Physics Impact of a Precise Determination of the Top Quark Mass at TESLA

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based on collaboration with
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1. Introduction
2. Electroweak Precision Observables
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4. RG running in mSUGRA
5. Dark matter constraints
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1. Introduction

TESLA : $\delta m_t \lesssim 100$ MeV

LHC : $\delta m_t = 1 - 2$ GeV

[*M. Martinez, R. Miquel '02*] [*A. Hoang et al., '00*]
[*M. Beneke, I. Efthymiopoulos, M. Mangano, J. Womersley et al. '00*]

Q: What can be learned from the improved accuracy obtainable at TESLA?

A: Many things . . . See below!

Disclaimer:

- no comments on different m_t definitions and possible implications
- assumption: conversion of different definitions will be precise enough so that the calculations used below are valid
- list of our examples is not exclusive!

2. Electroweak precision observables

EWPO: M_W, $\sin^2\theta_{\rm eff}$

1.) Theoretical prediction for M_W in terms of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\,\alpha}{\sqrt{2}\,G_\mu}\left(\frac{1}{1-\Delta r}\right)$$

$\Updownarrow$

loop corrections

SM prediction for Δr, one-loop:

[*A. Sirlin '80*] [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\rm loop} = \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{\rm rem}(m_h)$$

$$\sim \log\frac{M_Z}{m_f} \qquad \sim m_t^2$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

2.) Effective mixing angle:

$$\sin^2\theta_{\rm eff}=\frac{1}{4\,|Q_f|}\left(1-\frac{{\rm Re}\,g_V^f}{{\rm Re}\,g_A^f}\right)$$

m_t enters higher order contributions:

$$g_V^f \to g_V^f + \Delta g_V^f, \quad g_A^f \to g_A^f + \Delta g_A^f$$

Experimental errors:

	today	Tev./LHC	LC	GigaZ
$\delta \sin^2\theta_{\rm eff} (\times 10^5)$	17	17	–	1.3
δM_W [MeV]	34	15	10	7

Relevant SM parametric errors:

	ΔM_W	$\delta \sin^2\theta_{\rm eff}$
$\delta m_t = 2$ GeV	12 MeV	6×10^{-5}
$\delta m_t = 1$ GeV	6 MeV	3×10^{-5}
$\delta m_t = 0.1$ GeV	1 MeV	0.3×10^{-5}
$\delta(\Delta\alpha_{\rm had}) = 5 \times 10^{-5}$	1 MeV	1.8×10^{-5}
$\delta M_Z = 2.1$ MeV	2.5 MeV	1.4×10^{-5}

To keep the parametric error induced by m_t at/below the level of other uncertainties:

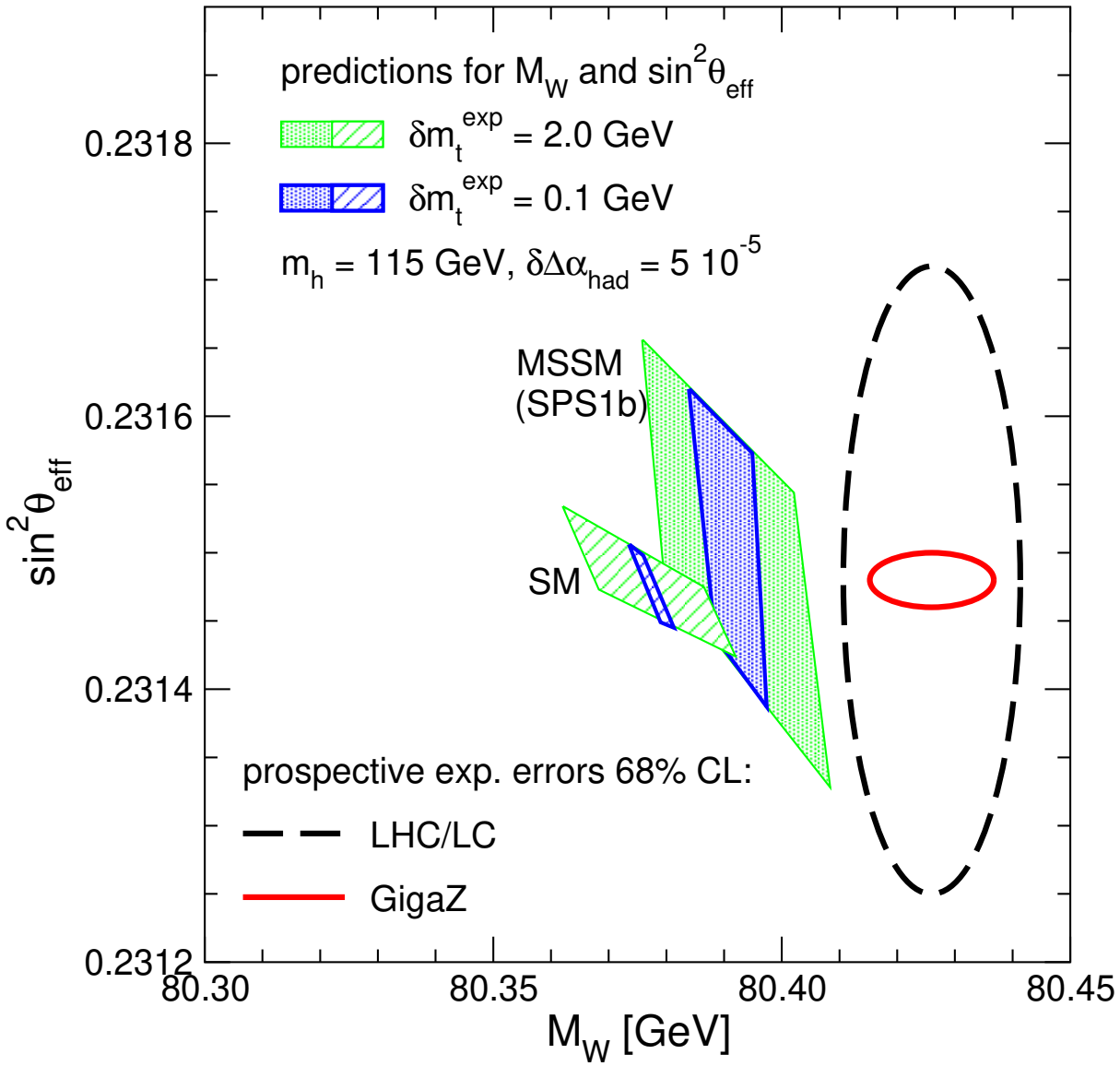
$\Rightarrow$ $\delta m_t \lesssim 0.2$ GeV for M_W

$\Rightarrow$ $\delta m_t \lesssim 0.5$ GeV for $\sin^2\theta_{\rm eff}$

Compare SM and MSSM:

SM: $m_h = 115$ GeV

MSSM: SPS1b, all SUSY parameters varied within realistic errors



$\delta m_t = 0.1$ GeV:

SM: Improvement by ~ 10

MSSM: Improvement by $\sim 2 - 3$

$\delta m_t = 0.1$ GeV necessary to establish new physics effects

3. MSSM Higgs and scalar top sector

Higgs sector of the MSSM:

Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}$$

Higgs potential:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2(\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) + \frac{g'^2 + g^2}{8}(H_1\bar{H}_1 - H_2\bar{H}_2)^2 + \frac{g^2}{2}|H_1\bar{H}_2|^2$$

Physical states: $h^0, H^0, A^0, H^\pm$

Goldstones: $G^0, G^\pm$

Input parameters:

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$$

$\tilde{t}$ sector of the MSSM:

(scalar partner of the top quark)

Mass matrix for $\tilde{t}_L, \tilde{t}_R$:

$$M^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{1t} & m_t X_t \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{2t} \end{pmatrix}$$

$$\Downarrow \leftarrow \text{Diagonalization, } \theta_{\tilde{t}}$$

$$\begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

$m_t X_t = m_t(A_t - \mu \cot\beta)$; large mixing possible

$\Rightarrow$ Physical parameters: $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$

Soft SUSY-breaking parameters: $M_{\tilde{t}_L}$, $M_{\tilde{t}_R}$, A_t

often used:

$$M_{\text{SUSY}} = m_{\tilde{q}} \equiv M_{\tilde{t}_L} = M_{\tilde{t}_R}$$

Higgs sector tree-level predictions:

$$m^2_{H,h} = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4 M_Z^2 M_A^2 \cos^2 2\beta}\right]$$

$$\Rightarrow m_h \leq M_Z \ , \qquad M_A \approx m_H \approx m_{H^\pm}$$

Dominant corrections from $t - \tilde{t}$-sector:

Yukawa couplings: $\frac{e\, y_t}{2 s_w \sin\beta}$, $\frac{e\, y_t^2\, M_W}{s_w \sin\beta}$, ...

$$\Delta m_h^2 \sim \frac{m_t^4}{M_W^2 \sin^2\beta} \log\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right)$$

$$\approx \frac{m_t^4}{M_W^2 \sin^2\beta} \log\left(\frac{(M_{\rm SUSY}^2 + m_t^2)^2 - m_t^2 X_t^2}{m_t^4}\right)$$

$\Rightarrow m_h$ is best precision observable

$\Rightarrow$ use m_h measurement to constrain SUSY parameters

$\Rightarrow \delta m_t$ has to be under control

Parametric uncertainty induced by δm_t:

$$\Delta m_h^{\delta m_t} \approx \delta m_t$$

(numerical coincidence)

Experimental error:

Prospective accuracies:

$$\Delta m_h^{\rm exp} \approx 200 \text{ MeV} \quad \text{(LHC)}$$
$$\Delta m_h^{\rm exp} \approx 50 \text{ MeV} \quad \text{(TESLA)}$$

Parametric errors from other SM input:

– M_W: negligible

– α_s:

today: $\delta\alpha_s(M_Z) \approx 0.002 \Rightarrow \Delta m_h^{\alpha_s} \approx 0.3$ GeV

future: $\delta\alpha_s(M_Z) \lesssim 0.001 \Rightarrow \Delta m_h^{\alpha_s} \approx 0.1\text{–}0.2$ GeV

– m_b:

today: $\delta m_b(m_b) \approx 100$ MeV

$\Rightarrow \Delta m_h^{m_b} \approx 0.1/1$ GeV (small/large $\tan\beta$)

future: $\delta m_b(m_b) \approx 50$ MeV

$\Rightarrow \Delta m_h^{m_b} \approx 0.05$ GeV (small $\tan\beta$)

$\Rightarrow \Delta m_h^{m_b} \approx 0.5$ GeV (large $\tan\beta$)

Intrinsic error: (from unknown higher-order corr.)

today: $\Delta m_h^{\rm intr} \approx 3$ GeV

[*G. Degrassi, S.H., W. Hollik, P. Slavich, G. Weiglein '02*]

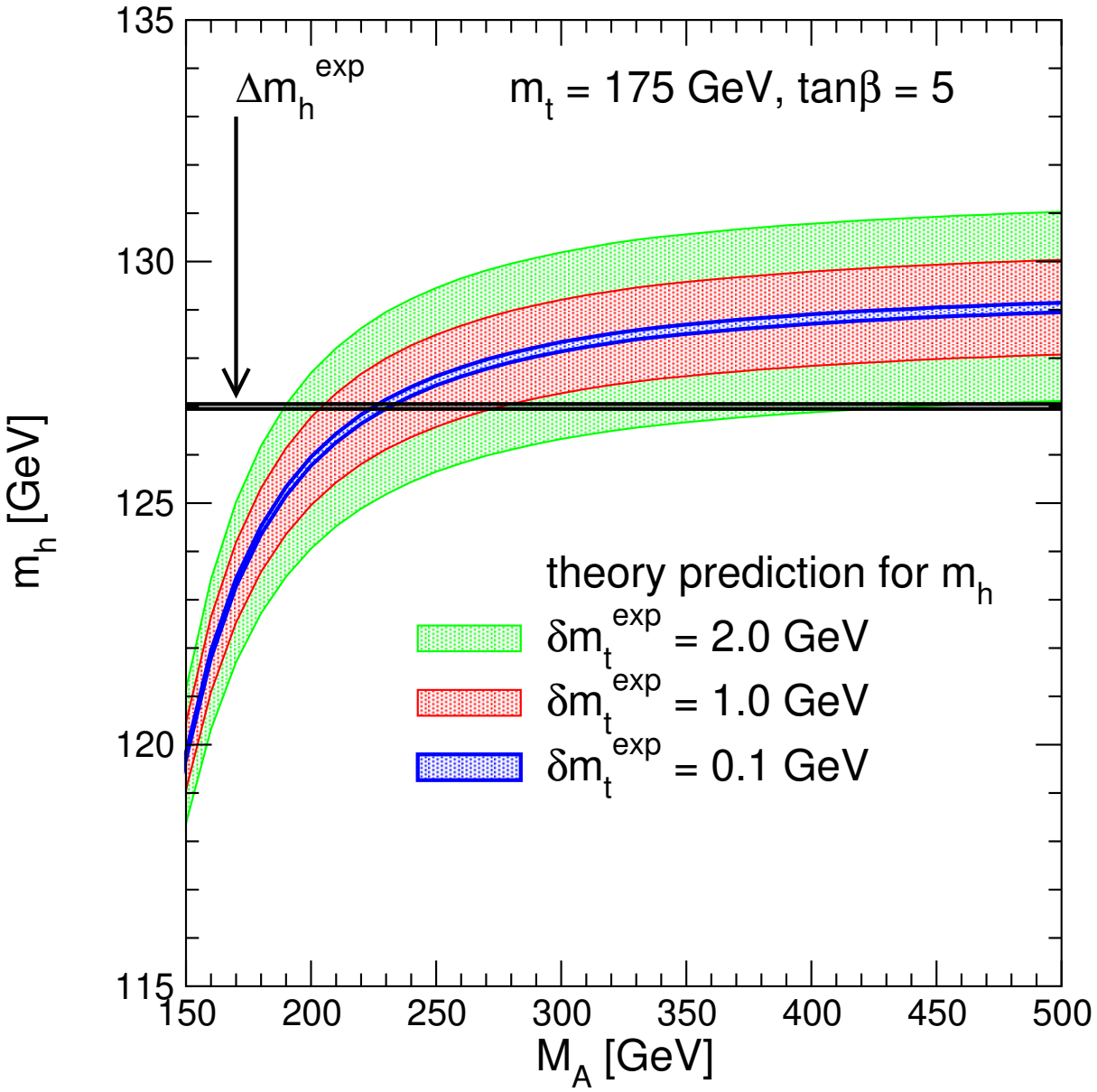
needed for future: $\Delta m_h^{\rm intr} \lesssim 0.5 - 0.1$ GeV

$\Delta m_h^{\rm intr}$ crucial (but future is hard to tell . . .)

$\Rightarrow$ $\Delta m_h^{\rm intr}$ mostly not included in plots

Example for m_h theory prediction:

$m_h^{\max}$ scenario, no SUSY experimental errors:

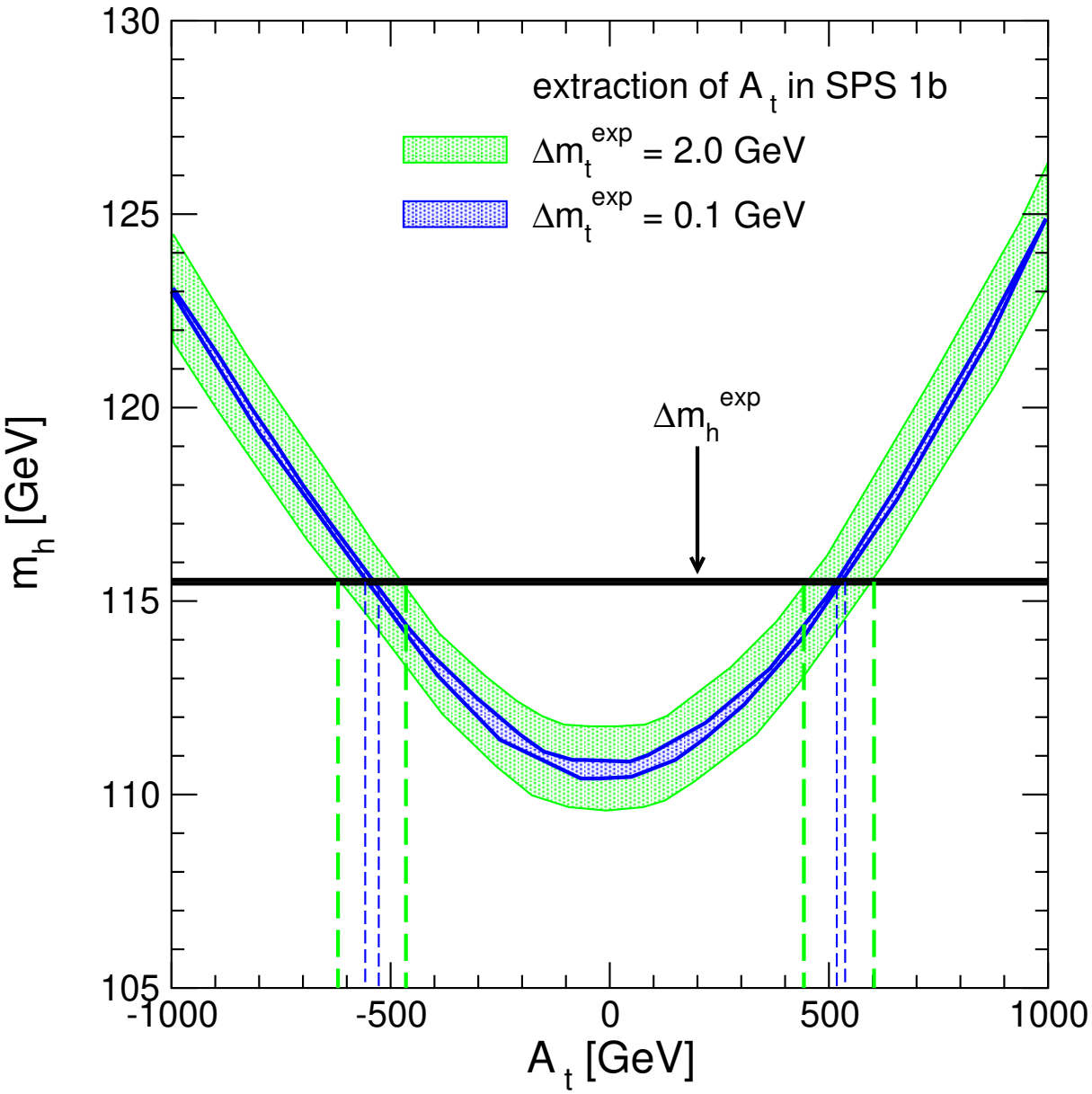


$\Rightarrow$ impact of δm_t obvious

"Realistic" SUSY example I:

SPS1b:

$m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$ known, A_t unknown
$\tan\beta, M_A$ known, realistic errors assumed



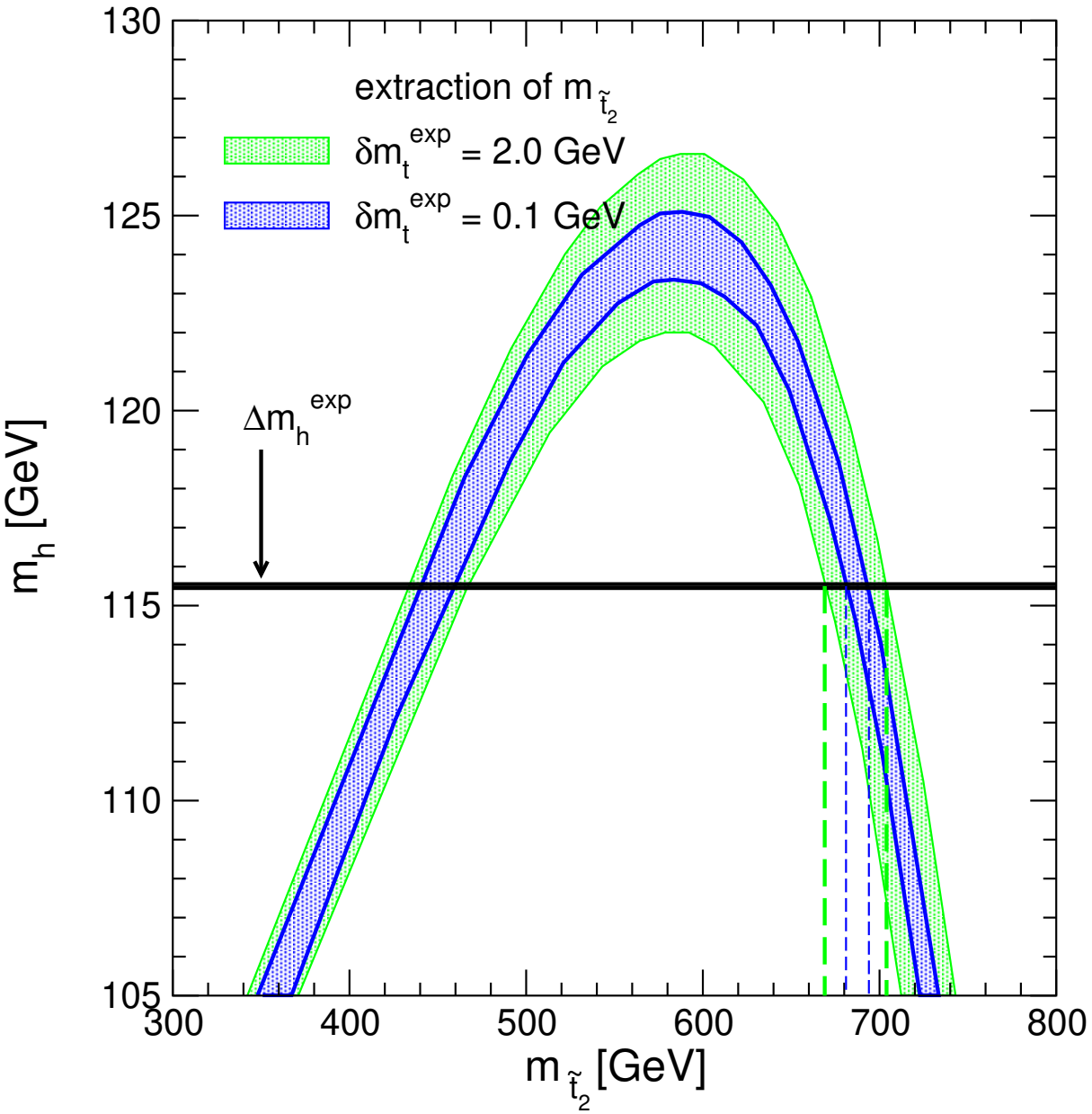
$\Rightarrow$ extraction of A_t possible

$\Rightarrow$ $\delta m_t = 0.1$ GeV gives improvement $\sim 3-4$

"Realistic" SUSY example II:

Light $\tilde{t}$ scenario (TESLA TDR):

$m_{\tilde{t}_1} = 180$ GeV, $\theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$ known, $m_{\tilde{t}_2}$ unknown
M_A known, $\tan\beta > 10$, realistic errors assumed



$\Rightarrow$ extraction of $m_{\tilde{t}_2}$ possible

$\Rightarrow$ $\delta m_t = 0.1$ GeV gives improvement ~ 3

4. RG running in mSUGRA

mSUGRA:

$$\{m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ \mathrm{sign}(\mu)\}$$

m_0 : common scalar mass at the GUT scale
$m_{1/2}$: common gaugino mass at the GUT scale
A_0 : common trilinear coupling at the GUT scale
$\tan\beta$: low energy value
$\mathrm{sign}(\mu)$: low energy value

$\Rightarrow$ prediction of all SUSY parameters in terms of $4\frac{1}{2}$ input parameters

Two examples:

– Bottom-up determination of GUT scale parameters

– Neutralino/chargino mass prediction

4A) Bottom-up determination of GUT scale parameters

Assume:

measurement of low-energy parameters
by TESLA and LHC

Then:

Use RGEs to evolve to the GUT scale

$\Rightarrow$ Check for unification etc.

Here:

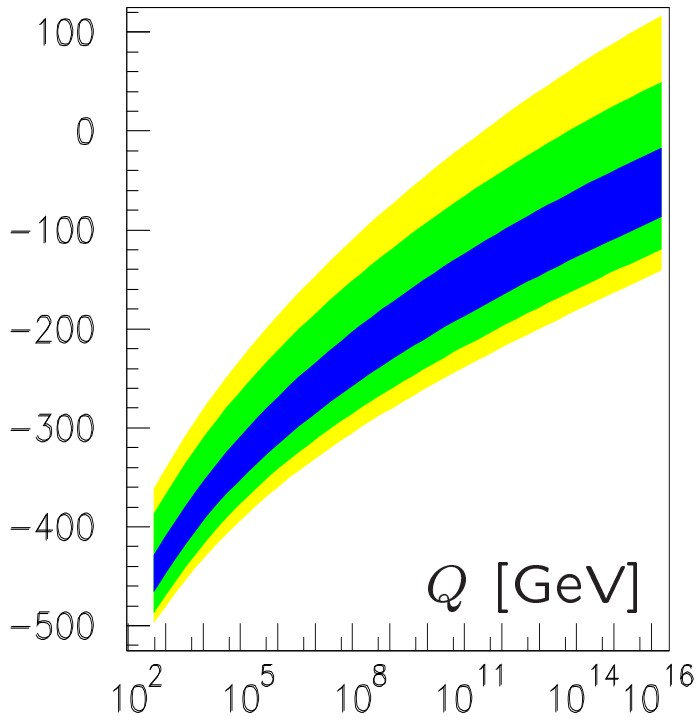
Focus on δm_t dependence of RG evolution

Example point given by:

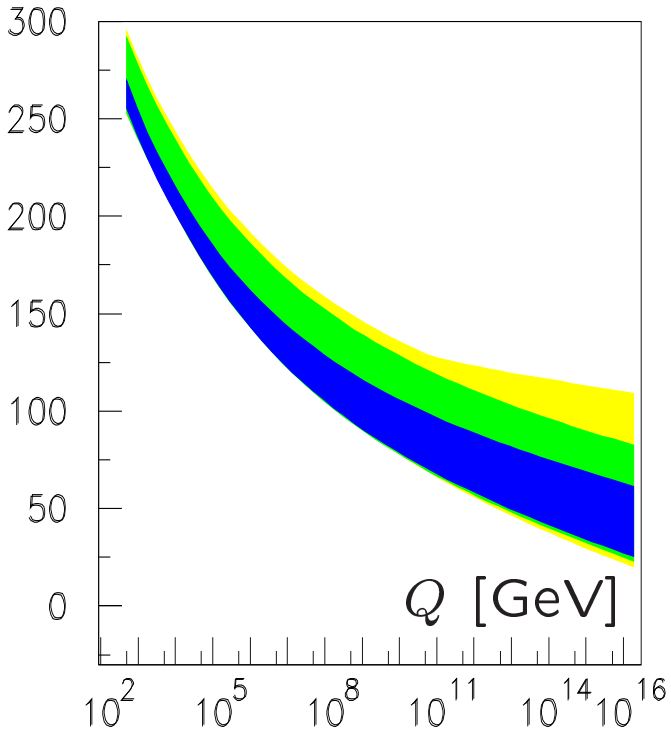
$m_0 = 200$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -100$ GeV
$\tan\beta = 10, \mu > 0$ $(A_t' = A_t y_t)$
$\Delta m_h^{\text{intr}} = 0.5$ GeV (used as constraint) $\rightarrow$ T

$\Rightarrow$ precision of GUT scale parameters
improved by ~ 4
(sign of A_t determined)

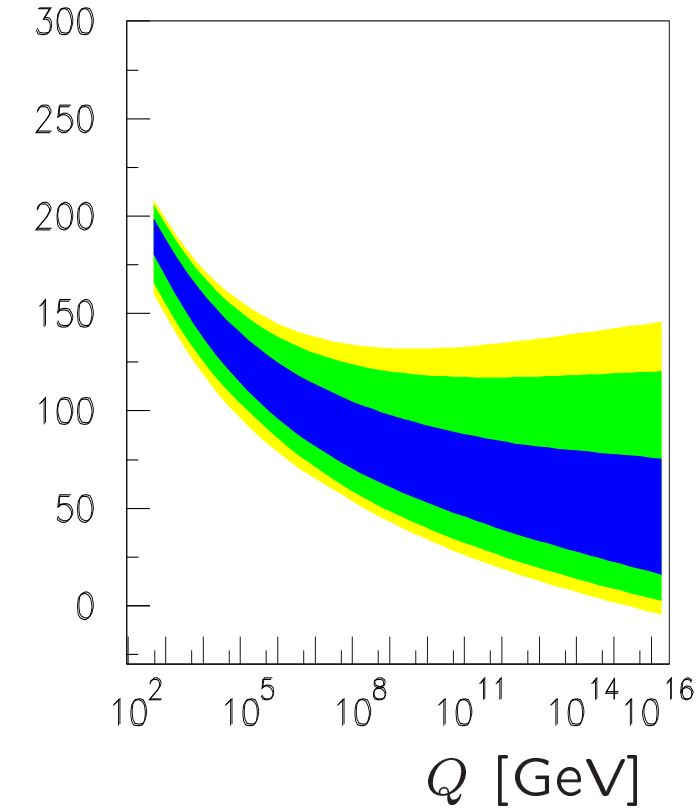
(a) A'_t [GeV]



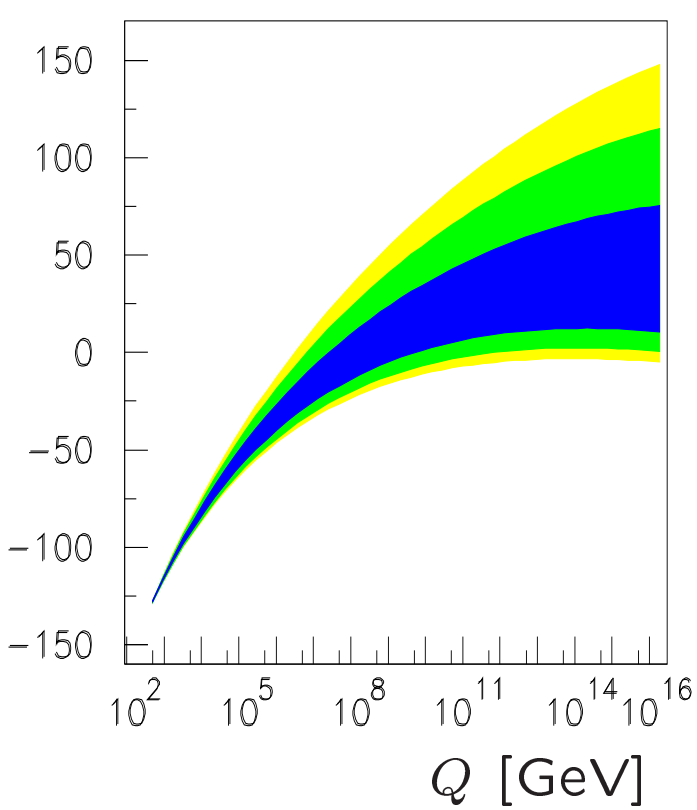
(b) $M^2_{\tilde{t}_L}$ [10^3 GeV^2]



(c) $M^2_{\tilde{t}_R}$ [10^3 GeV^2]



(d) $M^2_{\tilde{H},2}$ [10^3 GeV^2]



$\delta m_t = 0.1$ GeV, $\delta m_t = 1.0$ GeV, $\delta m_t = 2.0$ GeV

4B) Neutralino/chargino mass prediction

mSUGRA:

GUT scale parameters $\Leftrightarrow$ low-energy parameters

low-energy parameters used for data analysis

Q: precision of higgsino mass scale μ ?
(e.g. from measurement of $m_{\chi^0_{1,2}}, m_{\chi^\pm_1}$)

A: depends (sometimes strongly) on δm_t

Our example:

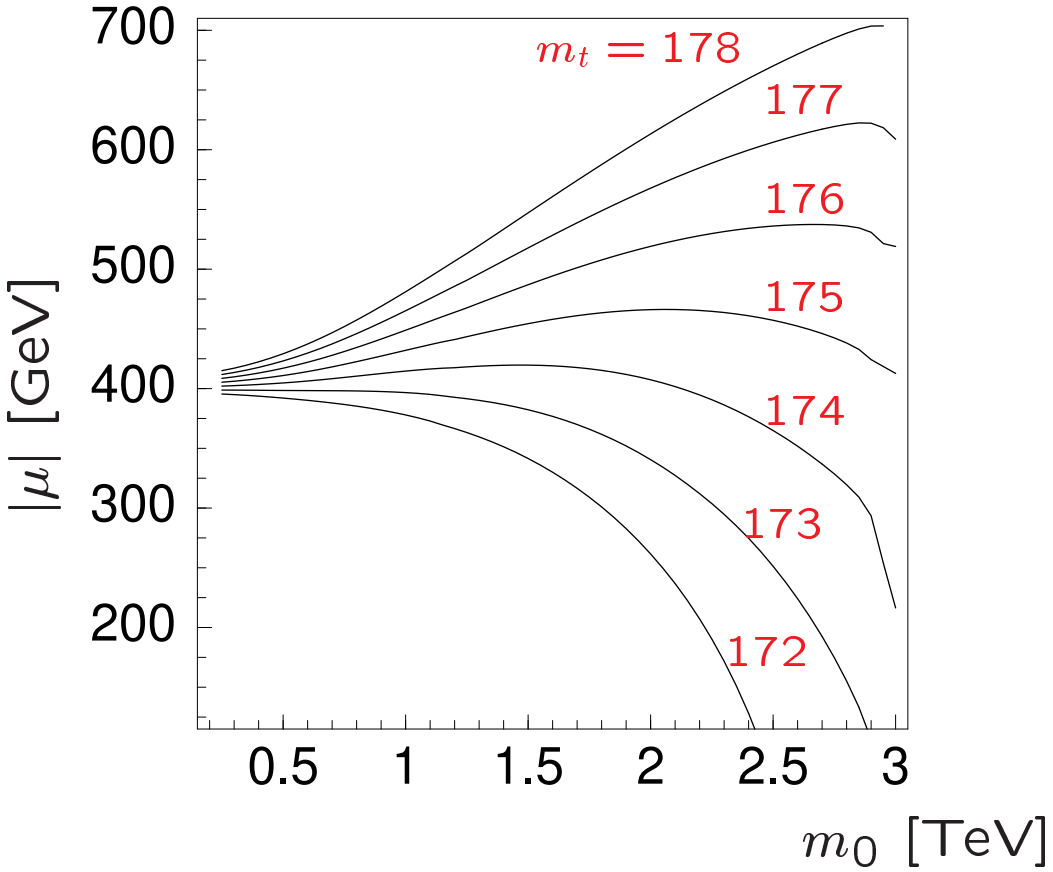
Set of mSUGRA parameters:

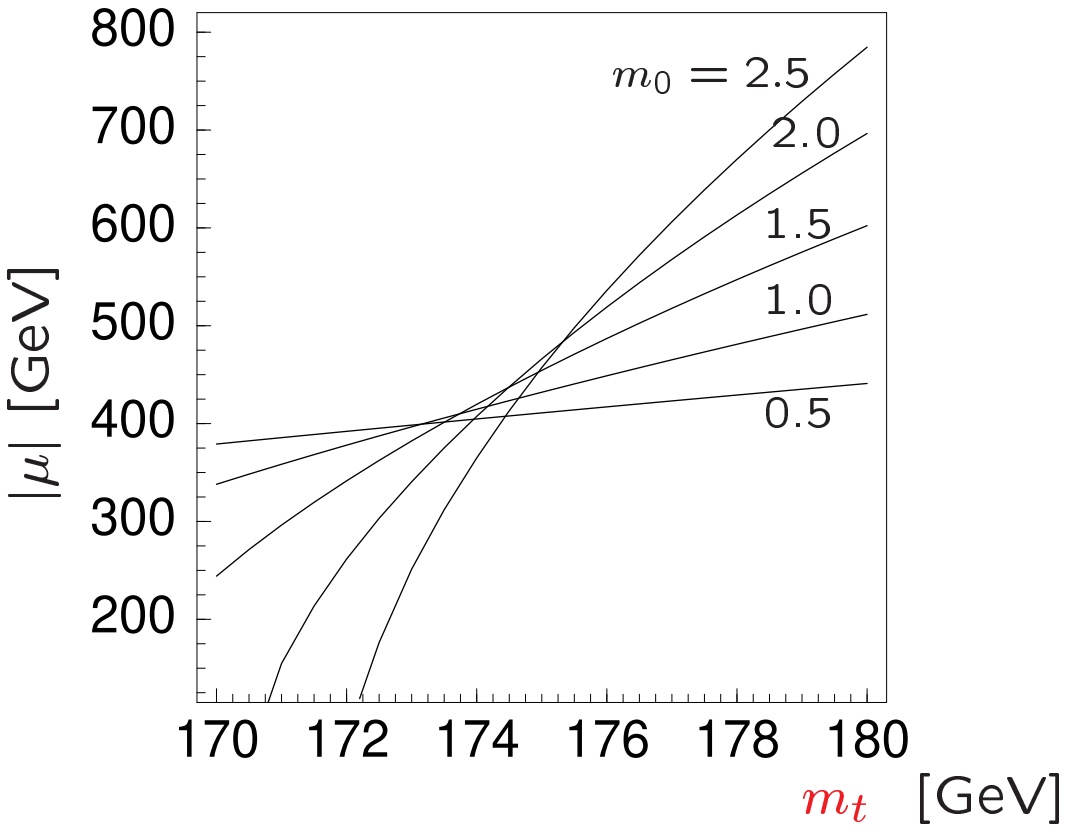
$m_{1/2} = 300$ GeV, $A_0 = 0$, $\tan\beta = 10$, $\mu > 0$

Keep m_0 free

$\Rightarrow$ derive higgsino mass scale μ
depending on m_0 and δm_t $\rightarrow$ T

$\Rightarrow$ Prediction of μ depends strongly on δm_t
LC precision crucial to derive required accuracy


700
600
500
400
300
200
$|\mu|$ [GeV]
$m_t = 178$
177
176
175
174
173
172
0.5
1
1.5
2
2.5
3
m_0 [TeV]


800
700
600
500
400
300
200
$|\mu|$ [GeV]
$m_0 = 2.5$
2.0
1.5
1.0
0.5
170
172
174
176
178
180
m_t [GeV]

5. Dark matter constraints

Lightest SUSY particle (LSP) is a good candidate for cold dark matter (CDM)

mSUGRA: LSP is lightest neutralino

Latest results on dark matter density:

$$0.094 \leq \Omega_\chi h^2 \leq 0.129$$

[*WMAP '03*]

mSUGRA: evaluate $\Omega_\chi h^2$ as a function of

$$\{m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ \mathrm{sign}(\mu)\}$$

$\Rightarrow$ derive bounds on mSUGRA parameters
(determine CDM allowed regions)

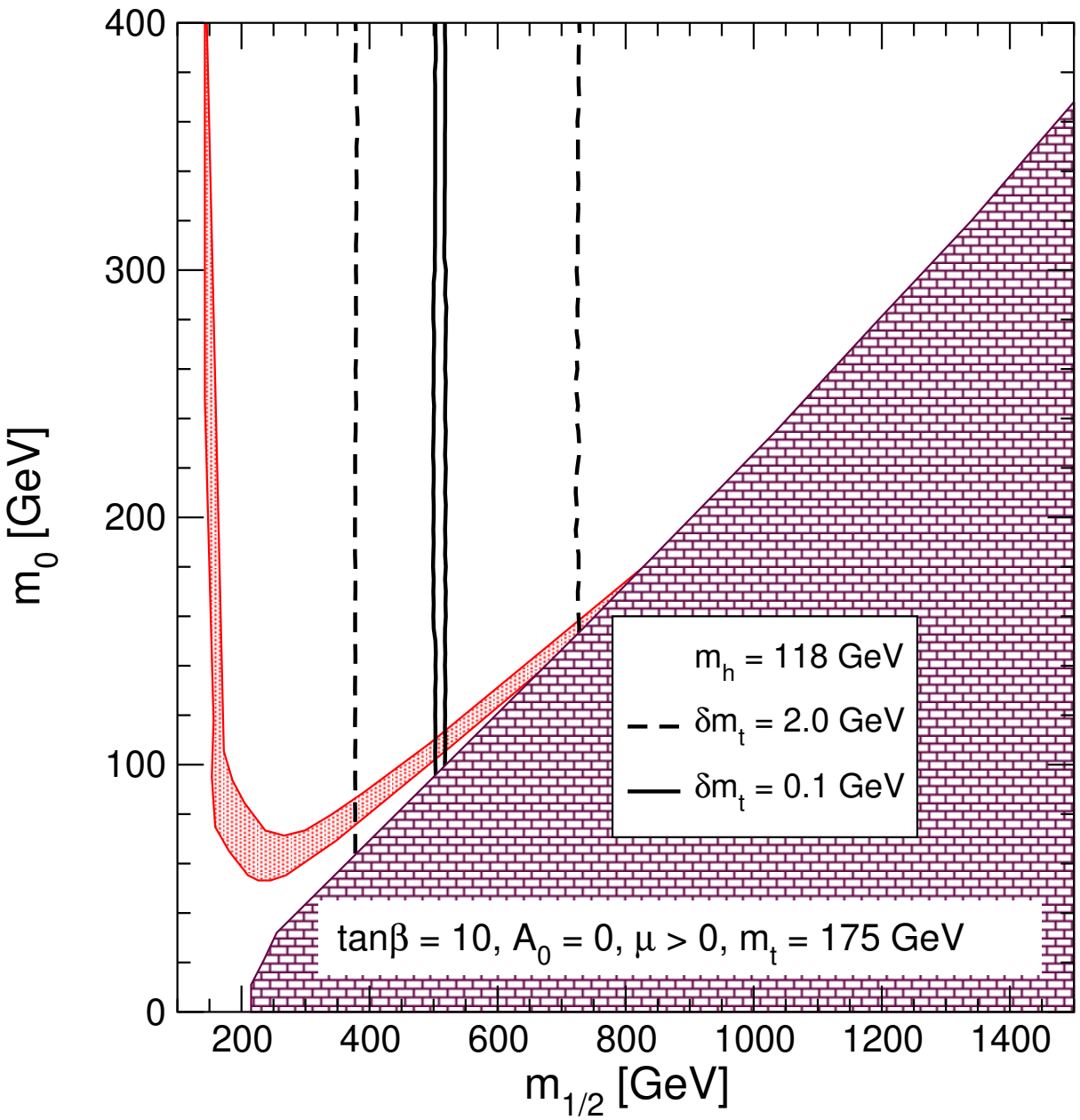
Impact of δm_t precision ?

Example I: low $\tan\beta$

$\rightarrow$ CDM allowed region depends weakly on m_t

$\rightarrow$ m_h depends strongly on m_t

$\Rightarrow$ use both constraints to determine mSUGRA parameters



$\delta m_t = 2$ GeV $\Rightarrow$ 400 GeV $\lesssim m_{1/2} \lesssim$ 750 GeV

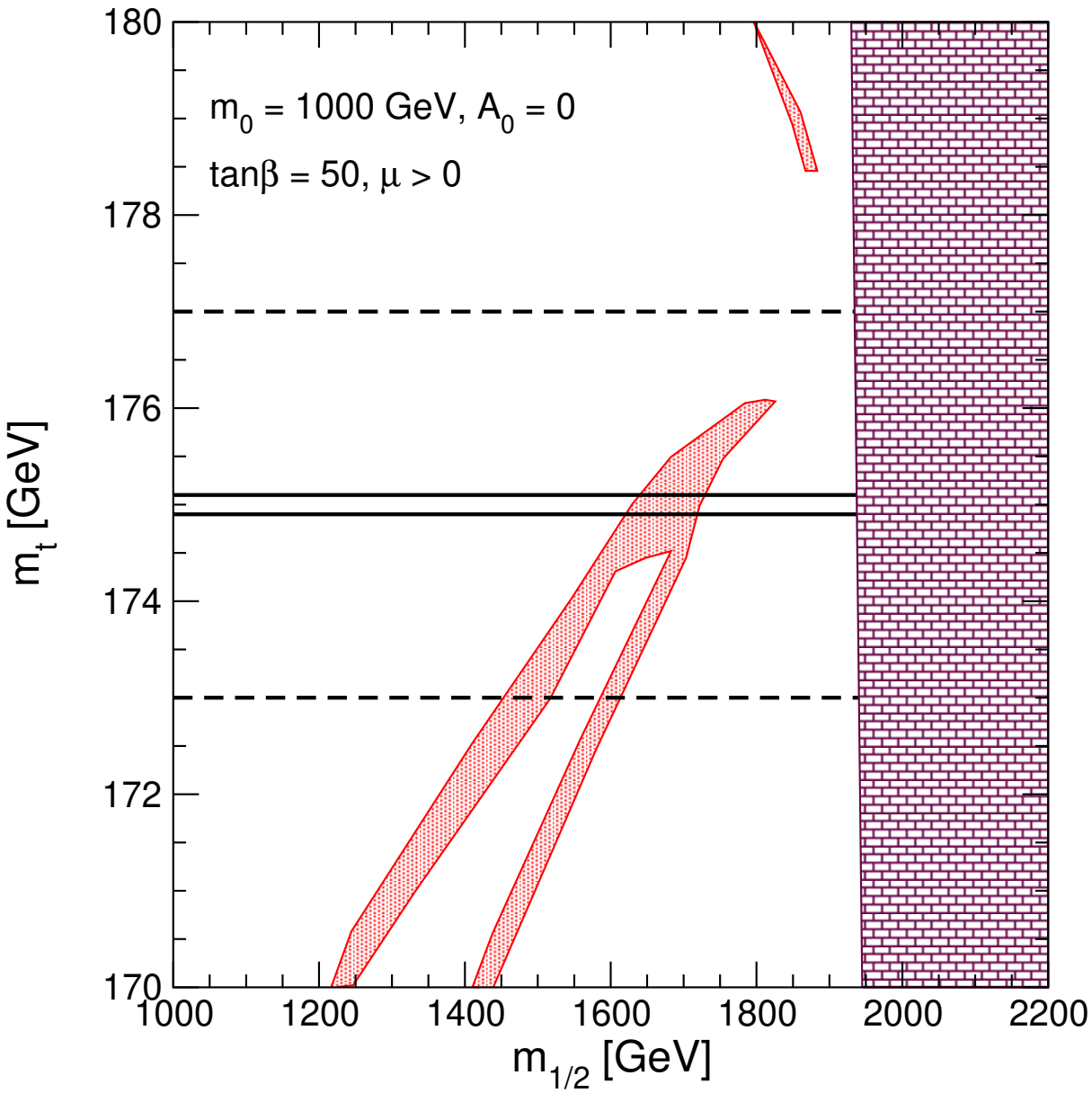
$\delta m_t = 0.1$ GeV $\Rightarrow$ 500 GeV $\lesssim m_{1/2} \lesssim$ 520 GeV

Example II: high $\tan\beta$

$\rightarrow$ CDM allowed region itself depends strongly on m_t

Fix mSUGRA parameters, keep $m_{1/2}$ free

$\Rightarrow$ use CDM constraint to determine $m_{1/2}$



$\delta m_t = 2$ GeV $\Rightarrow$ 1450 GeV $\lesssim m_{1/2} \lesssim$ 1850 GeV

$\delta m_t = 0.1$ GeV $\Rightarrow$ 1625 GeV $\lesssim m_{1/2} \lesssim$ 1725 GeV

6. Conclusinos

- LHC: $\delta m_t = 1 - 2$ GeV
  TESLA: $\delta m_t = 0.1$ GeV $\Rightarrow$ physics impact?

- m_t enters prediction of EWPO: M_W, $\sin^2\theta_{\rm eff}$
  δm_t yields leading parametric error
  $\delta m_t \lesssim 0.2$ GeV needed to match other unc.
  $\delta m_t \approx 0.1$ GeV needed to set constraints on new physics effects

- MSSM Higgs sector: $\Delta m_h^{\delta m_t} \approx \delta m_t$
  δm_t yields leading parametric error

- MSSM $\tilde{t}$ sector: use m_h as precision obs. to determine unknown parameters
  $\delta m_t = 0.1$ GeV gives improvement of ~ 3 in $m_{\tilde{t}_2}/A_t$ determination

- RG running in mSUGRA:
  - determination of GUT scale parameters:
    $\delta m_t = 0.1$ GeV gives impr. of ~ 4
  - strong m_t of μ parameter dependence

- Dark matter constraints in mSUGRA:
  $\delta m_t = 0.1$ GeV crucial to pin down mSUGRA parameters $\Rightarrow$ improvement in $m_{1/2}$ determination by more than ~ 4