Tree and one-loop tests

of the minimal LR-symmetric model

Basic issue of the talk, also important for fitting New Physics Models in high energies (e.g. using TESLA)

- Tree level processes: restoration of the results of the Standard Model for $M_{\text{heavy,NS}} \to \infty$
- One-loop (and beyond) processes: is the same true there?

Muon decay, SM:

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2 s_W^2 M_W^2}(1 + \Delta r_{SM}).$$

Muon decay, LR:

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2 s_W^2 M_W^2}(1 + \Delta r_{LR}),$$

$$\Delta r_{LR} \stackrel{?}{=} \Delta r_{SM} + {}''\text{subleading effects from h.o.c.}''$$

W_R (Right-Handed W Boson) MASS LIMITS

Assuming a light right-handed neutrino, except for BEALL 82, LANGACKER 89B, and COLANGELO 91. $g_R = g_L$ assumed. [Limits in the section MASS LIMITS for W' below are also valid for W_R if $m_{\nu_R} \ll m_{W_R}$.] Some limits assume manifest left-right symmetry, *i.e.*, the equality of left- and right Cabibbo-Kobayashi-Maskawa matrices. For a comprehensive review, see LANGACKER 89B. Limits on the W_L-W_R mixing angle ζ are found in the next section. Values in brackets are from cosmological and astrophysical considerations and assume a light right-handed neutrino.

VALUE (GeV)	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
> 715	90	[11] CZAKON	99	RVUE	Electroweak
> 310	90	[12] THOMAS	01	CNTR	β^+ decay
> 137	95	[13] ACKERSTAFF	99D	OPAL	τ decay
>1400	68	[14] BARENBOIM	98	RVUE	Electroweak, Z-Z' mixing
> 549	68	[15] BARENBOIM	97	RVUE	μ decay
> 220	95	[16] STAHL	97	RVUE	τ decay
> 220	90	[17] ALLET	96	CNTR	β^+ decay
> 281	90	[18] KUZNETSOV	95	CNTR	Polarized neutron decay
> 282	90	[19] KUZNETSOV	94B	CNTR	Polarized neutron decay
> 439	90	[20] BHATTACH...	93	RVUE	Z-Z' mixing
> 250	90	[21] SEVERIJNS	93	CNTR	β^+ decay
		[22] IMAZATO	92	CNTR	K^+ decay
> 475	90	[23] POLAK	92B	RVUE	μ decay
> 240	90	[24] AQUINO	91	RVUE	Neutron decay
> 496	90	[24] AQUINO	91	RVUE	Neutron and muon decay
> 700		[25] COLANGELO	91	THEO	$m_{K^0_L} - m_{K^0_S}$
> 477	90	[26] POLAK	91	RVUE	μ decay
[none 540–23000]		[27] BARBIERI	89B	ASTR	SN 1987A; light ν_R
> 300	90	[28] LANGACKER	89B	RVUE	General
> 160	90	[29] BALKE	88	CNTR	$\mu \rightarrow e\nu\overline{\nu}$
> 406	90	[30] JODIDIO	86	ELEC	Any ζ
> 482	90	[30] JODIDIO	86	ELEC	$\zeta = 0$
> 800		MOHAPATRA	86	RVUE	$SU(2)_L \times SU(2)_R \times U(1)$
> 400	95	[31] STOKER	85	ELEC	Any ζ
> 475	95	[31] STOKER	85	ELEC	ζ <0.041
		[32] BERGSMA	83	CHRM	$\nu_\mu e \rightarrow \mu\nu_e$
> 380	90	[33] CARR	83	ELEC	μ^+ decay
>1600		[34] BEALL	82	THEO	$m_{K^0_L} - m_{K^0_S}$
[> 4000]		STEIGMAN	79	COSM	Nucleosynthesis; light ν_R

1. Short introduction to the MLRM
2. Muon decay at tree level
3. Muon decay at one loop level
4. Conclusions and Outlook: how to fit [some] beyond SM

Muon decay, MLRM:

M. Czakon, J. Gluza, M. Zrałek, Nucl. Phys. B573, 57 (2000),

M. Czakon, J. Gluza, J. Hejczyk, Nucl. Phys. B642, 157 (2002).

"CONFRONTING ELECTROWEAK PRECISION MEASUREMENTS WITH NEW PHYSICS MODELS",

M. Czakon, J. Gluza, F. Jegerlehner, M. Zrałek, Eur. Phys. J. C13, 275 (2000).

1. Introduction to the LRM

The model has been constructed in 1973-1974,

Pati, Salam, Senjanovic, Mohapatra

$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge group

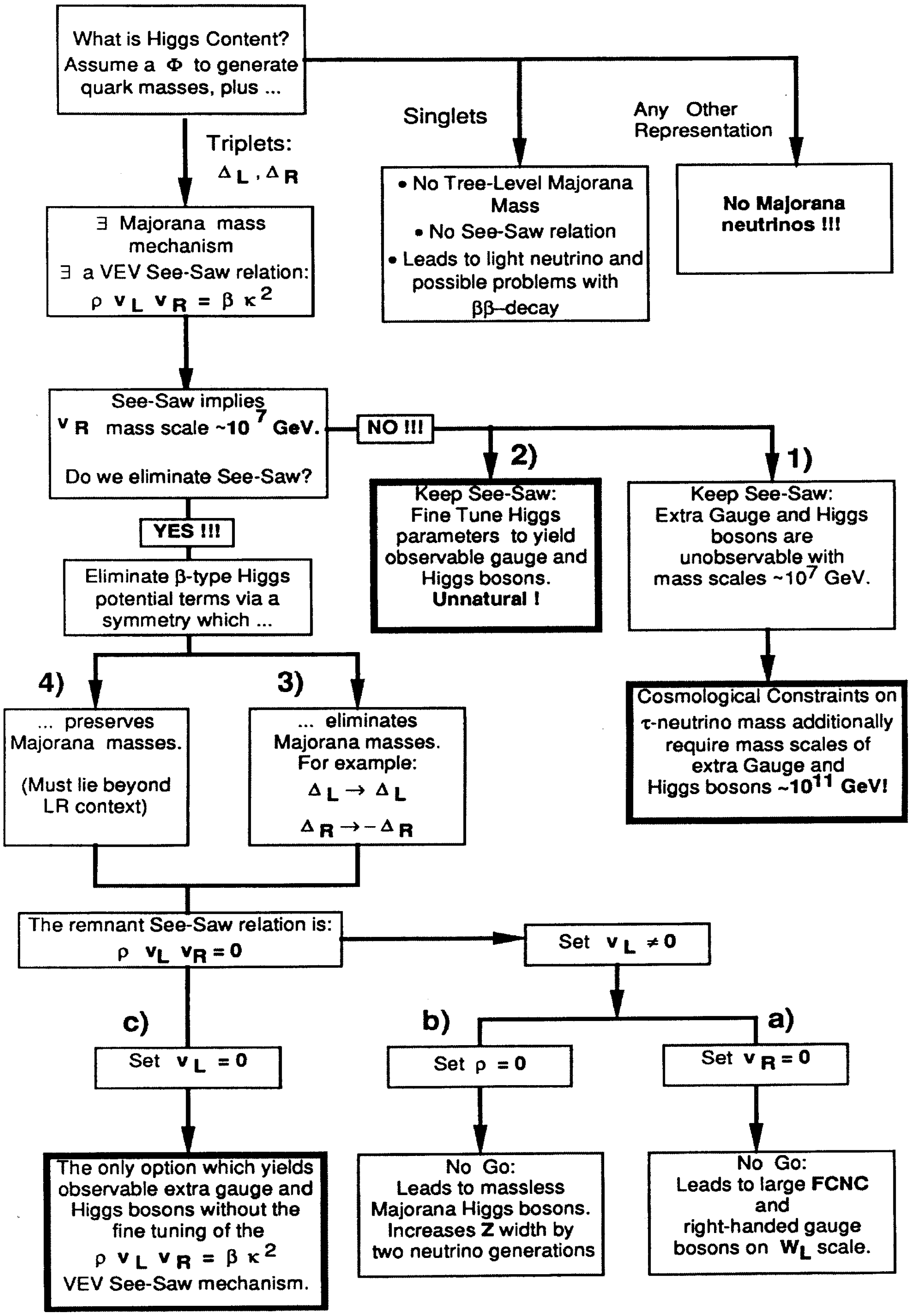
(i) restore lepton-quark symmetry to the weak interactions:

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}, \quad \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

(ii) Interpretation of the hypercharge as the difference of the baryon and lepton numbers:

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2}$$

$$\begin{array}{ccc} W_L^{\pm}, W_L^0 & & W_1^{\pm}, W_2^{\pm} \\ W_R^{\pm}, W_R^0 & \rightarrow [SSB?] & Z_1, Z_2 \\ B^0 & & \gamma \end{array}$$


What is Higgs Content? Assume a Φ to generate quark masses, plus ...
Triplets: Δ_L, Δ_R
Singlets
Any Other Representation
• No Tree-Level Majorana Mass
• No See-Saw relation
• Leads to light neutrino and possible problems with ββ–decay
No Majorana neutrinos !!!
∃ Majorana mass mechanism
∃ a VEV See-Saw relation: $\rho\, v_L\, v_R = \beta\, \kappa^2$
See-Saw implies v_R mass scale ~10^7 GeV.
Do we eliminate See-Saw?
NO !!!
2)
1)
Keep See-Saw: Fine Tune Higgs parameters to yield observable gauge and Higgs bosons. Unnatural !
Keep See-Saw: Extra Gauge and Higgs bosons are unobservable with mass scales ~10^7 GeV.
YES !!!
Eliminate β-type Higgs potential terms via a symmetry which ...
Cosmological Constraints on τ-neutrino mass additionally require mass scales of extra Gauge and Higgs bosons ~10^{11} GeV!
4)
3)
... preserves Majorana masses. (Must lie beyond LR context)
... eliminates Majorana masses. For example: $\Delta_L \to \Delta_L$ $\Delta_R \to -\Delta_R$
The remnant See-Saw relation is: $\rho\, v_L\, v_R = 0$
Set $v_L \neq 0$
c)
b)
a)
Set $v_L = 0$
Set $\rho = 0$
Set $v_R = 0$
The only option which yields observable extra gauge and Higgs bosons without the fine tuning of the $\rho\, v_L\, v_R = \beta\, \kappa^2$ VEV See-Saw mechanism.
No Go: Leads to massless Majorana Higgs bosons. Increases Z width by two neutrino generations
No Go: Leads to large FCNC and right-handed gauge bosons on W_L scale.

The minimal Higgs sector consists of two triplets and one bidoublet

$$\Delta_{L,R} = \begin{pmatrix} \delta^{+}_{L,R}/\sqrt{2} & \delta^{++}_{L,R} \\ \delta^{0}_{L,R} & -\delta^{+}_{L,R}/\sqrt{2} \end{pmatrix},$$

$$\Phi = \begin{pmatrix} \phi^0_1 & \phi^+_1 \\ \phi^-_2 & \phi^0_2 \end{pmatrix}.$$

with vacuum expectation values allowed for the neutral particles

$$\frac{v_L}{\sqrt{2}} = \langle \delta^0_L \rangle,$$

$$\text{new HE scale}: \frac{v_R}{\sqrt{2}} = \langle \delta^0_R \rangle,$$

$$\text{SM VEV scale}: \sqrt{\kappa_1^2 + \kappa_2^2}$$

$$\frac{\kappa_1}{\sqrt{2}} = \langle \phi^0_1 \rangle,$$

$$\frac{\kappa_2}{\sqrt{2}} = \langle \phi^0_2 \rangle.$$

- The result is 20 real scalar fields, of which 14 are physical (the rest are Goldstone bosons):
  - 4 neutral scalars: $H_0^0, H_1^0, H_2^0, H_4^0$, (the first can be considered to be the light Higgs of the SM at tree level),
  - 2 neutral pseudo-scalars: A_1^0, A_2^0,
  - 2 charged scalars: $H_1^{\pm}, H_2^{\pm}$,
  - 2 doubly-charged scalars: $H_1^{\pm\pm}, H_2^{\pm\pm}$.
- see-saw mechanism for the generation of light neutrino masses, with specific SB sectors. The neutrino mass matrix

$$M_\nu = \begin{pmatrix} M_L(v_L) & M_D(\kappa_{1,2}) \\ M_D^T & M_R(v_R) \end{pmatrix}$$

with $M_L \ll M_D \ll M_R$.

$$\begin{aligned} m_N &\sim M_R \\ m_{\text{light}} &\sim M_D^2/M_R \end{aligned}$$

$M_D \sim \mathcal{O}(1)\ \mathrm{GeV} \rightarrow \mathrm{M_R} \sim 10^{15}\ \mathrm{GeV}$, if light neutrino masses of the order of 0.1 eV.

- The SSB leads to the following gauge-boson mass matrices

$$\frac{g^2}{4}\begin{pmatrix} \kappa_+^2 & -2\kappa_1\kappa_2 \\ -2\kappa_1\kappa_2 & \kappa_+^2+2v_R^2 \end{pmatrix},$$

$$\frac{1}{2}\begin{pmatrix} \frac{1}{2}g^2\kappa_+^2 & -\frac{1}{2}g^2\kappa_+^2 & 0 \\ -\frac{1}{2}g^2\kappa_+^2 & \frac{1}{2}g^2(\kappa_+^2+4v_R^2) & -2gg'v_R^2 \\ 0 & -2gg'v_R^2 & 2g'^2v_R^2 \end{pmatrix}$$

- The diagonalization introduces various mixing angles. It is possible to write the respective matrices as

$$\begin{pmatrix} W_L^\pm \\ W_R^\pm \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix}\begin{pmatrix} W_1^\pm \\ W_2^\pm \end{pmatrix},$$

$$\begin{pmatrix} W_L^3 \\ W_R^3 \\ B \end{pmatrix} = \begin{pmatrix} c_W c & c_W s & s_W \\ -s_W s_M c - c_M s & -s_W s_M s + c_M c & c_W s_M \\ -s_W c_M c + s_M s & -s_W c_M s - s_M c & c_W c_M \end{pmatrix}\begin{pmatrix} Z_1 \\ Z_2 \\ A \end{pmatrix},$$

where:

$$c_W \equiv \cos\theta_W, \quad s_W \equiv \sin\theta_W, \quad s \equiv \sin\phi, \quad c \equiv \cos\phi$$

$$c_M \equiv \frac{\sqrt{\cos 2\theta_W}}{\cos\theta_W}. \quad s_M \equiv \tan\theta_W$$

2. Tree level muon decay

Goal: getting contraints on heavy sector (masses of heavy neutrinos, masses of gauge boson masses their mixings).

Tree level diagrams to the muon decay:

- exchanged W_1,
- exchanged W_2,
- exchanged charged Higgs particles.

We assume:

- no mixing in the charged gauge sector: $\kappa_2 = 0, \quad \xi = 0$,
- mixings among light and heavy neutrino states are neglected

Effects of heavy neutrinos remain:

- $Z_1 N_i N_i$, $Z_2 N_i N_i$ (build up gauge-boson self energies);
- $\bar{e} N_1 H_2^-$: $\quad m_{N_4} \frac{g}{2M_{W_2}} \frac{\kappa_1}{v_R} P_L$,
- $\bar{e} N_1 H_1^-$: $\quad \sqrt{2} \frac{m_{N_4}}{v_R} P_R$

Nice feature: constraints on the heavy (right) sector come directly from 1-loop level!

$$
\begin{aligned}
-\mathcal{L} &= \bar{c}_{LL}\bar{e}\gamma_\alpha P_L \nu_e \bar{\nu}_\mu \gamma^\alpha P_L \mu + \bar{c}_{RR}\bar{e}\gamma_\alpha P_R \nu_e \bar{\nu}_\mu \gamma^\alpha P_R \mu \\
&+ \bar{c}_{LR}\bar{e}\gamma_\alpha P_L \nu_e \bar{\nu}_\mu \gamma^\alpha P_R \mu + \bar{c}_{RL}\bar{e}\gamma_\alpha P_R \nu_e \bar{\nu}_\mu \gamma^\alpha P_L \mu.
\end{aligned}
$$

where:

$$
\begin{aligned}
\bar{c}_{LL} &= \frac{g^2}{2M_{W_1}^2}(\cos^2\xi + \beta\sin^2\xi), \\
\bar{c}_{RR} &= \frac{g^2}{2M_{W_1}^2}(\sin^2\xi + \beta\cos^2\xi), \\
\bar{c}_{RL} = \bar{c}_{LR} &= \frac{g^2}{2M_{W_1}^2}(-1+\beta)\sin\xi\cos\xi.
\end{aligned}
$$

$\beta = \frac{M_{W_1}^2}{M_{W_2}^2}$,

To have neutrino mixings properly included:

$$
U = \begin{pmatrix} K_L^T \\ K_R^\dagger \end{pmatrix} = \begin{pmatrix} O(1) & O(1/M_N) \\ O(1/M_N) & O(1) \end{pmatrix}.
$$

$$
\begin{aligned}
c_{LL} &= \bar{c}_{LL}(K_L^\dagger)_{ea}(K_L)_{b\mu}, \\
c_{RR} &= \bar{c}_{RR}(K_R^\dagger)_{ea}(K_R)_{b\mu}, \\
c_{LR} &= \bar{c}_{LR}(K_L^\dagger)_{ea}(K_R)_{b\mu}, \\
c_{RL} &= \bar{c}_{RL}(K_R^\dagger)_{ea}(K_L)_{b\mu}.
\end{aligned}
$$

then: $c_{LL} \gg c_{RR}, c_{LR}, c_{RL}$

and

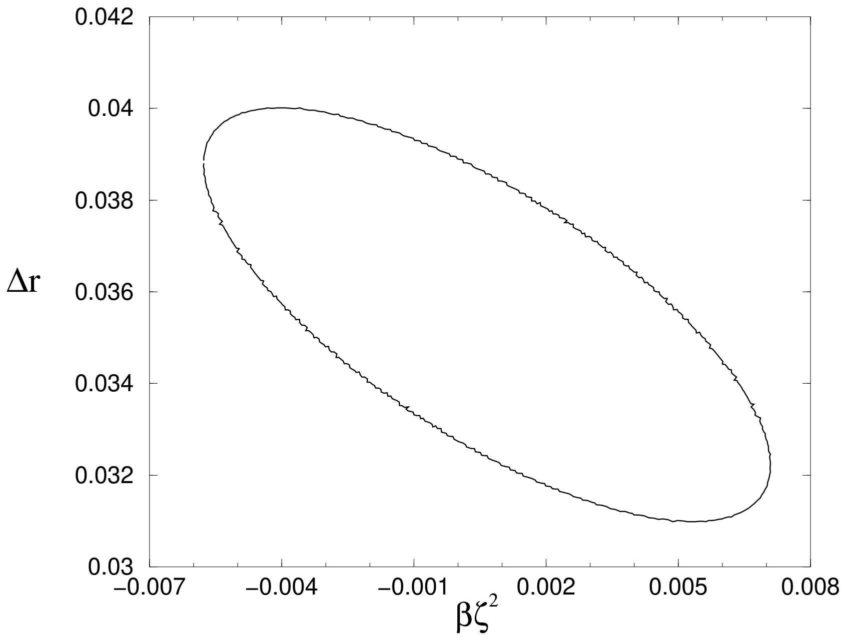
$$
\begin{aligned}
\frac{G_F}{\sqrt{2}} &\simeq \frac{|c_{LL}|}{4} \\
&= \frac{\pi\alpha}{2s_W^2 M_{W_1}^2 (1-\Delta r)}(cos^2\xi + \beta \sin^2\xi) \\
&\simeq \frac{\pi\alpha}{2s_W^2 M_{W_1}^2 (1-\Delta r)}(1+\beta\xi^2).
\end{aligned}
\tag{1}
$$

Standard approach: to make the fitting procedure of the ξ and β parameters possible at all at the tree-level, we have to naively rely on SM corrections:

$M_{W_1} = 80.446 \pm 0.040$ GeV, $\Delta r = 0.0355 \pm 0.0021$ and

$$
s_W^2 \equiv (s_W^2)_{SM} = 1 - \frac{M_{W_1}^2}{M_{Z_1}^2} = 0.2228 \pm 0.0004
$$

.



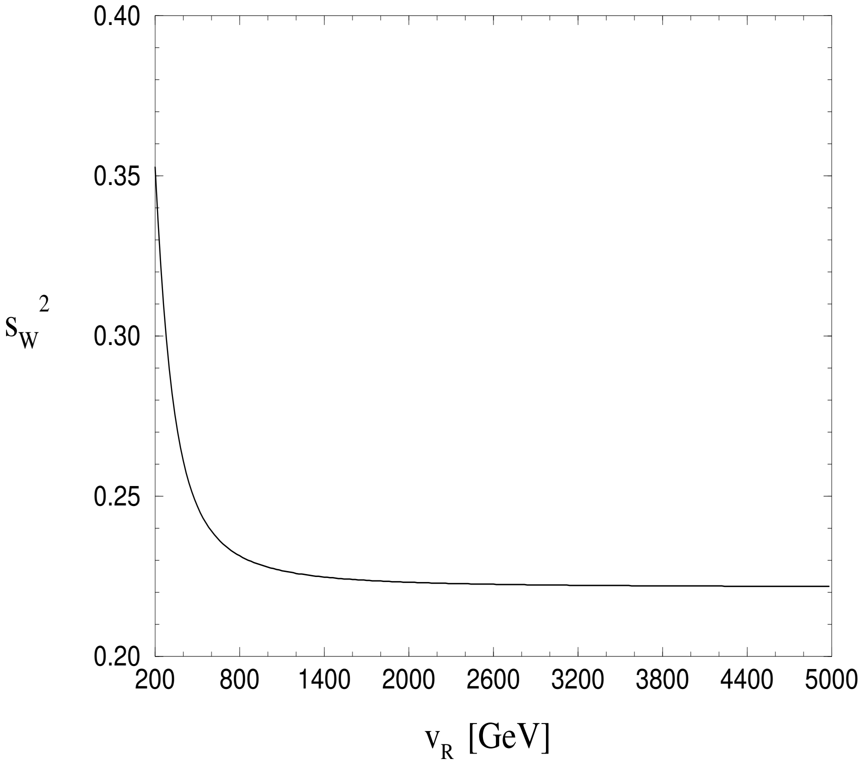
with the very worst bound $\zeta < .1$, it gives $\beta < 1 \quad \rightarrow M_{W_2} \geq M_{W_1}$

without heavy neutrino mixing (means $\sum_{i=1}^{6} m_{N_i} \leq m_\mu$):

$$\frac{G_F}{\sqrt{2}} = |c_{LL}|^2 + |c_{LR}|^2 + |c_{RL}|^2 + |c_{RR}|^2$$
$$= \frac{\pi\alpha}{2s_W^2 M_W^2, (1-\Delta_R)}(1+\beta^2)$$

and $M_{W_2} \geq 500 \text{ GeV}(!)$

3. One loop level calculations



In MLRM s_W^2 is expressable by M_{W_1},M_{Z_1}, and a heavy scale v_R,

SM:

$$s_W^2 = 1 - \frac{M_{W_1}^2}{M_{Z_1}^2}$$

We include the change of s_W^2 from the SM to the LRM in Δr,

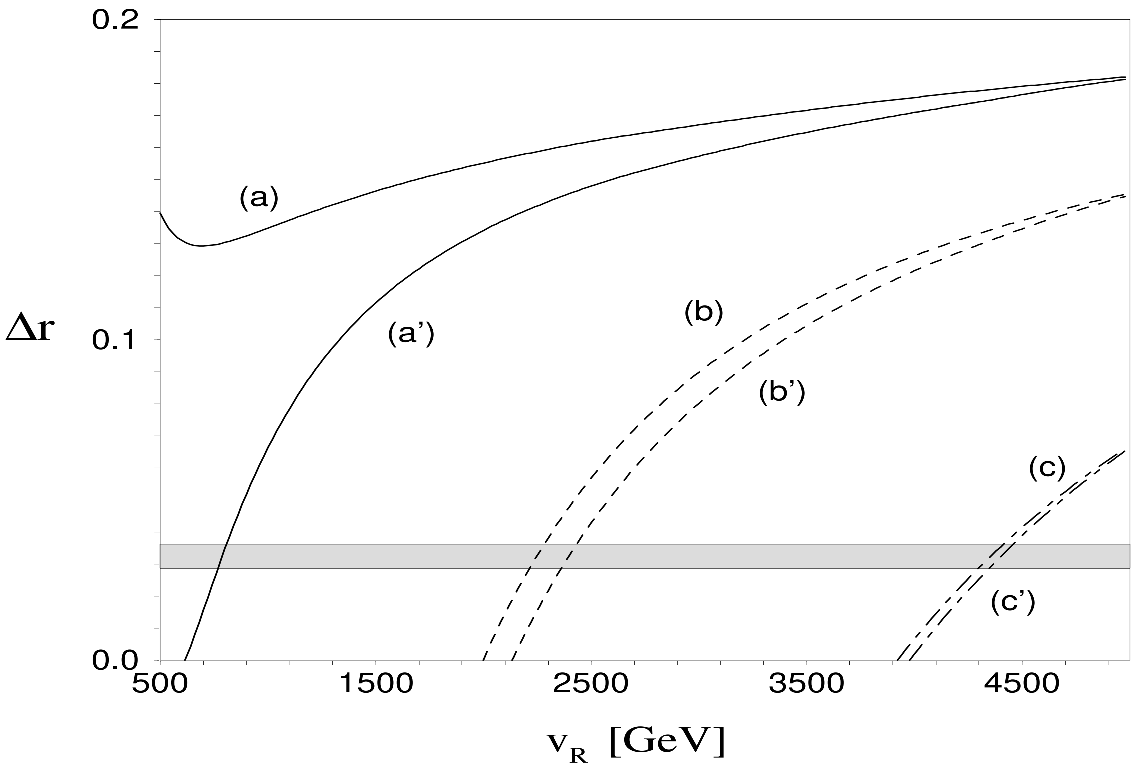
$$\frac{e^2}{(8M_{W_1}^2(s_W)_{SM}^2)}(1+\Delta r) = \frac{e^2}{(8M_{W_1}^2(s_W)_{LR}^2)}(1+\Delta r_{LR})$$

then:

$$\begin{aligned}
\Delta r &= \frac{(s_W^2)_{SM}}{(s_W^2)_{LR}}(\Delta r)_{LR} + \frac{(s_W^2)_{SM}}{(s_W^2)_{LR}} - 1, \\
(\Delta r)_{LR} &= \frac{-\Pi_{W_1}^T(0) - \delta M_{W_1}^2}{M_{W_1}^2} + 2\frac{\delta e}{e} - \frac{(\delta s_W^2)_{LR}}{(s_W^2)_{LR}} + \delta
\end{aligned}$$

δ - complete vertex, box and external line corrections within MLRM.

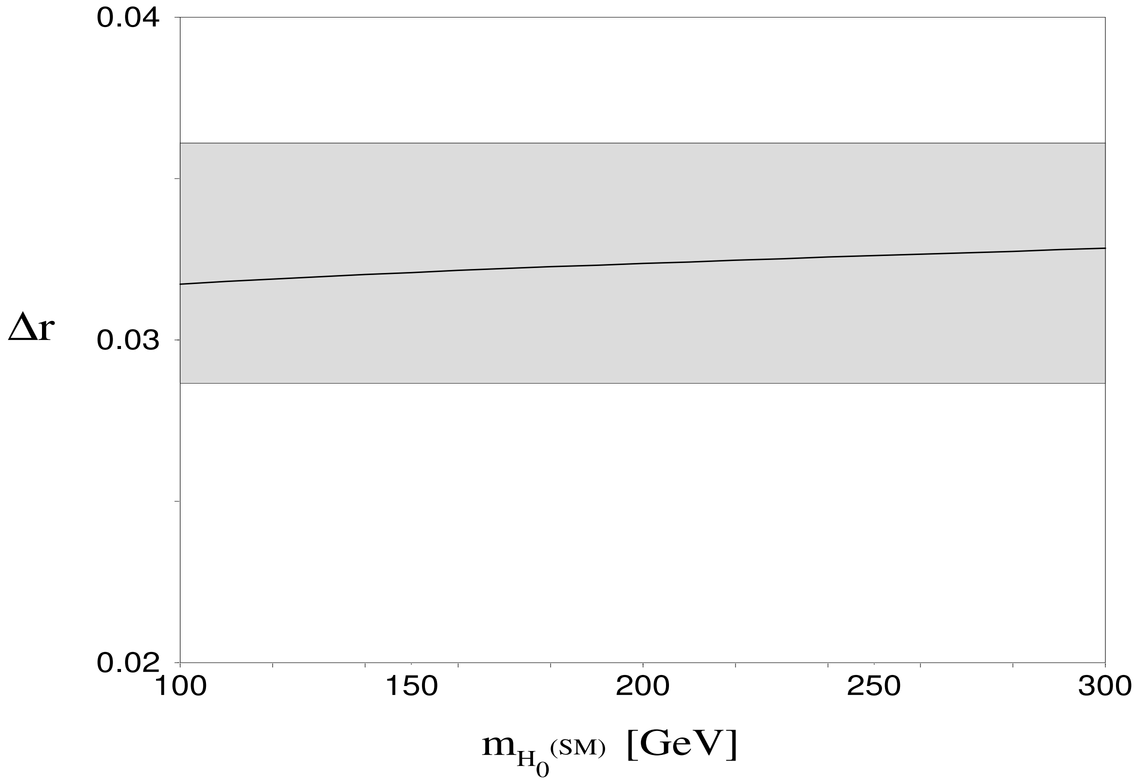
All Higgs particle masses equal (apart from H_0^0):



Sets with and without primes show results for three heavy neutrino masses with $m_N = 100$ GeV and $m_N = 2$ TeV respectively.

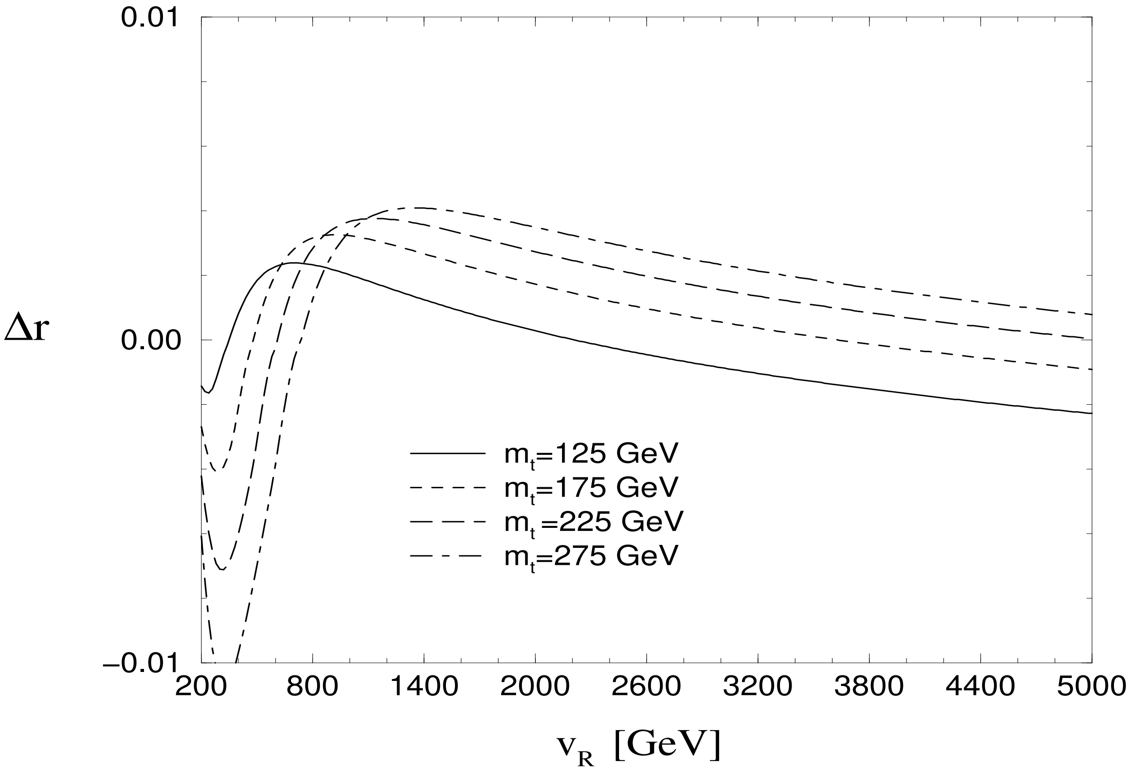
for $M_H \simeq 2v_R$ (and large v_R), Δr can be accomodated, remark: fine tuned masses

Variaton of $m(H_0^0)$ is negligible:



Δr as function of the lightest Higgs scalar mass $M_{H_0^0}$. The gray area shows the experimentally allowed values of Δr and the heavy particle spectrum is chosen to fit approximately to this region, namely, $v_R = 2390$ GeV, $m_N = 2$ TeV, $m_H = 5$ TeV.

m_t is effectively light and is not predictable by Δr in LRM:



The contribution of the third quark family to Δr as function of v_R for different top quark masses.

Leading terms:

$$(\Delta r)^{top}_{SM} \simeq \frac{m_t^2}{M_W^2}$$

$$(\Delta r)^{top}_{LRM} \simeq \frac{m_t^2}{M_{W_2}^2 - M_{W_1}^2}$$

4. Conclusions and Outlook: how to fit [some] beyond SM

LRM can be fitted to muon decay width, however:

- heavy neutrino masses and Higgs masses must be very tuned to get proper Δr
- corrections coming from SM particles within LR model do not constitute a structure equivalent to their SM structure, e.g. top quark and $\Delta\rho$, the lightest Higgs;

$$\Delta r_{LR} \neq \Delta r_{SM} + \text{"subleading effects from h.o.c."}$$

Also dramatic consequences for [some] NPM at high energy processes:

Czakon, Gluza, Jegerlehner, Zrałek, EPJ, C13, 275 (2000),
see also: Lynn & Nardi, NPB381, 1992, ...

[some] non-standard models means "unconstrained" extended models:

$\rho_{\rm tree} \equiv M_W^2/(M_Z^2 \cos^2\Theta_W) \neq 1$

e.g.: $SU(2)_L \times U(1) \times U(1)'$,
$SU(2)_L \times SU(2)_R \times U(1)$,...

LEP physics:

$$\begin{aligned} v_f &= \sqrt{\rho_{\rm eff}^f}\,(T_f^3 - 2Q_f \sin^2\Theta_{\rm eff}^f)(\cos\phi - \frac{\sin\phi}{\sqrt{\cos 2\Theta_W}}) \\ a_f &= \sqrt{\rho_{\rm eff}^f}\,T_f^3\,(\cos\phi + \sin\phi\sqrt{\cos 2\Theta_W}\,)\,. \end{aligned}$$

Relations for $\sin^2\Theta_W$ and $\sin^2\Theta_{\rm eff}^f$ in the SM
$[A_Z = f(\alpha, G_F, M_Z)]$:

$$\begin{aligned} \sin^2\Theta_W &= \frac{1}{2}\left[1 - \sqrt{1 - \frac{4A_Z}{\rho M_Z^2}(1+\Delta r_{\rm rem})}\,\right] \\ \sin^2\Theta_{\rm eff}^f &= \frac{1}{2}\left[1 - \sqrt{1 - \frac{4A_Z}{\rho_{\rm eff}^f M_Z^2}\left(1+\Delta r_{\rm rem}^f\right)}\,\right]\,, \end{aligned}$$

$$\Delta r = \Delta\alpha - c_W^2/s_W^2 \Delta\rho + \Delta r_{\rm rem}$$

$$\begin{aligned} \rho &= \frac{\rho_{\rm tree}}{(1-\Delta\rho)} \\ \rho_{\rm eff}^f &= \rho\,(1+\Delta\rho_{\rm rem}^f)\ . \end{aligned}$$

$$\Delta\rho = \Delta\rho^{\rm top} + \Delta\rho_{\rm rem}\ , \quad \Delta\rho^{\rm top} = \frac{3\sqrt{2}G_F}{16\pi^2} m_t^2$$

The chain of relations leading to the succes of the SM in prediction for m_t break down as in the MLRM:

$$\Delta\rho_{\rm LR}^{\rm top} = \frac{\sqrt{2}G_F}{8\pi^2} c_W^2 \left(\frac{c_W^2}{s_W^2} - 1\right) \frac{M_{W_1}^2}{M_{W_2}^2 - M_{W_1}^2} 3m_t^2$$

Can we make reasonable fits of the new parameters without any knowledge of the radiative corrections in the NPM?

Solution: parameters depending strongly on m_t must be free parameters: $\rho^f_{\rm eff}$, $\sin^2\Theta^f_{\rm eff}$ (no SM corrections),

Physical data (M_Z, Γ_Z, σ^0_h, R_ℓ, $A^{0,\ell}_{FB}$, A_ℓ, R^0_b, R^0_c, $A^{0,b}_{FB}$, $A^{0,c}_{FB}$, A_b, A_c, $\sin^2\Theta^\ell_{\rm eff}$, $\sin^2\Theta^b_{\rm eff}$, $\sin^2\Theta_W$, α_s, M_W) as a function of free parameters $\rho^\ell_{\rm eff}$, $\rho^b_{\rm eff}$,ϕ and $\sin^2\Theta^\ell_{\rm eff}$.

Result:

$$\begin{aligned}
|\phi| &\leq 0.003 \rightarrow M_{Z_2} \geq 1420 \text{ GeV} \\
\rho^\ell_{\rm eff} &= 1.005 \pm 0.004 \\
\rho^b_{\rm eff} &= 0.998 \pm 0.009.
\end{aligned}$$

Constraint on m_t comes from Zbb vertex, namely:

$$\frac{\rho^b_{\rm eff}}{\rho^e_{\rm eff}} = (1 - 2\frac{\sqrt{2}G_F}{16\pi^2}m_t^2)^2 \geq 0.98 \rightarrow m_t \leq 225 \text{ GeV}.$$