Electroweak 1-loop Radiative Corrections to $e^+e^- \to b\bar{b}(\gamma)$

ECFA/DESY Linear Collider Workshop
Amsterdam

1 – 4 April 2003

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work done in collaboration with

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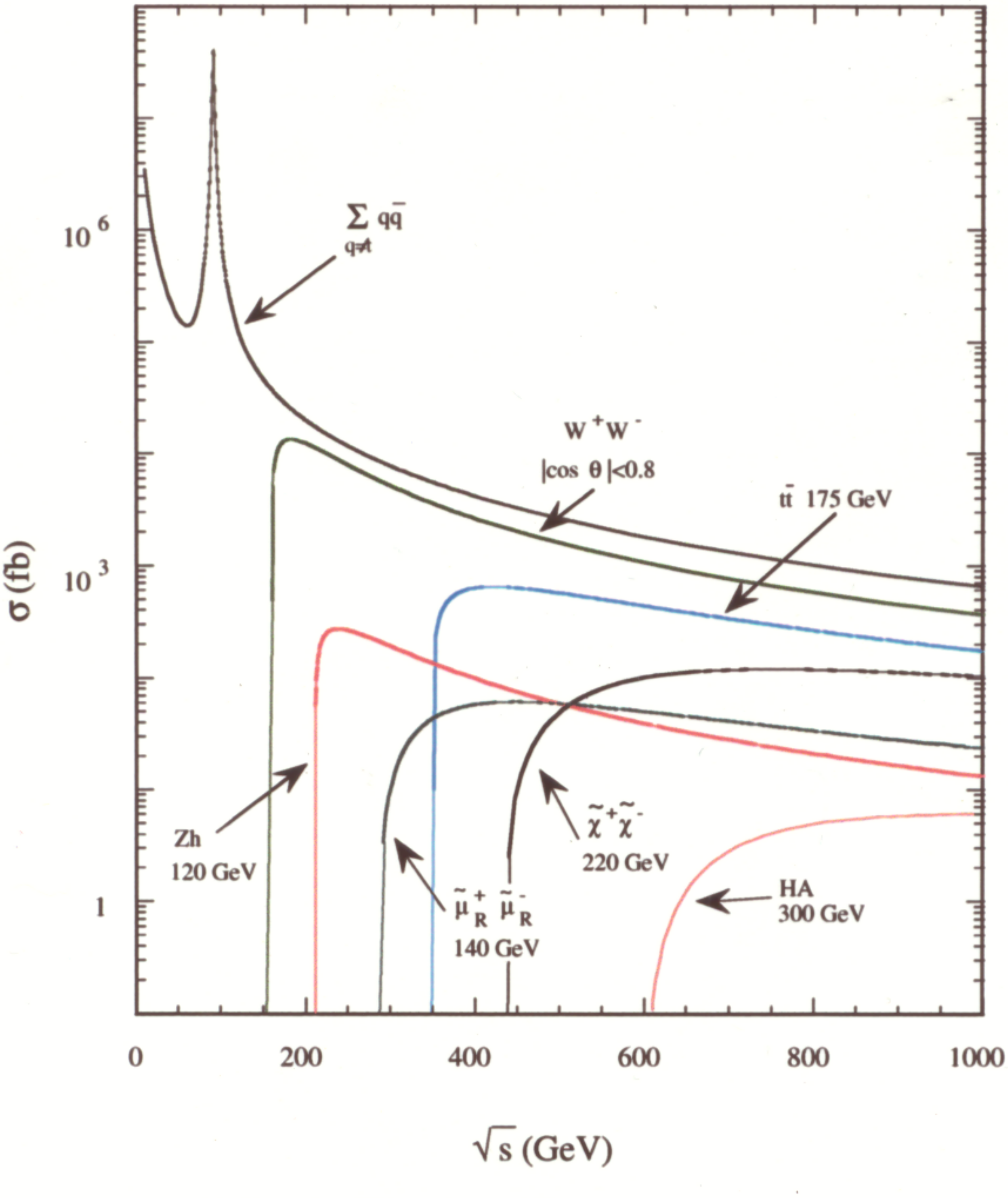
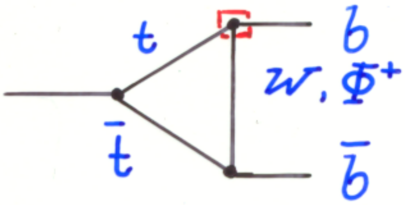
Motivation: LC production channels



TESLA Design Report, 2001

Motivation

- Next generation of colliders: $\sqrt{s} \approx 1\,\mathrm{TeV}$
- need to control theoretical uncertainties beyond leading order in α
- disentangle 'new physics' effects ↔ perturbative ones
- fermion pair production large cross section (Tesla TDR)
- ▷ missing ingredient to complete topfit project
  ew. one loop corrections to fermion pair production
- $b\bar{b}$ of particular interest: isospin partner of top-quark
- sensitive probe into physics beyond SM → mass-effects
- coupling to charged bosons might deviate from Standard Model predictions



To have reliable theoretical predictions for next generation of colliders it is indispensable to have various independent calculations relying on different tools

REALIZATION

- SM broken gauge theory but photon remains massless and U(1) unbroken

renormalization and evaluation of QED contributions is independent of details of full electroweak theory

→ gauge invariant subset of diagrams

program topfit.F contains QED and weak radiative corrections as separate libaries
usable independently or together

- QED (only photons and leptons)

→ virtual corrections
interference with born amplitude

→ real 'bremsstrahlungs' corrections
interference with other real emission amplitudes

IR (Infra Red = small photon momentum)
divergencies only cancel if real emission diagrams are taken into account

- WEAK (full SM particle content but photon)
→ virtual corrections

- focus here on virtual corrections
  choice of gauge: feynman gauge

Contributing diagrams:

- Tadpoles (One Point functions)

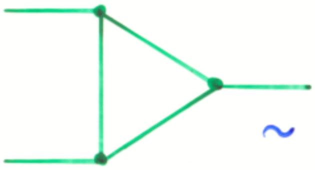
do not really contribute in feynman gauge

$$\sim \int \frac{d^N k}{[k^2 - m^2]}$$

- Selfenergies (Two Point functions)

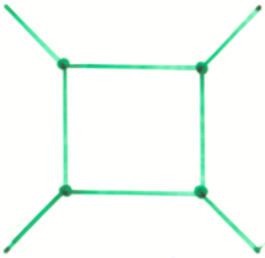
$$\sim \int \frac{d^N k \; (1, k_\mu, k_\mu k_\nu)}{[k^2 - m^2][(k - q_1)^2 - m_1^2]}$$

- Vertices (Three Point functions)



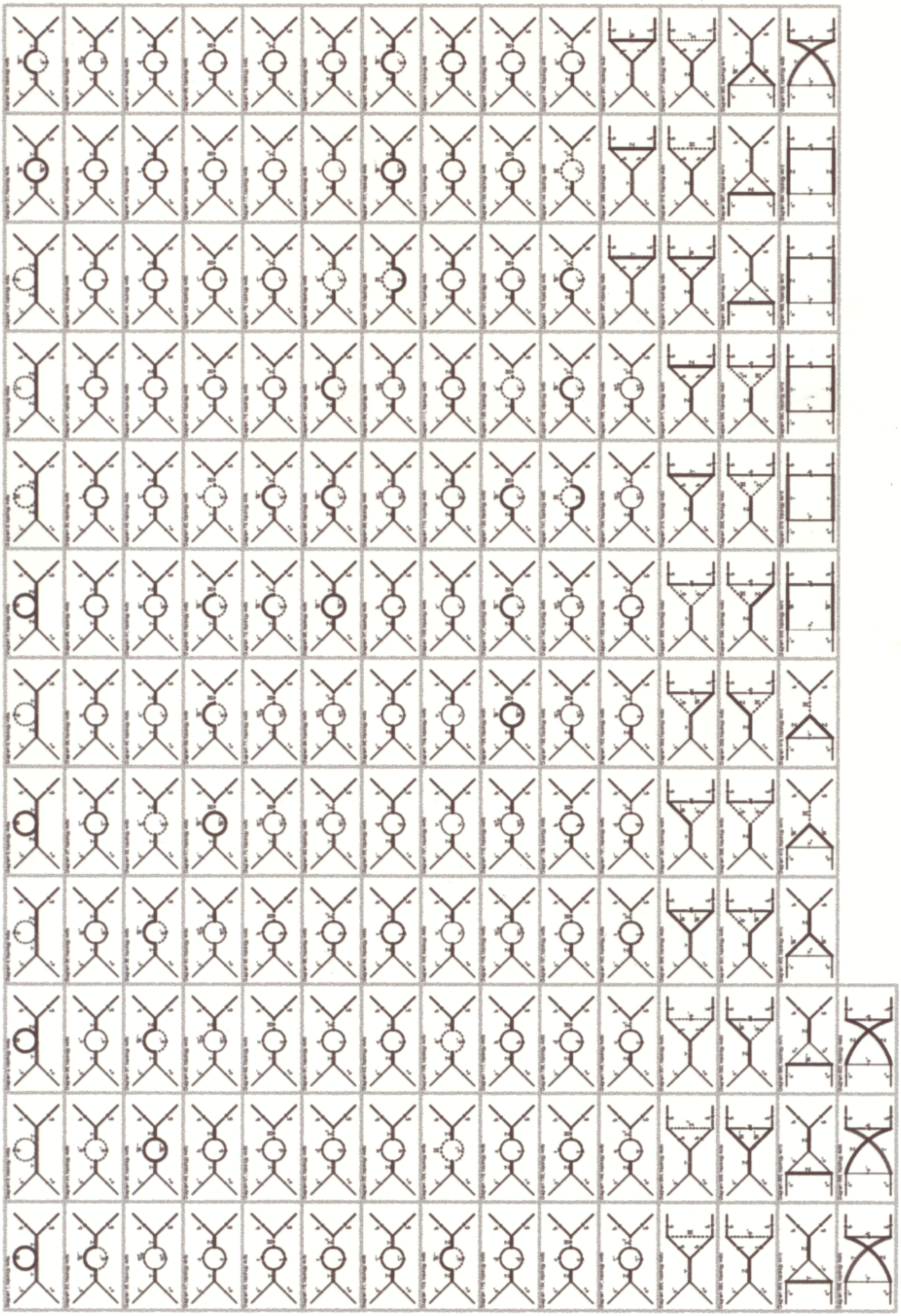
$$\sim \int \frac{d^N k \; (1, k_\mu, k_\mu k_\nu)}{[k^2 - m^2][(k - q_1)^2 - m_1^2][(k - q_2)^2 - m_2^2]}$$

- Boxes (Four Point functions)



$$\sim \int \frac{d^N k \; (1, k_\mu, k_\mu k_\nu)}{[k^2 - m^2][(k - q_1)^2 - m_1^2][(k - q_2)^2 - m_2^2][(k - q_3)^2 - m_3^2]}$$

Numerical evaluation: FF J. Vermaeseren and LoopTools T. Hahn



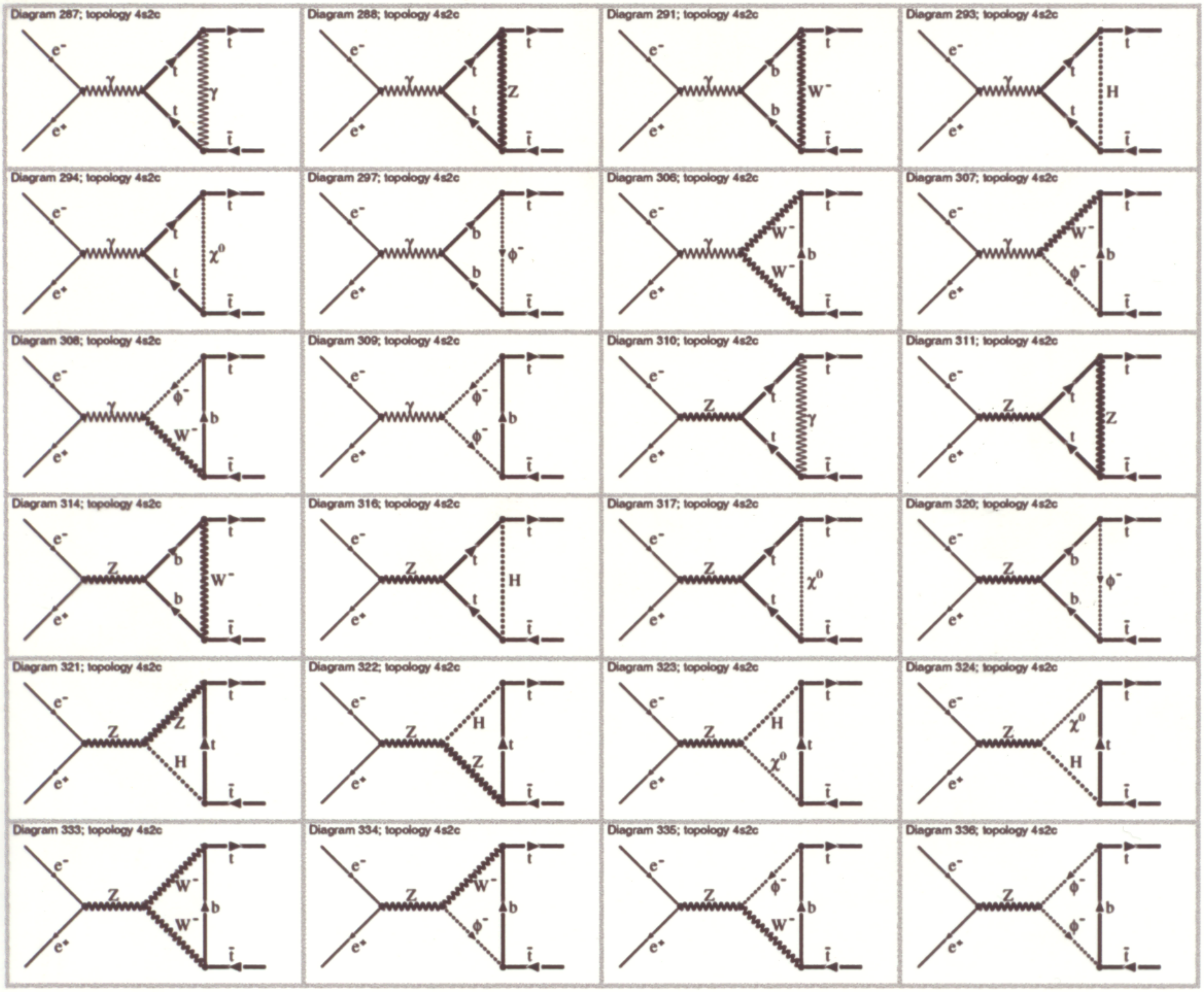


Diagram 287; topology 4s2c
e⁻ e⁺ γ t t γ t t̄
Diagram 288; topology 4s2c
e⁻ e⁺ γ t t Z t t̄
Diagram 291; topology 4s2c
e⁻ e⁺ γ b b W⁻ t t̄
Diagram 293; topology 4s2c
e⁻ e⁺ γ t t H t t̄
Diagram 294; topology 4s2c
e⁻ e⁺ γ t t χ⁰ t t̄
Diagram 297; topology 4s2c
e⁻ e⁺ γ b b φ⁻ t t̄
Diagram 306; topology 4s2c
e⁻ e⁺ γ W⁻ W⁻ b t t̄
Diagram 307; topology 4s2c
e⁻ e⁺ γ W⁻ φ⁻ b t t̄
Diagram 308; topology 4s2c
e⁻ e⁺ γ φ⁻ W⁻ b t t̄
Diagram 309; topology 4s2c
e⁻ e⁺ γ φ⁻ φ⁻ b t t̄
Diagram 310; topology 4s2c
e⁻ e⁺ Z t t γ t t̄
Diagram 311; topology 4s2c
e⁻ e⁺ Z t t Z t t̄
Diagram 314; topology 4s2c
e⁻ e⁺ Z b b W⁻ t t̄
Diagram 316; topology 4s2c
e⁻ e⁺ Z t t H t t̄
Diagram 317; topology 4s2c
e⁻ e⁺ Z t t χ⁰ t t̄
Diagram 320; topology 4s2c
e⁻ e⁺ Z b b φ⁻ t t̄
Diagram 321; topology 4s2c
e⁻ e⁺ Z Z H t t t̄
Diagram 322; topology 4s2c
e⁻ e⁺ Z H Z t t t̄
Diagram 323; topology 4s2c
e⁻ e⁺ Z H χ⁰ t t t̄
Diagram 324; topology 4s2c
e⁻ e⁺ Z χ⁰ H t t t̄
Diagram 333; topology 4s2c
e⁻ e⁺ Z W⁻ W⁻ b t t̄
Diagram 334; topology 4s2c
e⁻ e⁺ Z W⁻ φ⁻ b t t̄
Diagram 335; topology 4s2c
e⁻ e⁺ Z φ⁻ W⁻ b t t̄
Diagram 336; topology 4s2c
e⁻ e⁺ Z φ⁻ φ⁻ b t t̄

$e^+e^- \to t\bar{t}$

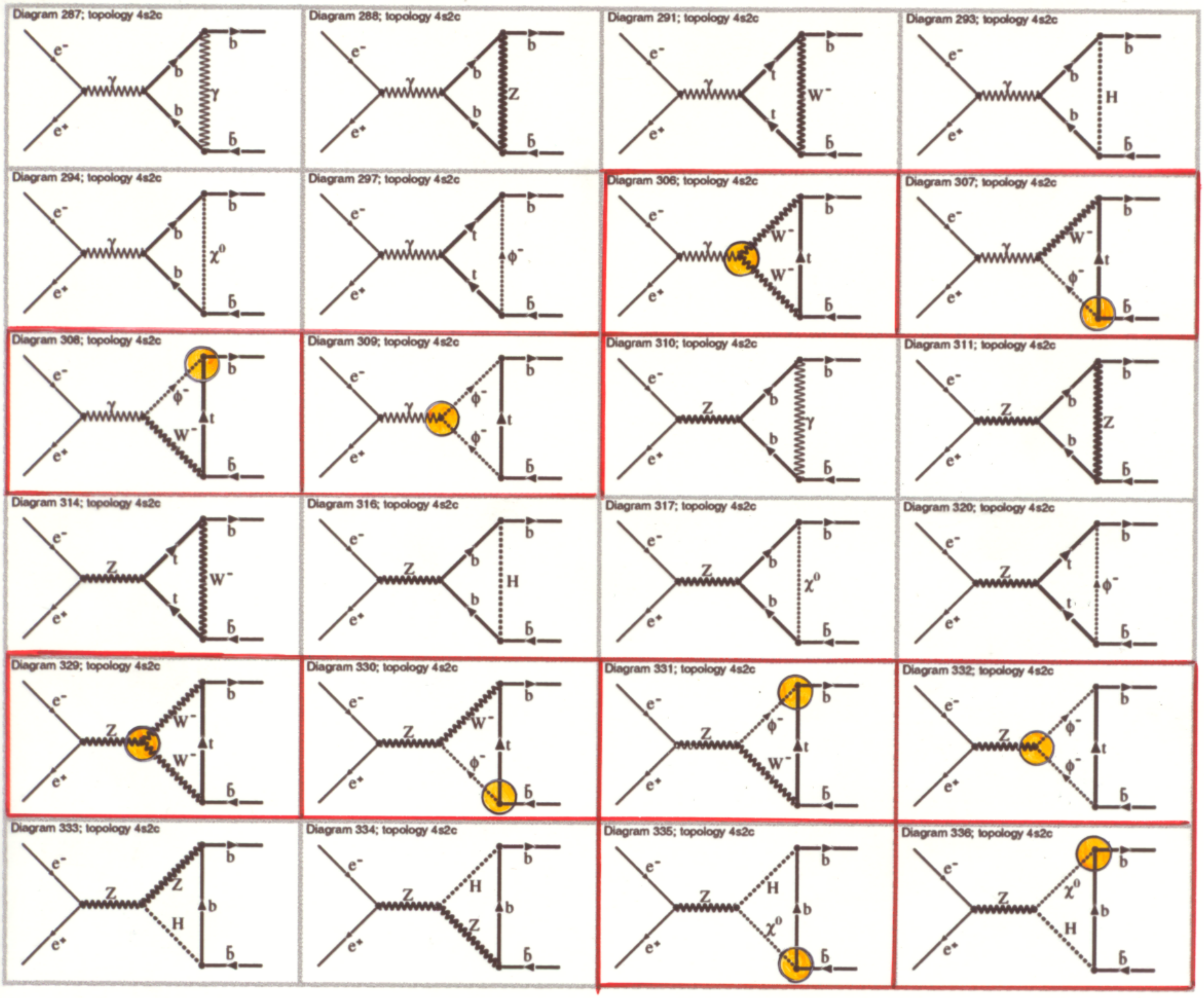

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Diagram 297; topology 4s2c
Diagram 306; topology 4s2c
Diagram 307; topology 4s2c
Diagram 308; topology 4s2c
Diagram 309; topology 4s2c
Diagram 310; topology 4s2c
Diagram 311; topology 4s2c
Diagram 314; topology 4s2c
Diagram 316; topology 4s2c
Diagram 317; topology 4s2c
Diagram 320; topology 4s2c
Diagram 329; topology 4s2c
Diagram 330; topology 4s2c
Diagram 331; topology 4s2c
Diagram 332; topology 4s2c
Diagram 333; topology 4s2c
Diagram 334; topology 4s2c
Diagram 335; topology 4s2c
Diagram 336; topology 4s2c

$e^+ e^- \to b\bar{b}$

PARAMETERIZATION

- Introduce basis for decomposition of the scattering matrix element into amplitudes: W. Hollik, W. Beenakker

$$M_1^{\rho\kappa} = [\bar{u}_t(p_1)\gamma^\mu\omega_\rho v_t(p_2)]\,[\bar{v}_e(k_2)\gamma_\mu\omega_\kappa u_e(k_1)]$$
$$M_2^{\rho\kappa} = [\bar{u}_t(p_1)\not{k}_2\omega_\rho v_t(p_2)]\,[\bar{v}_e(k_2)\not{p}_1\omega_\kappa u_e(k_1)]$$
$$M_3^{\rho\kappa} = [\bar{u}_t(p_1)\omega_\rho v_t(p_2)]\,[\bar{v}_e(k_2)\not{p}_1\omega_\kappa u_e(k_1)]$$
$$M_4^{\rho\kappa} = [\bar{u}_t(p_1)\gamma^\mu\not{k}_2\omega_\rho v_t(p_2)]\,[\bar{v}_e(k_2)\gamma_\mu\omega_\kappa u_e(k_1)]$$

For $m_e = 0$ (as assumed here) only 6 independent amplitudes of the 16 introduced above remain

→ 6 corresponding form factors $f_1^{\rho\kappa}$ and $f_3^{\rho(-\rho)}$

- differential cross section

$$\frac{d\sigma}{d\cos\theta} = \frac{N_c\beta}{128\pi s}\left[\left(M_1^{\rho\kappa} f_{1\,\mathrm{Born}}^{\rho\kappa}\right)^* \left(M_1^{\rho'\kappa'} f_1^{\rho'\kappa'} + M_3^{\rho'(-\rho')} f_3^{\rho'(-\rho')}\right) + \mathrm{h.c.}\right]$$

$$f_1^{\rho'\kappa'} = f_{1\,\mathrm{Born}}^{\rho'\kappa'} + f_{1\,\mathrm{real}}^{\rho'\kappa'} + f_{1\,\mathrm{virtuell}}^{\rho'\kappa'} + f_{1\,\mathrm{renormierung}}^{\rho'\kappa'}$$

→ libaries (containing form factors) are designed to be implemented in general purpos Monte Carlo programm

higher order corrections
QCD corrections etc.

Regularization and Renormalization

Some Integrals appear to be divergent

→ predictive power of perturbative calculation is lost!

- UV divergencies: high energetic gauge bosons
- IR divergencies: soft photons

dimensional regularisation

replace minkowski space-time dimension by $N < 4$

$\epsilon := \frac{4 \pm N}{2}$ parametrises divergencies

→ defines integral mathematically

- IR divergencies appearing from real and virtual corrections cancel order by order!

checked this analytically in our calculation

→ IR divergencies are under controll!

- UV divergencies are localised in $1/\epsilon$ poles

Theory needs to be renormalized

Physical observables are finite

- on-shell renormalization scheme relates the bare parameters which enter the Lagrangian (feynman rules) to the physical (i.e. measurable) parameters of the quantized renormalized Lagrangian

→ parameter (mass, charge, weak mixing angle) renormalization

→ wave function renormalization

RESULTS $e^+e^- \to b\,\bar{b}\,(\gamma)$

$\sqrt{s} = 500$ GeV

$\Gamma_Z = 2.49977\,\text{GeV}$, $\alpha = \frac{e^2}{4\pi} = 1/137.03599976$, $E_\gamma^{\max} = \sqrt{s}/10$,
$M_W = 80.4514958\,\text{GeV}$, $M_Z = 91.1867\,\text{GeV}$, $M_H = 120\,\text{GeV}$,
$m_e = 0.00051099907\,\text{GeV}$, $m_t = 173.8\,\text{GeV}$, $m_b = 4.7\,\text{GeV}$,
$m_\mu = 0.105658389\,\text{GeV}$, $m_u = 0.062\,\text{GeV}$, $m_d = 0.083\,\text{GeV}$,
$m_\tau = 1.77705\,\text{GeV}$, $m_c = 1.5\,\text{GeV}$, $m_s = 0.215\,\text{GeV}$.

$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{B+weak}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{All }\mathcal{O}(\alpha^3)}$	Group
−0.9	0.035 947 210 200	0.042 347 4	0.037 629 4	`topfit`
−0.9	0.035 947 210 200	0.042 347 8	0.037 629 8	*FeynArts*
−0.5	0.052 846 991 429	0.055 564 4	0.049 542 2	`topfit`
−0.5	0.052 846 991 429	0.055 564 7	0.049 542 4	*FeynArts*
0.0	0.134 448 437 257	0.135 139 0	0.121 176 2	`topfit`
0.0	0.134 448 437 256	0.135 139 3	0.121 176 5	*FeynArts*
0.5	0.283 246 237 852	0.291 227 2	0.264 541 2	`topfit`
0.5	0.283 246 237 851	0.291 227 8	0.264 541 8	*FeynArts*
0.9	0.450 665 853 761	0.482 564 5	0.447 083 2	`topfit`
0.9	0.450 665 853 760	0.482 565 4	0.447 084 1	*FeynArts*

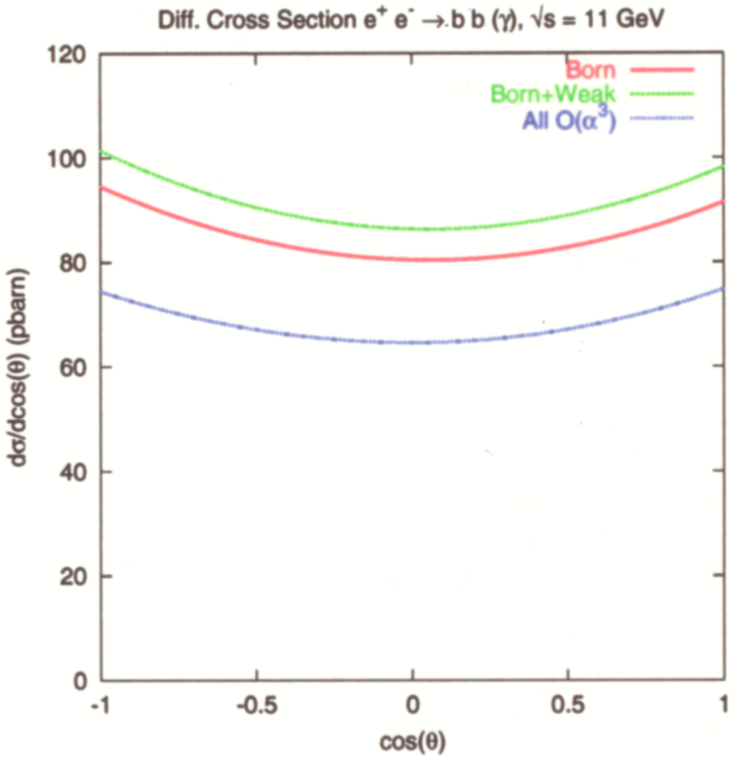
- independent agreement up to 6th digit!

FeynArts numbers gently supplied by Thomas Hahn Thanks

Results: Differential X Section

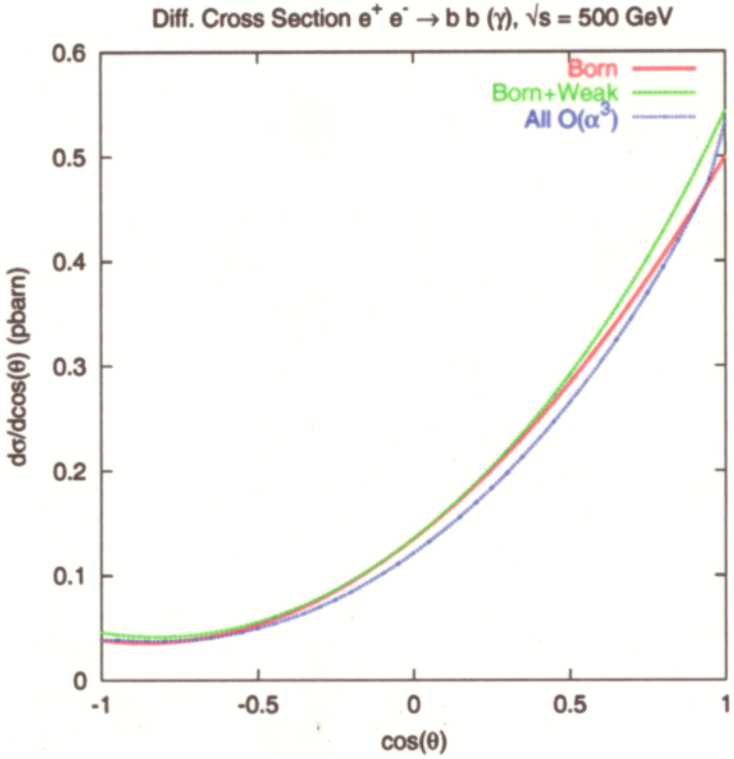
a) Max. threshold b−prod. (11 GeV)

(B-factories $E_{\rm max}(\gamma_{\rm soft}) = \sqrt{s}/100$)



b) High LC energy (500 GeV)

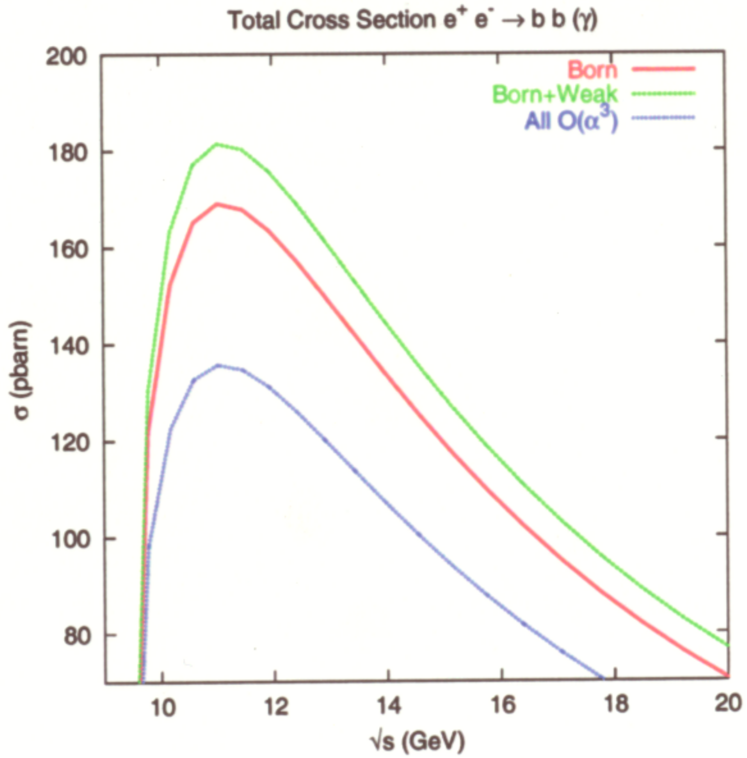
($E_{\rm max}(\gamma_{\rm soft}) = \sqrt{s}/10$)



Results: Total X Section

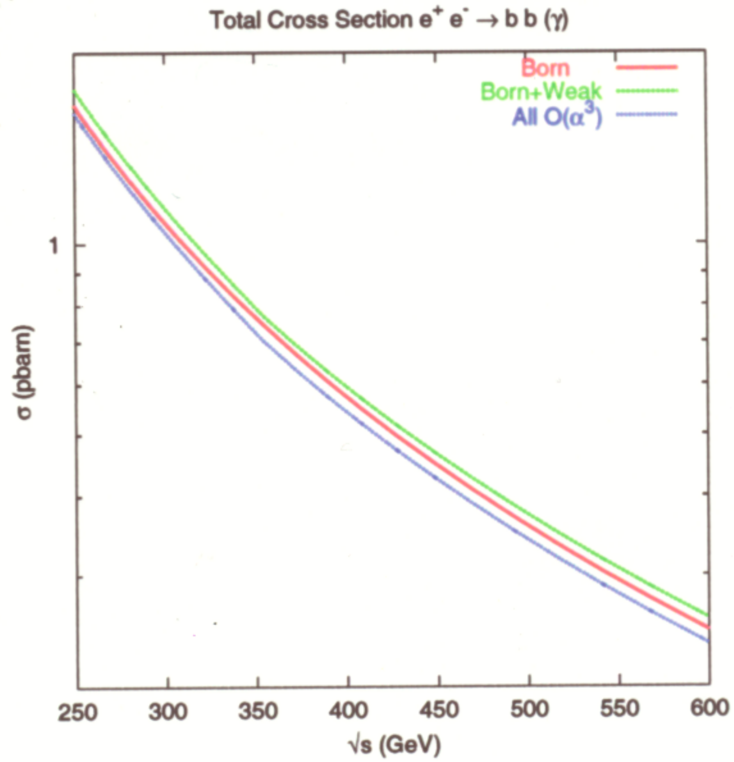
a) Threshold b–production pairs

(B-factories $E_{\max}(\gamma_{\rm soft}) = \sqrt{s}/100$)



b) Heavy weak box–vertex resonance

(Linear Collider $E_{\max}(\gamma_{\rm soft}) = \sqrt{s}/10$)



- **High radiative corrections at production threshold**
- **Around 2% at LC (experimentally reachable at GigaZ)**

Results: Numerical comparison

t–quarks pair production: $e^+ + e^- \to t + \bar{t}\ (+\gamma)$

at linear collider: $\sqrt{s} = 500$ GeV, $E_{max}(\gamma_{\rm soft}) = \sqrt{s}/10$

$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\rm Born}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\rm B+weak}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{{\rm All}\ \mathcal{O}(\alpha^3)}$	Group
−0.9	0.108 839 194 075	0.124 259 037 132	0.114 084 095 578	**topfit**
−0.9	0.108 839 194 075	0.124 259 037 133	0.114 084 095 578	*FeynArts*
−0.5	0.142 275 069 392	0.156 848 371 875	0.143 081 205 165	**topfit**
−0.5	0.142 275 069 392	0.156 848 371 875	0.143 081 205 164	*FeynArts*
0.0	0.225 470 464 033	0.240 266 804 030	0.217 188 009 766	**topfit**
0.0	0.225 470 464 032	0.240 266 804 029	0.217 188 009 765	*FeynArts*
0.5	0.354 666 470 332	0.368 886 506 993	0.329 337 273 949	**topfit**
0.5	0.354 666 470 332	0.368 886 506 991	0.329 337 273 947	*FeynArts*
0.9	0.491 143 715 767	0.503 337 511 605	0.442 908 167 346	**topfit**
0.9	0.491 143 715 766	0.503 337 511 601	0.442 908 167 343	*FeynArts*

Improvement of 3 more digits with respect to: hep-ph/0202109

FeynArts numbers gently supplied by Thomas Hahn, Thanks!

Results: Numerical agreement for c, b-quarks and τ lepton

- Born: ~ 12 **digits**
  - ✗ m_e not fully neglected at kinematics by Thomas.
  - ✓ Solved! Easy to restore by hand, full 16 digits agreement.
- Born+QED+Soft: ~ 11 **digits**
  - ✓ Achieved desired agreement.
- Born+Weak: ~ 6 **digits**
  - ✗ Higgs and unphysical gauge fixing fields $(\chi, \phi^{\pm})$ partially discarded in Thomas' comparisons:

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  Model -> "SMc", Restrictions -> NoLightFHCoupling]
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  - ? Other possible m_e effects (divergencies cancellations).
  - ✏ Waiting for further comparisons ...

Results: Numerical comparison

b–quarks pair production: $e^+ + e^- \to b + \bar{b}\ (+\gamma)$

at linear collider: $\sqrt{s} = 500$ GeV, $E_{max}(\gamma_{\text{soft}}) = \sqrt{s}/10$

$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{Born}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{B+weak}}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\text{All } \mathcal{O}(\alpha^3)}$	Group
−0.9	0.035 947 210 200	0.042 347 4	0.037 629 4	topfit
−0.9	0.035 947 210 200	0.042 347 8	0.037 629 8	*FeynArts*
−0.5	0.052 846 991 429	0.055 564 4	0.049 542 2	topfit
−0.5	0.052 846 991 429	0.055 564 7	0.049 542 4	*FeynArts*
0.0	0.134 448 437 257	0.135 139 0	0.121 176 2	topfit
0.0	0.134 448 437 256	0.135 139 3	0.121 176 5	*FeynArts*
0.5	0.283 246 237 852	0.291 227 2	0.264 541 2	topfit
0.5	0.283 246 237 851	0.291 227 8	0.264 541 8	*FeynArts*
0.9	0.450 665 853 761	0.482 564 5	0.447 083 2	topfit
0.9	0.450 665 853 760	0.482 565 4	0.447 084 1	*FeynArts*

Nice independent agreement up to 6th digit!

FeynArts numbers gently supplied by Thomas Hahn, Thanks!

Results: Numerical comparison

τ−lepton pair production: $e^+ + e^- \to \tau^- + \tau^+ \ (+\gamma)$

at linear collider: $\sqrt{s} = 500$ GeV, $E_{max}(\gamma_{\rm soft}) = \sqrt{s}/10$

$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\rm Born}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\rm B+weak}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{{\rm All}\ \mathcal{O}(\alpha^3)}$	Group
−0.9	0.094 591 021 718	0.108 606 0	0.092 419 0	topfit
−0.9	0.094 591 021 718	0.108 606 3	0.092 419 3	*FeynArts*
−0.5	0.089 298 531 177	0.100 256 8	0.866 994 8	topfit
−0.5	0.089 298 531 177	0.100 257 0	0.866 996 8	*FeynArts*
0.0	0.150 321 682 774	0.164 180 9	0.143 597 9	topfit
0.0	0.150 321 682 774	0.164 181 2	0.143 598 2	*FeynArts*
0.5	0.286 499 017 453	0.315 040 5	0.282 588 6	topfit
0.5	0.286 499 017 452	0.315 040 9	0.282 589 1	*FeynArts*
0.9	0.449 551 897 014	0.509 042 2	0.476 482 9	topfit
0.9	0.449 551 897 013	0.509 042 8	0.476 483 5	*FeynArts*

Nice independent agreement up to 6th digit!

FeynArts numbers gently supplied by Thomas Hahn, Thanks!

Results: Numerical comparison

c–quarks pair production: $e^+ + e^- \to c + \bar{c}\ (+\gamma)$

at linear collider: $\sqrt{s} = 500$ GeV, $E_{max}(\gamma_{\rm soft}) = \sqrt{s}/10$

$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\rm Born}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\rm B+weak}$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{{\rm All}\ \mathcal{O}(\alpha^3)}$	Group
−0.9	0.078 403 691 569	0.091 244 8	0.083 668 4	topfit
−0.9	0.078 403 691 569	0.091 245 0	0.083 668 5	*FeynArts*
−0.5	0.104 111 287 582	0.116 501 6	0.105 902 0	topfit
−0.5	0.104 111 287 581	0.116 501 7	0.105 902 2	*FeynArts*
0.0	0.247 708 288 845	0.262 558 0	0.234 481 6	topfit
0.0	0.247 708 288 844	0.262 558 2	0.234 481 8	*FeynArts*
0.5	0.515 152 519 273	0.530 949 6	0.463 714 2	topfit
0.5	0.515 152 519 271	0.530 949 9	0.463 714 5	*FeynArts*
0.9	0.818 277 908 613	0.830 434 3	0.700 270 0	topfit
0.9	0.818 277 908 611	0.830 434 9	0.700 270 2	*FeynArts*

Nice independent agreement up to 6th digit!

FeynArts numbers gently supplied by Thomas Hahn, Thanks!

Summary

Calculated the one-loop electroweak radiative correction to $e^+e^- \rightarrow b\bar{b}\,(\gamma)$

- Last building block for complete fermion-pair generator
- Based upon code topfit for $e^+e^- \rightarrow t\bar{t}\,(\gamma)$
- Diana to create feynman diagrams in symbolic notation
- Form to manipulate the files and simplify the γ-structure

  extract the form factors for each helicity amplitude
- dimensional regularization: parametrize the divergencies
- on-shell renormalization to cancel UV divergencies
- finite 1-loop amplitudes containing kinematics of process and various Passarino-Veltmann integrals
- Looptools to evaluate the One-, Two-, Three- and Four Point functions numerically
- Fortran to 'square' the amplitudes and obtain cross section (differential or total)

Our calculation in agreement with FeynArts team