# Gauge boson self couplings at the photon collider

O. Nachtmann, F. Nagel and M. Pospischil

**Institut für Theoretische Physik**

**Universität Heidelberg**

# Outline

- Effective Lagrangian
- Diagonalisation and parameters
- Limits from electroweak precision data
- Gauge boson couplings in $\gamma\gamma \to WW$
- Summary

# Effective Lagrangian - 1/3

New physics effects at scale $\Lambda$
parameterised by effective Lagrangian $\mathscr{L}_{\text{eff}}$:

- built from Standard Model (SM) fields
- $SU(3) \times SU(2) \times U(1)$ gauge invariant
- lepton and baryon number conserving
- expanded in powers of $(1/\Lambda)$ where $\Lambda \gg G_{\text{F}}^{-1/2}$

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_0 + \frac{1}{\Lambda}\mathscr{L}_1 + \frac{1}{\Lambda^2}\mathscr{L}_2 + \ldots$$

W. Buchmüller and D. Wyler, Nucl. Phys. B **268**, 621 (1986).

# Effective Lagrangian - 2/3

focus on SM Lagr. $\mathscr{L}_0$ and ew. gauge boson sector of $\mathscr{L}_2$:

$$\begin{aligned}
\mathscr{L}'_{\text{eff}} &= \mathscr{L}_0 + \frac{1}{\Lambda^2}\mathscr{L}'_2 \\
\frac{1}{\Lambda^2}\mathscr{L}'_2 &= h_W \mathcal{O}_W + h_{\tilde{W}} \mathcal{O}_{\tilde{W}} + h_{WB} \mathcal{O}_{WB} + h_{\tilde{W}B} \mathcal{O}_{\tilde{W}B} \\
&\quad + h_{\varphi W} \mathcal{O}_{\varphi W} + h_{\varphi \tilde{W}} \mathcal{O}_{\varphi \tilde{W}}
\end{aligned}$$

contains all dimension six operators
with gauge boson self couplings:

$$\mathcal{O}_W = \epsilon^{ijk}\, W_\mu^{i\,\nu} W_\nu^{j\,\lambda} W_\lambda^{k\,\mu} \qquad \mathcal{O}_{\tilde{W}} = \epsilon^{ijk}\, \tilde{W}_\mu^{i\,\nu} W_\nu^{j\,\lambda} W_\lambda^{k\,\mu}$$

$$\mathcal{O}_{WB} = (\phi^\dagger \tau^i \phi)\, W_{\mu\nu}^i B^{\mu\nu} \qquad \mathcal{O}_{\tilde{W}B} = (\phi^\dagger \tau^i \phi)\, \tilde{W}_{\mu\nu}^i B^{\mu\nu}$$

$$\mathcal{O}_{\varphi W} = \frac{1}{2}(\phi^\dagger \phi)\, W_{\mu\nu}^i W^{i\,\mu\nu} \qquad \mathcal{O}_{\varphi \tilde{W}} = (\phi^\dagger \phi)\, \tilde{W}_{\mu\nu}^i W^{i\,\mu\nu}$$

# Effective Lagrangian - 3/3

$$\mathcal{O}_W = \epsilon^{ijk}\, W_\mu^{i\,\nu} W_\nu^{j\,\lambda} W_\lambda^{k\,\mu} \qquad 3,\ 4,\ \ldots$$

$$\mathcal{O}_{\tilde{W}} = \epsilon^{ijk}\, \tilde{W}_\mu^{i\,\nu} W_\nu^{j\,\lambda} W_\lambda^{k\,\mu} \qquad 3,\ 4,\ \ldots$$

$$\mathcal{O}_{WB} = (\phi^\dagger \tau^i \phi)\, W^i_{\mu\nu} B^{\mu\nu} \qquad 3,\ \text{kin.},\ \gamma\gamma h,\ \ldots$$

$$\mathcal{O}_{\tilde{W}B} = (\phi^\dagger \tau^i \phi)\, \tilde{W}^i_{\mu\nu} B^{\mu\nu} \qquad 3,\ \gamma\gamma h,\ \ldots$$

$$\mathcal{O}_{\varphi W} = \frac{1}{2}(\phi^\dagger \phi)\, W^i_{\mu\nu} W^{i\,\mu\nu} \qquad 3,\ 4,\ \text{kin.},\ \gamma\gamma h,\ \ldots$$

$$\mathcal{O}_{\varphi \tilde{W}} = (\phi^\dagger \phi)\, \tilde{W}^i_{\mu\nu} W^{i\,\mu\nu} \qquad \gamma\gamma h,\ \ldots$$

operators contribute to:

| | | |
|---|---|---|
| 3 | = | triple gauge couplings |
| 4 | = | quartic gauge couplings |
| kin. | = | kinetic terms of gauge bosons |

# Diagonalisation and parameters - 1/4

Gauge boson kinetic and mass terms:

$$\mathscr{L}^{(2)}_{V'} = -\frac{1}{4}\,(Z'_{\mu\nu}, A'_{\mu\nu})\,T'\begin{pmatrix} Z'^{\mu\nu} \\ A'^{\mu\nu} \end{pmatrix} + \frac{1}{2}\,(Z'_{\mu}, A'_{\mu})\,M'\begin{pmatrix} Z'^{\mu} \\ A'^{\mu} \end{pmatrix}$$

$$\mathscr{L}^{(2)}_{W'} = -\frac{1}{2}(1 - h_{\varphi W} v^2) W^{+'}_{\mu\nu} W^{-'\mu\nu} + m'^2_W W^{+'}_{\mu} W^{-'\mu}$$

non-diagonal kinetic matrix: mass matrix:

$$T'(s'_W, h_{WB}, h_{\varphi W}) = \begin{pmatrix} a & b \\ b & d \end{pmatrix} \qquad M' = \begin{pmatrix} m'^2_Z & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{aligned} a,\, d &= 1 + O(hv^2) \\ b &= O(hv^2) \end{aligned} \qquad \begin{aligned} W^3_\mu &= c'_W\, Z'_\mu + s'_W\, A'_\mu \\ B_\mu &= c'_W\, A'_\mu - s'_W\, Z'_\mu \end{aligned}$$

# Diagonalisation and parameters - 2/4

physical charged boson $W_\mu^\pm$ and mass $m_W^2$:

$$W_\mu^{\pm\prime} = \frac{1}{\sqrt{1 - h_{\varphi W} v^2}} W_\mu^\pm \qquad m_W'^2 = (1 - h_{\varphi W} v^2) m_W^2$$

physical neutral gauge bosons $A_\mu$ and $Z_\mu$:

$$\begin{pmatrix} Z'^\mu \\ A'^\mu \end{pmatrix} = A \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

non-orthogonal transformation matrix $A$ defined such that:

$$T = A^{\mathrm{T}} T' A = \mathbb{1}$$

$$M = A^{\mathrm{T}} M' A = \begin{pmatrix} m_Z^2 & 0 \\ 0 & 0 \end{pmatrix} \qquad m_Z^2 = \frac{d}{ad - b^2} m_Z'^2$$

# Diagonalisation and parameters - 3/4

free parameters: $m_Z$, $e$, $\sin^2\theta_{\rm eff}^{\rm lept}$

interaction Lagrangian (in the SM form):

$$\mathscr{L}^{\rm int} = -e'\left\{\frac{1}{s'_W c'_W} Z'_\mu \mathcal{J}'^{\mu}_{\rm NC} + A'_\mu \mathcal{J}^{\mu}_{\rm em} + \frac{1}{\sqrt{2}s'_W}\left(W^{+\prime}_\mu \mathcal{J}^{\mu}_{\rm CC} + {\rm h.c.}\right)\right\}$$

inserting the physical fields:

$$\begin{aligned}\mathscr{L}^{\rm int} = & -\overbrace{\frac{e'}{\sqrt{d}}}^{e}\left\{\frac{1}{s'_W c'_W}\frac{d}{\sqrt{ad-b^2}} Z_\mu \overbrace{\left(\mathcal{J}'^{\mu}_{\rm NC} - \frac{b}{d}s'_W c'_W \mathcal{J}^{\mu}_{\rm em}\right)}^{\mathcal{J}^{\mu}_{\rm NC}}\right. \\ & \left. + A_\mu \mathcal{J}^{\mu}_{\rm em} + \frac{1}{\sqrt{2}s'_W}\frac{\sqrt{d}}{\sqrt{1-h_{\varphi W}v^2}}\left(W^{+}_\mu \mathcal{J}^{\mu}_{\rm CC} + {\rm h.c.}\right)\right\}\end{aligned}$$

# Diagonalisation and parameters - 4/4

as usual: $$s^2_{\rm eff} \equiv \sin^2\theta^{\rm lept}_{\rm eff} = \frac{1}{4}\left(1 - \frac{g^\ell_V}{g^\ell_A}\right) = s'^2_W + \frac{b}{d} s'_W c'_W$$

$$\longrightarrow \qquad s'_W \approx s_{\rm eff} - \frac{h_{WB} v^2}{2}(c^2_{\rm eff} - s^2_{\rm eff}) c_{\rm eff} + \frac{h_{\varphi W} v^2}{2} s_{\rm eff} c^2_{\rm eff}$$

interaction Lagrangian in terms of physical fields and free parameters:

$$\begin{aligned}\mathscr{L}^{\rm int}(e, s_{\rm eff}, h_{WB}, h_{\varphi W}) \;\; &= \;\; -e\Bigg\{\frac{1}{s'_W(s_{\rm eff})c'_W(s_{\rm eff})}\frac{d}{\sqrt{ad - b^2}} Z_\mu \mathcal{J}^\mu_{\rm NC} \\ &+ A_\mu \mathcal{J}^\mu_{\rm em} + \frac{1}{\sqrt{2} s'_W(s_{\rm eff})}\frac{\sqrt{d}}{\sqrt{1 - h_{\varphi W} v^2}}\left(W^+_\mu \mathcal{J}^\mu_{\rm CC} + {\rm h.c.}\right)\Bigg\}\end{aligned}$$

# Limits from ew. precision data - 1/2

No impact on forward-backward asymmetries
and hadronic $Z$ pole cross section:

Combination of ew. measurements, hep-ex/0212036

$$A_{\mathrm{FB}}^{0,\mathrm{b}} \equiv \frac{3}{4}\mathcal{A}_{\mathrm{e}}\mathcal{A}_{\mathrm{b}}, \quad \mathcal{A}_{\mathrm{f}} \equiv \frac{2g_V^{\mathrm{f}}/g_A^{\mathrm{f}}}{1+(g_V^{\mathrm{f}}/g_A^{\mathrm{f}})^2}, \quad \sigma_{\mathrm{had}}^0 \equiv \frac{12\pi}{m_Z^2}\frac{\Gamma_{\mathrm{ee}}\Gamma_{\mathrm{had}}}{\Gamma_Z^2}$$

$\rho$-parameter and total widths of $W$ and $Z$ modified:

$$\rho \equiv \frac{m_W^2}{m_Z^2 c_{\mathrm{eff}}^2} \approx 1 - h_{WB}v^2 \ \tan_{\mathrm{eff}}, \qquad \frac{\Gamma_W^{\mathrm{an}}}{\Gamma_W^{\mathrm{SM}}} \approx 1 + h_{WB}v^2 \ (\cot_{\mathrm{eff}} - \frac{1}{2}\tan_{\mathrm{eff}})$$

$$\frac{\Gamma_Z^{\mathrm{an}}}{\Gamma_Z^{\mathrm{SM}}} \approx 1 + h_{WB}v^2 \ (\tan_{\mathrm{eff}} + \cot_{\mathrm{eff}})$$

# Limits from ew. precision data - 2/2

| | SM prediction [GeV] | exp. value [GeV] |
|---|---|---|
| $\Gamma_W$ | 2.0921$\pm$0.0025 | 2.114$\pm$0.043 |
| $\Gamma_Z$ | 2.4961$\pm$0.0012 | 2.4952$\pm$0.0023 |
| $\rho$ | 1 | 1.014$\pm$0.037 |

PDG 2002

weighted average: $h_{WB}v^2 = -0.00014 \pm 0.0006$

at GigaZ: $\delta h_{WB}v^2 = 0.00053$

(using $\delta\rho^{\mathrm{exp}} = 0.0042$ and $\delta\Gamma_Z^{\mathrm{exp}} = 1 \ \mathrm{MeV}$)

TDR

# Constraints from TGCs - 1/2

Using relations:

$$\lambda_\gamma = \lambda_Z = h_W \; 6 s_{\text{eff}} m_W^2 / e$$

$$\Delta\kappa_\gamma = h_{WB} v^2 \; \cot_{\text{eff}}$$

$$\Delta\kappa_Z = h_{WB} v^2 \; (\cot_{\text{eff}} - \tan_{\text{eff}})/2$$

$$\Delta g_1^Z = h_{WB} v^2 \; (\cot_{\text{eff}} + \tan_{\text{eff}})/2$$

Combination of ew. measurements, hep-ex/0212036

we obtain:

$$h_W v^2 = -\,0.020 \pm 0.024$$

$$h_{WB} v^2 = -\,0.0085 \pm 0.019$$

$$\tilde{\lambda}_\gamma = \tilde{\lambda}_Z = h_{\tilde{W}} \; 6 s_{\text{eff}} m_W^2 / e$$

$$\tilde{\kappa}_\gamma = -\tilde{\kappa}_Z \cot^2_{\text{eff}} = \; h_{\tilde{W}B} v^2 \; \cot_{\text{eff}}$$

ALEPH, OPAL

$$h_{\tilde{W}} v^2 = 0.026 \pm 0.088$$

$$h_{\tilde{W}B} v^2 = 0.29 \pm 0.13$$

# Constraints from TGCs - 2/2

Setting $h_i = 1/\Lambda_i^2$ gives constraints on the scale of new physics:

$$\Lambda_W \gtrsim 1.2\,\mathrm{TeV} \qquad \Lambda_{WB} \gtrsim 1.5\,\mathrm{TeV}$$

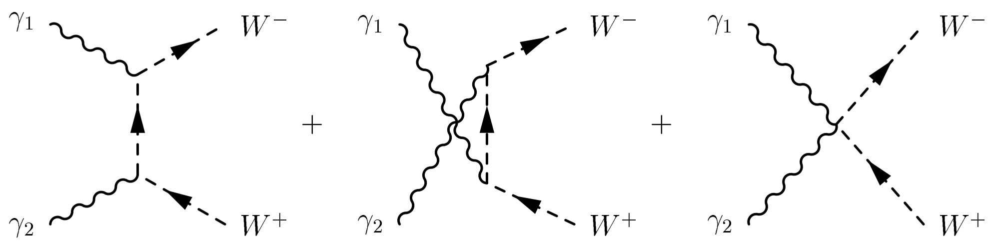
$$\Lambda_{\tilde{W}} \gtrsim 730\,\mathrm{GeV} \qquad \Lambda_{\tilde{W}B} \gtrsim 380\,\mathrm{GeV}$$

from the $\rho$-parameter, $Z$- and $W$-widths:

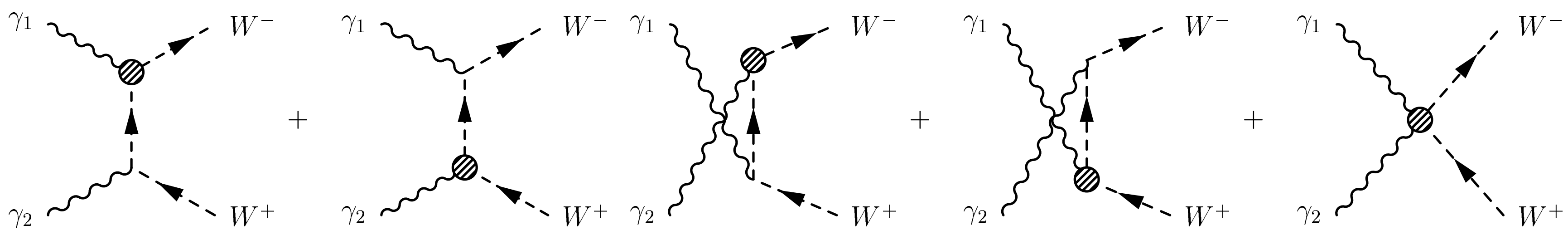
$$\Lambda_{WB} \gtrsim 9\,\mathrm{TeV}$$

# Gauge boson couplings in $\gamma\gamma \to WW$

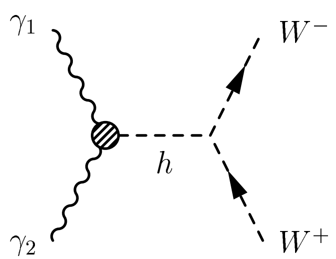
SM diagrams:



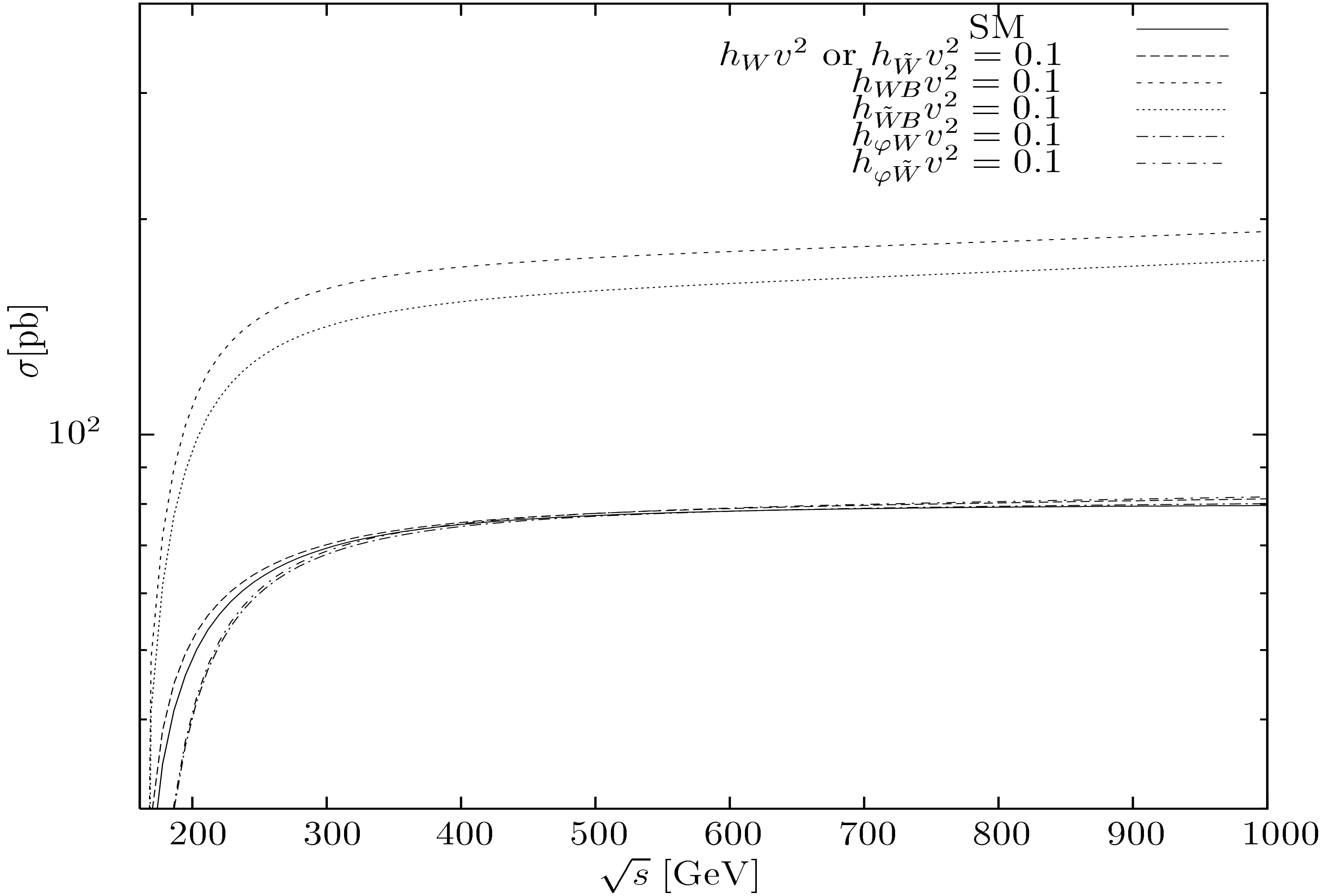

anomalous triple and quartic gauge couplings:



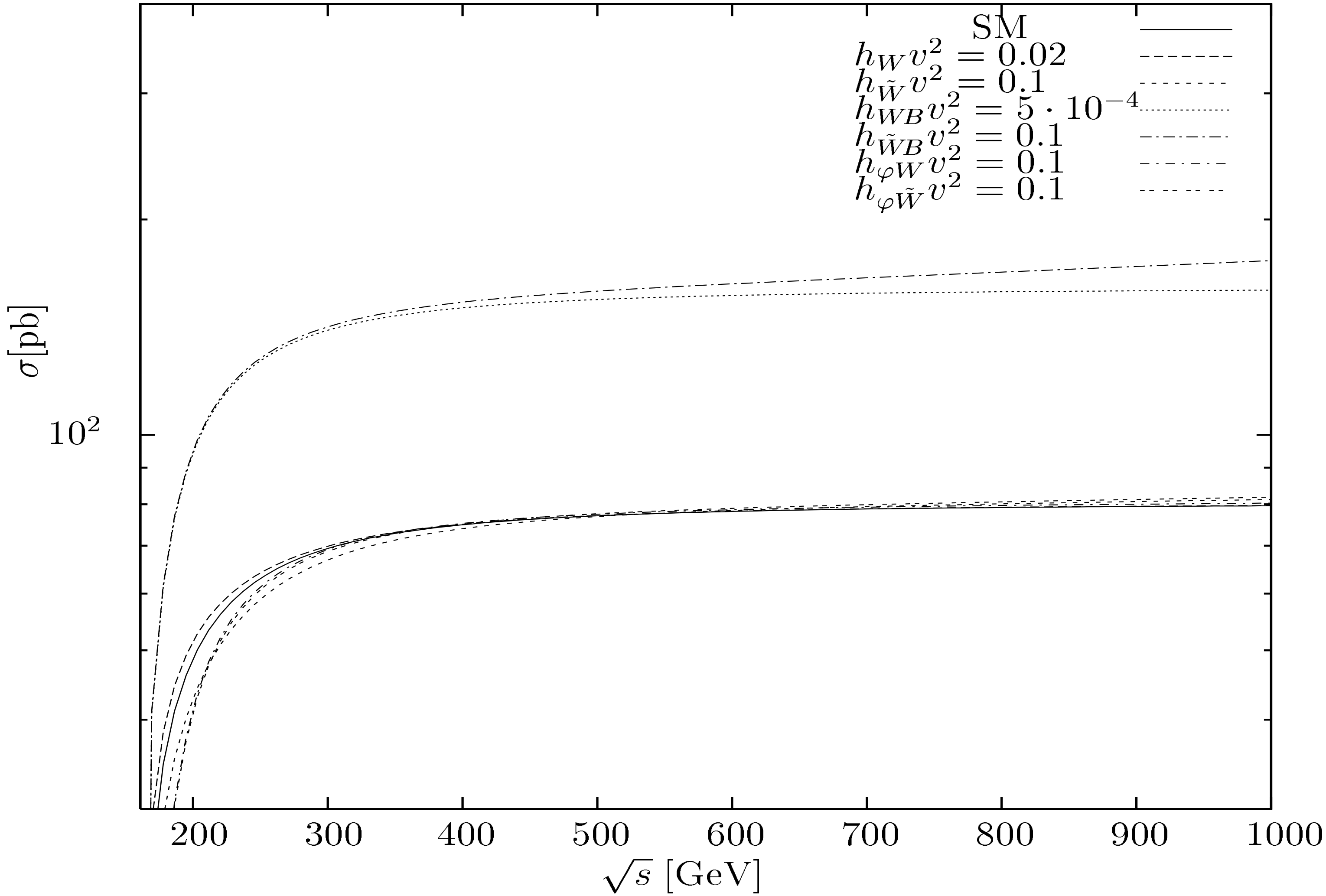

Higgs exchange:



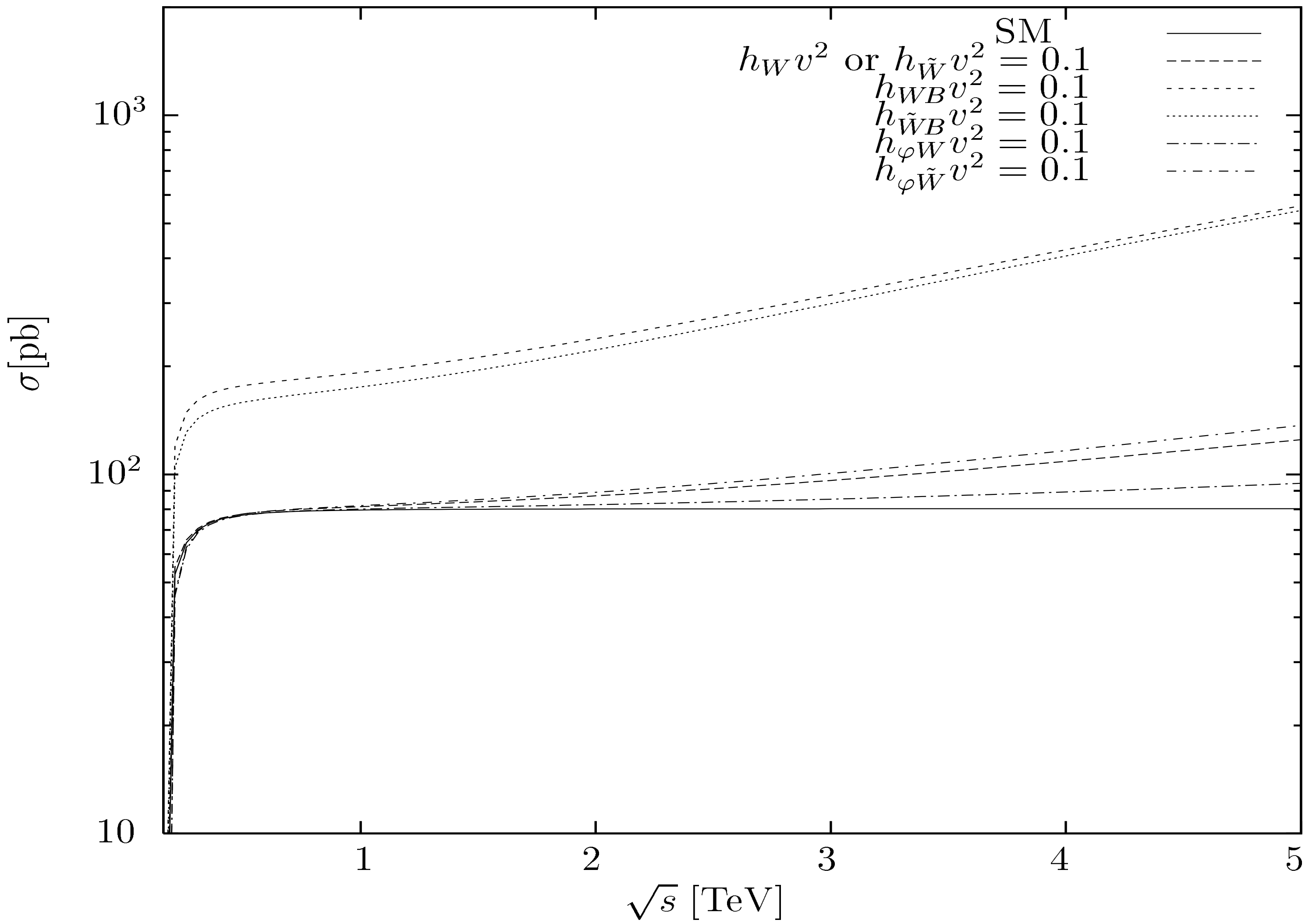
# Total cross section $\gamma\gamma \to WW$



# Total cross section $\gamma\gamma \to WW$



# Total cross section $\gamma\gamma \to WW$



# Optimal observables

differential cross section:

$$d\sigma/d\phi = S_0 + \sum_i S_{1i} h_i + \sum_{i,j} S_{2ij} h_i h_j$$

where $\phi$ denotes the set of all phase space variables of the final state. Then the observables

$$\mathcal{O}_i = \frac{S_{1i}(\phi)}{S_0(\phi)}$$

are "optimal" in the sense that for $h_i \to 0$ the errors on the couplings

$$V(h) = V(\mathcal{O})^{-1}/N$$

are minimal.

# Errors and correlations on the couplings $h_i$

$\sqrt{s} = $ 500 GeV, unpolarised beams, $m_H = 120$ GeV

| $h$ | $\delta h \times 10^3$ | $h_W$ | $h_{WB}$ | $h_{\varphi W}$ |
|---|---|---|---|---|
| $h_W$ | $0.167$ | 1 | 0.273 | 0.274 |
| $h_{WB}$ | $0.771$ | | 1 | 0.987 |
| $h_{\varphi W}$ | $7.78$ | | | 1 |

| $h$ | $\delta h \times 10^3$ | $h_{\tilde{W}}$ | $h_{\tilde{W}B}$ | $h_{\varphi\tilde{W}}$ |
|---|---|---|---|---|
| $h_{\tilde{W}}$ | $0.243$ | 1 | 0.691 | 0.690 |
| $h_{\tilde{W}B}$ | $4.14$ | | 1 | 0.999 |
| $h_{\varphi\tilde{W}}$ | $20.5$ | | | 1 |

$\sqrt{s} = $ 800 GeV

| $h$ | $\delta h \times 10^3$ | $h_W$ | $h_{WB}$ | $h_{\varphi W}$ |
|---|---|---|---|---|
| $h_W$ | $0.0623$ | 1 | 0.162 | 0.168 |
| $h_{WB}$ | $0.450$ | | 1 | 0.994 |
| $h_{\varphi W}$ | $4.65$ | | | 1 |

| $h$ | $\delta h \times 10^3$ | $h_{\tilde{W}}$ | $h_{\tilde{W}B}$ | $h_{\varphi\tilde{W}}$ |
|---|---|---|---|---|
| $h_{\tilde{W}}$ | $0.0912$ | 1 | 0.662 | 0.663 |
| $h_{\tilde{W}B}$ | $3.81$ | | 1 | 0.999 |
| $h_{\varphi\tilde{W}}$ | $19.5$ | | | 1 |

$\sqrt{s} = $ 3 TeV

| $h$ | $\delta h \times 10^3$ | $h_W$ | $h_{WB}$ | $h_{\varphi W}$ |
|---|---|---|---|---|
| $h_W$ | $0.0084$ | 1 | 0.0284 | 0.0312 |
| $h_{WB}$ | $0.179$ | | 1 | 0.999 |
| $h_{\varphi W}$ | $1.87$ | | | 1 |

| $h$ | $\delta h \times 10^3$ | $h_{\tilde{W}}$ | $h_{\tilde{W}B}$ | $h_{\varphi\tilde{W}}$ |
|---|---|---|---|---|
| $h_{\tilde{W}}$ | $0.0104$ | 1 | 0.388 | 0.388 |
| $h_{\tilde{W}B}$ | $3.54$ | | 1 | 0.999 |
| $h_{\varphi\tilde{W}}$ | $18.5$ | | | 1 |

# Summary

- Gauge invariant effective Lagrangian:
  restriction to gauge boson self interactions
- Simultaneous diagonalisation of kinetic and mass terms
  $\Rightarrow$ modification of interaction Lagrangian
- Limits from electroweak precision data:
  $h_{WB}$ comparitively highly constrained ($Z$ width!)
- Helicity amplitudes for $\gamma\gamma \to WW$
- Max. sensitivity at the photon collider estimated
  by means of optimal observables