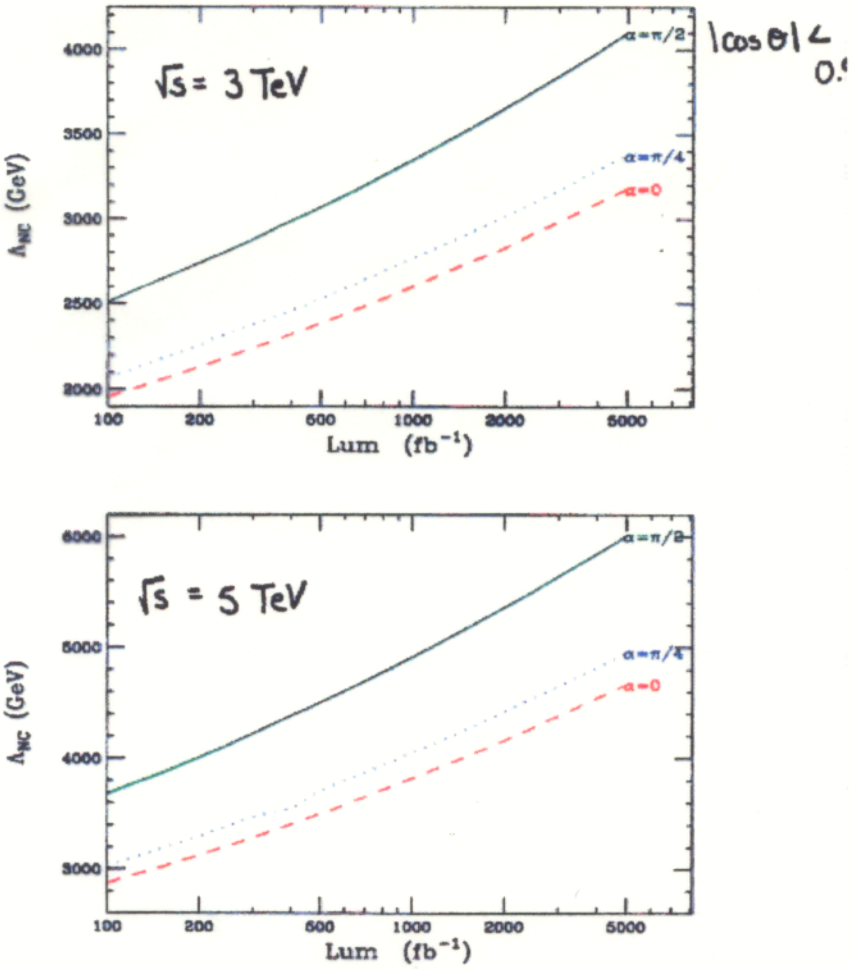


Figure 2: 95% CL search reaches for Λ_{NC} from $e^+e^- \to \gamma\gamma$ as a function of luminosity for $\sqrt{s} = 3$ TeV (top) and $\sqrt{s} = 5$ TeV (bottom).

$\gamma\gamma \to \gamma\gamma$ - 4 NC Contributions

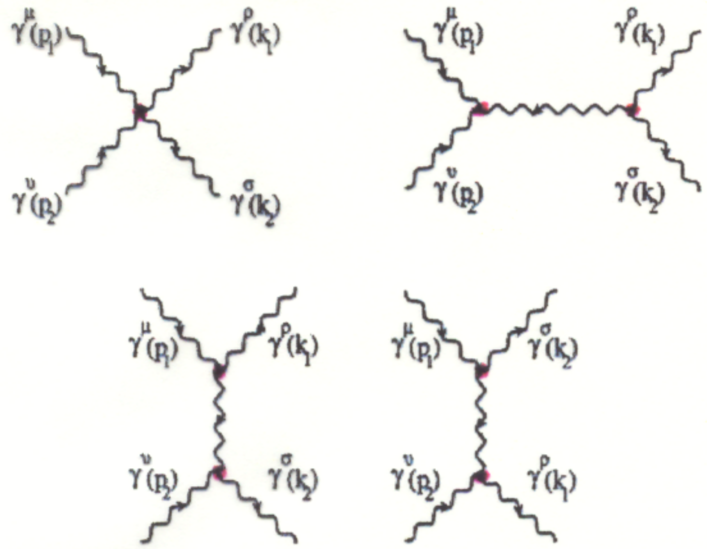
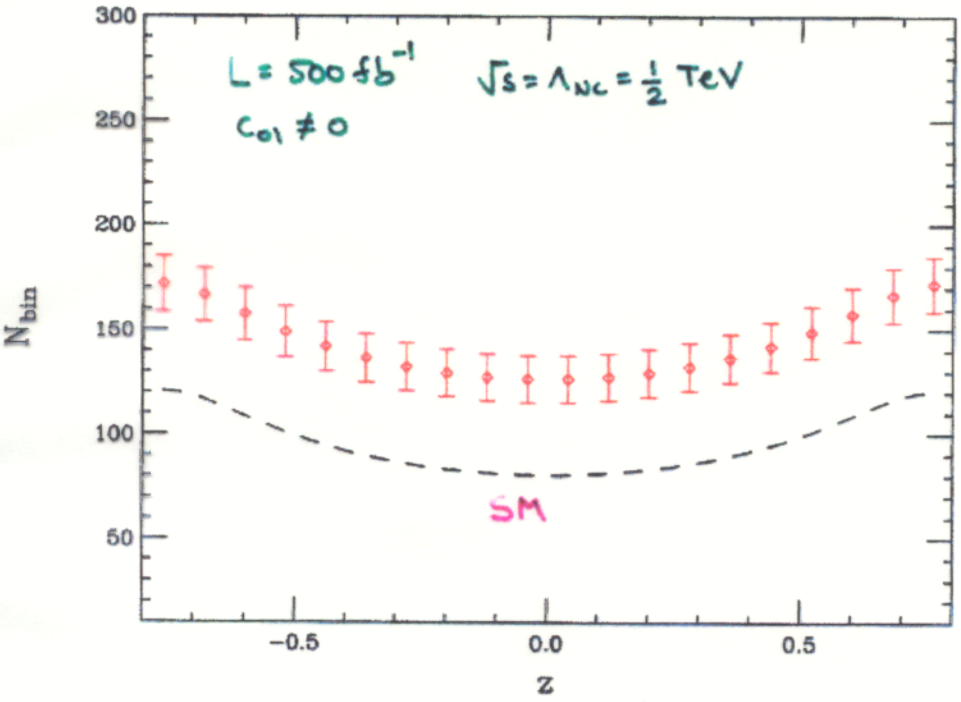


Figure 6: The tree level contributions to $\gamma\gamma \to \gamma\gamma$ in NCQED.

Vs the SM where this arises due to loops

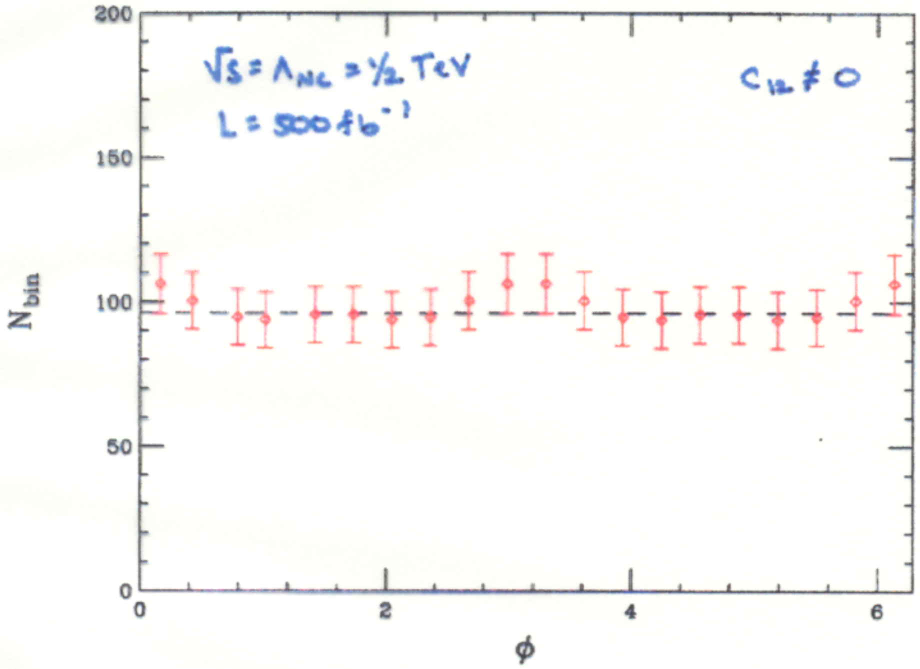
$\gamma\gamma \to \gamma\gamma$



(+ − + −)

polarization

$\gamma\gamma \to \gamma\gamma$



(+ − + −)
polarization

95 % CL Search Reach for Λ_{NC}

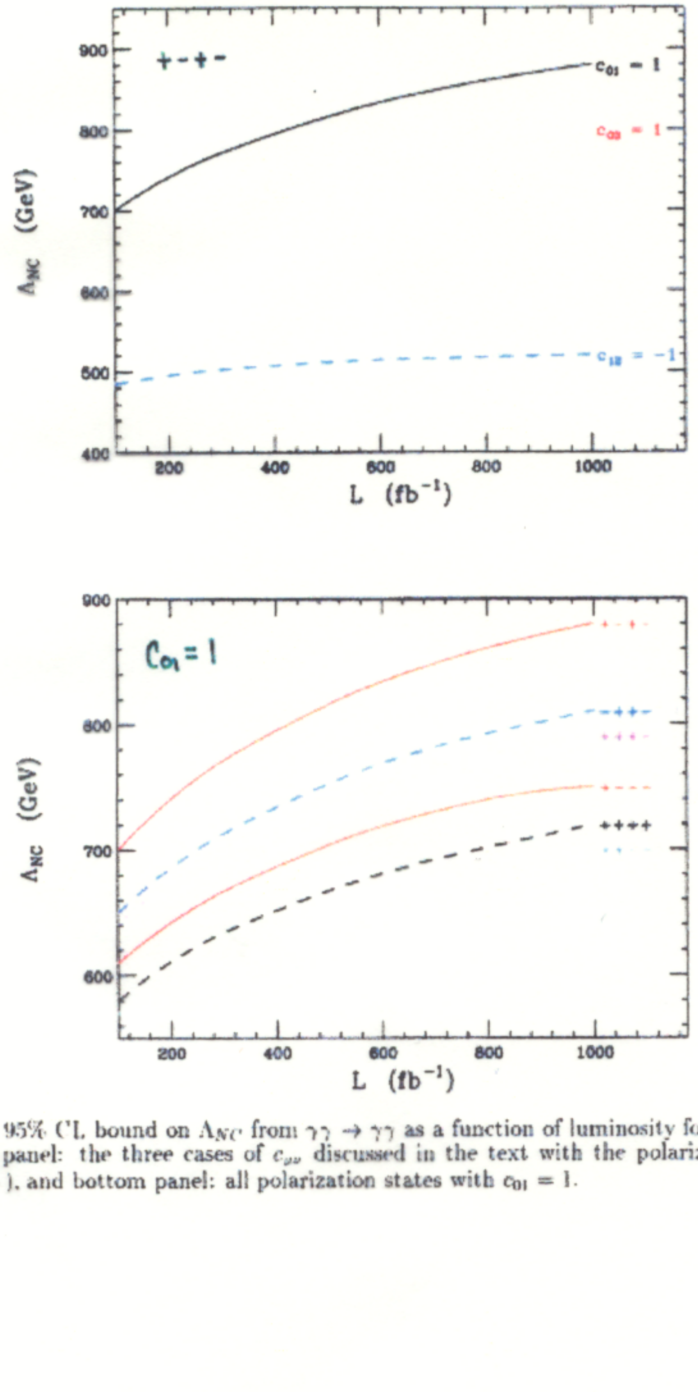


Figure 23: 95% CL bound on Λ_{NC} from $\gamma\gamma \to \gamma\gamma$ as a function of luminosity for $\sqrt{s} = 500$ GeV. Top panel: the three cases of $c_{\mu\nu}$ discussed in the text with the polarization state $(+, -, +, -)$, and bottom panel: all polarization states with $c_{01} = 1$.

Problems Building the NCSM

(i) we need (at least) $U(3) \times U(2) \times U(1)$ which must be broken at some $\gtrsim$ TeV scale to SM

(ii) $U(1)$ charges must be $0, \pm 1$ $\therefore$ $U(1) \neq U(1)_Y$

Furthermore : No-Go Thm {Chaichian et al}

if $G = \prod_i G_i$, matter fields can only transform non-trivially under 2 groups (at most)

{gauge inv.}

SM: Q_L transforms under $SU(3)$, $SU(2)$ + $U(1)$!!

* Can we embed the SM fields [+ extra?] into G in an anomaly free way satisfying all these constraints and recover the SM couplings in the $\Lambda_{E,B} \to \infty$ limit ??

Chaichian et al hep-th/0107055 say yes

- How does $U(3) \times U(2) \times U(1) \to$ SM ?

* Higgs in (F, Adj) breaks $U(N) \to U(N-1)$

<u>not</u> $U(N) \to SU(N)$

Chaichion et al employ "Higgsac" field coupling to $\sim$ Tr Adj [U(N) Adjoint] "$U(N) \to SU(N) \times U(1)$"

$\therefore$ $U(3) \times U(2) \times U(1)$

$\hookrightarrow$ "$SU(3) \times SU(2) \times U(1)_3 \times U(1)_2 \times U(1)$"

Higgsac 1

Higgsac 2

$U(1)_Y$

2 heavy Z′-like gauges

w/ the SM breaking in the usual way…

<u>Unfortunately</u> {Hewett, Petriello + TGR} showed that $W_L^+ W_L^- \to W_L^+ W_L^-$ is <u>not</u> unitary in this mod due to <u>Higgsac breaking</u> : only F + Adj's lead to unitary σ BUT <u>they can't break the symmetry</u>! <u>Problem</u>

The two requirements pull us in [opposite] directions… not solvable within this context.

Conclusions

- NCQFT are complex but phenomenologically interesting w/ distinctive signatures [i.e., ϕ dependence]
- NC sm not yet constructed in Moyal Approach [possible?]
- Use of common astronometric co-ordinate system is necessary when data taking over extended periods + to compare various experiments
- perhaps we gain some incite into strings ?
- Lots to do