High $\tan\beta$ study in the stau/stop/sbottom sectors

Gudrid Moortgat-Pick

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1. MSSM: parameter determination
   $\rightarrow$ The crucial case of high tan β
2. Strategy: Use of τ and t polarisation
3. Simulation of $\tan\beta$ and A_f determination
4. Results

$\rightarrow$ Boos, GMP, Martyn, Sachwitz, Vologdin, hep-ph/0211040
$\rightarrow$ Boos, GMP, Martyn, Sachwitz, Sherstnev, Zerwas, hep-ph/0303110

Step–by–step Strategy

General MSSM parameter from $\tilde{\chi}^{\pm}$, $\tilde{\chi}^{0}$, $\tilde{\tau}$

a) charginos: M_2, μ, Φ_μ, $\tan\beta$
b) neutralinos: $+$ M_1, Φ_1

Remember:
only light system, $\tilde{\chi}^{\pm}_1$, $\tilde{\chi}^{0}_1$, $\tilde{\chi}^{0}_2$ would be sufficient!
$\rightarrow$ and even $m_{\tilde{\chi}^{\pm}_2}$ predictable!...

Choi, Kalinowski, GMP, Zerwas, hep-ph/0108117+0202039

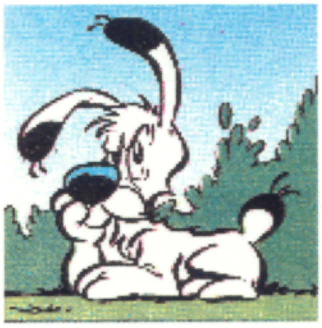
However, all $m_{\tilde{\chi}_i}$ and couplings $= f(\cos 2\beta, \sin 2\beta)$

High $\tan\beta$: $\sin 2\beta \rightarrow 2\frac{1}{\tan\beta}$ and $\cos 2\beta \rightarrow -1$

$\Rightarrow$ weak dependend on high $\tan\beta$

No precise determination possible via $\tilde{\chi}_i$ sector!

What to do?



Possible channels for determining high $\tan\beta$

Higgs sector, e.g.:

- J. Gunion, T. Han, J. Jiang, S. Mrenna, A. Sopczak '01,'02
  $e^+e^- \to HZ \to \bar{b}b\bar{b}b$, $e^+e^- \to b\bar{b}h$, $b\bar{b}A \to \bar{b}b\bar{b}b$
  rates and width $\Rightarrow \Delta\tan\beta > 10\%$ (small m_A)

- V. Barger, T. Han, J. Jiang '00:
  $e^+e^- \to Ht\bar{t}$, $Hb\bar{b}$, $At\bar{t}$, $A, b\bar{b}$
  J.L. Feng, T. Moroi '97:
  $e^+e^- \to Zh$, AH, $t\bar{b}H^-$, $\bar{t}bH^+$
  rates and BR

$\tan\beta$	BHJ	FM
3	2.4–3.6	< 5.2
5	4.3–6.3	3–6
10	6.2–12.7	> 6.5
20	14–32	7.5–90
30	18–80	> 8

$\Rightarrow$ high $\tan\beta$ is a problem ...

$\Rightarrow$ Help from $\tilde{\tau}$, $\tilde{t}$, $\tilde{b}$ sector possible?

$\Rightarrow$ Yukawa couplings: $Y_{\tau,b} = \frac{m_{\tau,b}}{m_W}/(\sqrt{2}\cos\beta)$,
$Y_t = \frac{m_t}{m_W}/(\sqrt{2}\sin\beta)$

What was already done in the $\tilde{\tau}$, $\tilde{b}$, $\tilde{t}$ sector?

- Masses, rates, BR's, e.g.:

Bartl, Eberl, Kraml, Majerotto, Porod	'97, '00
Bartl, Eberl, Kraml, Majerotto, Porod, Sopczak	'97
Eberl, Kraml, Majerotto	'99
Bartl, Hidaka, Kernreiter, Porod	'02

- Use of τ Polarisation, e.g.:

Nojiri	'94
Nojiri, Fujii, Tsukamoto	'96
Guchait, Roy	'01,'02

- Extraction of t polarisation, e.g.:

Jezabek, Kühn	'89
Boos, Sherstnev	'02

What are we doing now?

Interplay between $\tilde{\tau}_i, \tilde{t}_i, \tilde{b}_i \leftrightarrow \tilde{\chi}_j^{0,\pm}$ sector:

$\Rightarrow$ detailed simulation of $\tilde{\tau}$, t polarisation

$\Rightarrow$ measurement of (high) $\tan\beta$

$\Rightarrow$ analysis concerning determination of A_f

Analysis of the $\tilde{\tau}$ sector

Mixing matrix of $\tilde{\tau}_{1,2}$:

$$\mathcal{M}_{\tilde{\tau}} = \begin{pmatrix} M_L^2 + m_\tau^2 + L\cos(2\beta)m_z^2 & m_\tau(A_\tau - \mu\tan\beta) \\ m_\tau(A_\tau - \mu\tan\beta) & M_E^2 + m_\tau^2 - R\cos(2\beta)m_z^2 \end{pmatrix}$$

- off–diagonal terms depend strongly on $\tan\beta$ but also on A_τ!
- diagonal terms depend only slightly on (high) $\tan\beta$
- Mixing:

$$\begin{pmatrix}\tilde{\tau}_1 \\ \tilde{\tau}_2\end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde{\tau}} & \sin\theta_{\tilde{\tau}} \\ -\sin\theta_{\tilde{\tau}} & \cos\theta_{\tilde{\tau}} \end{pmatrix} \begin{pmatrix}\tilde{\tau}_L \\ \tilde{\tau}_R\end{pmatrix}$$

Rates and mixing angle:

$$\sigma(\tilde{\tau}_i\tilde{\tau}_i) = f(\cos^2 2\theta_{\tilde{\tau}}, \cos 2\theta_{\tilde{\tau}})$$
$$\sigma(\tilde{\tau}_1\tilde{\tau}_2) = f(\sin^2 2\theta_{\tilde{\tau}}, \sin 2\theta_{\tilde{\tau}})$$

$\Rightarrow$ $\cos 2\theta_{\tilde{\tau}}$ via light system $\sigma(\tilde{\tau}_1\tilde{\tau}_1)$ with 2-fold ambiguity

$\Rightarrow$ Resolving ambiguity via 2nd measurement
$\rightarrow$ use of beam polarization!

Parameters from the $\tilde{\tau}$ sector

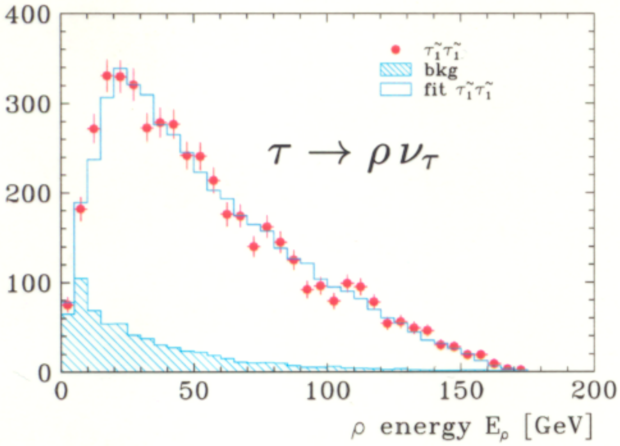
Reference scenario:

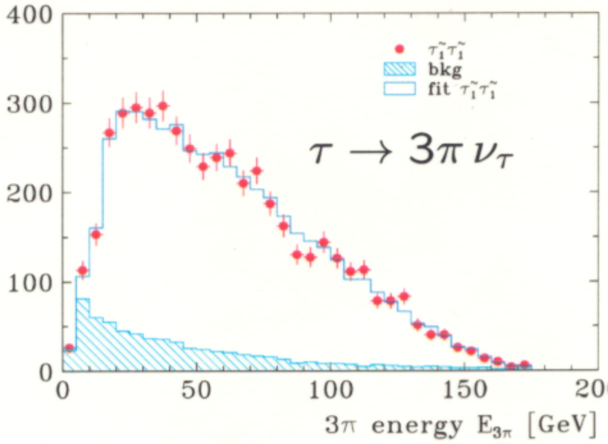
$m_{\tilde{\tau}_1}$	$m_{\tilde{\tau}_2}$	M_1	M_2	μ	$\tan\beta$	A_τ
151 GeV	305 GeV	99 GeV	193 GeV	140	20	−254.2

Process: $e^+e^- \to \tilde{\tau}_1\tilde{\tau}_1 \to \tau^+\tau^- + 2\tilde{\chi}^0_1$

Expected accuracy for masses and rates:

$\sqrt{s} = 500$ GeV, $P_{e^-} = +80\%$, $P_{e^+} = -60\%$, $\mathcal{L} = 250$ fb^{-1}





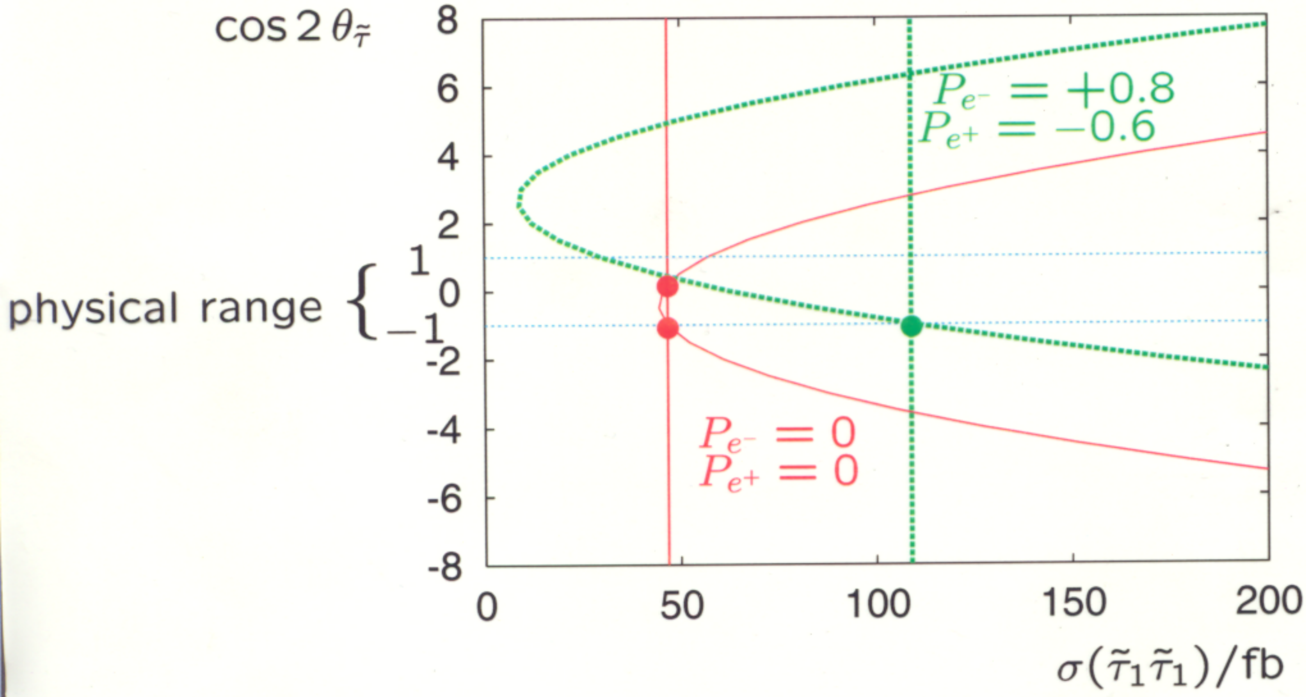
$m_{\tilde{\tau}_1} = 155.2 \pm 0.8$ GeV $m_{\tilde{\tau}_1} = 154.8 \pm 0.5$ GeV

- QED and beamstrahlung included
- Energy and angle cuts
  $\to$ Background $WW \sim 6\%$, $\tilde{\chi}^\pm_1\tilde{\chi}^\mp_1 \sim 3\%$, $\tilde{\chi}^0_2\tilde{\chi}^0_1 \sim 7\%$
- Determination of $\delta\sigma(\tilde{\tau}_1\tilde{\tau}_1)/\sigma(\tilde{\tau}_1\tilde{\tau}_1) \sim 3\%$!

The $\tilde{\tau}$ mixing angle

Process: $e^+e^- \to \tilde{\tau}_1^+ \tilde{\tau}_1^-$

(with $P(e^-) = +80\%, P(e^+) = -60\%$)



$\Rightarrow$ $P_{e^\pm} = 0$: $\sigma(\tilde{\tau}_1\tilde{\tau}_1) = 47$ fb

$\Rightarrow$ 2-fold ambiguity!

$\Rightarrow$ $P_{e^-} = +80\%, P_{e^+} = -60\%$: $\sigma(\tilde{\tau}_1\tilde{\tau}_1) = 109$ fb

$\Rightarrow$ no ambiguity: $\cos 2\theta_{\tilde{\tau}} = -.987 \pm 0.08$!

Beam polarisation = simple and elegant method for resolving ambiguities!

Masses known $\to$ predictable, which polarisation would be sufficient!

Tau polarisation $P(\tilde{\tau}_i \to \tau)$

Process: $\tilde{\tau}_i \to \tau \tilde{\chi}^0_j$

Lagrangian: $\mathcal{L} = \sum \tilde{\tau}_i \bar{\tau} (P_L a^R_{ij} + P_R a^L_{ij}) \tilde{\chi}^0_j$

Polarization with complete $\tilde{\chi}^0_i$ mixing:

$$_{\tilde{\tau}_1 \to \tau \tilde{\chi}^0_1} = \frac{(4 - n_g^2) - (4 + n_g^2 - 2 n_h^2 \mu_\tau^2 / \cos^2\beta) \cos 2\theta_{\tilde{\tau}} + 2(2 + n_g) \sin 2\theta_{\tilde{\tau}} n_h \mu_\tau / \cos\beta}{(4 + n_g^2 + 2 n_h^2 \mu_\tau^2 / \cos^2\beta) - (4 - n_g^2) \cos 2\theta_{\tilde{\tau}} + 2(2 - n_g) \sin 2\theta_{\tilde{\tau}} n_h \mu_\tau / \cos\beta}$$

- n_g: gaugino components from $\tilde{\chi}^0_i$
- n_h: higgsino components from $\tilde{\chi}^0_i$
  crucial contribution!

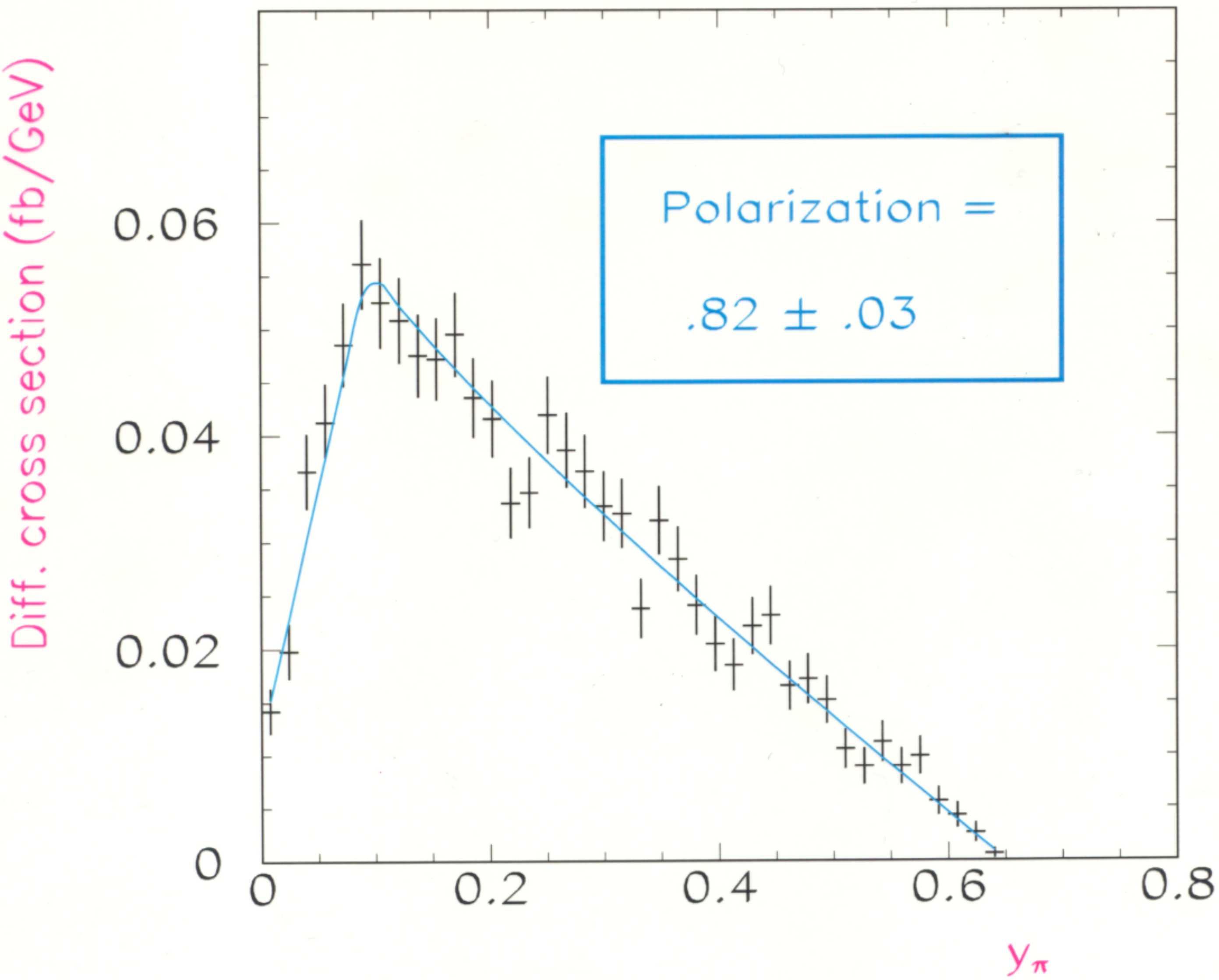
$\tan\beta$ dependence of $P(\tilde{\tau}_i \to \tau)$:

- strong dependence via $\theta_{\tilde{\tau}}$
- weak dependence via neutralino contributions n_g, n_h
- strong dependence via $\tau \leftrightarrow \tilde{\chi}^0_i$ interplay: $n_h \leftrightarrow \mu_\tau / \cos\beta$!

Measurement of the $P(\tilde{\tau}_1 \to \tau)$

Process: $e^+e^- \to \tilde{\tau}_1^+\tilde{\tau}_1^- \to \tilde{\tau}_1^+\tau\tilde{\chi}_1^0 \to \tilde{\tau}_1^+\nu_\tau\pi^-\tilde{\chi}_1^0$

Energy spectra of π ($\tan\beta = 20$)

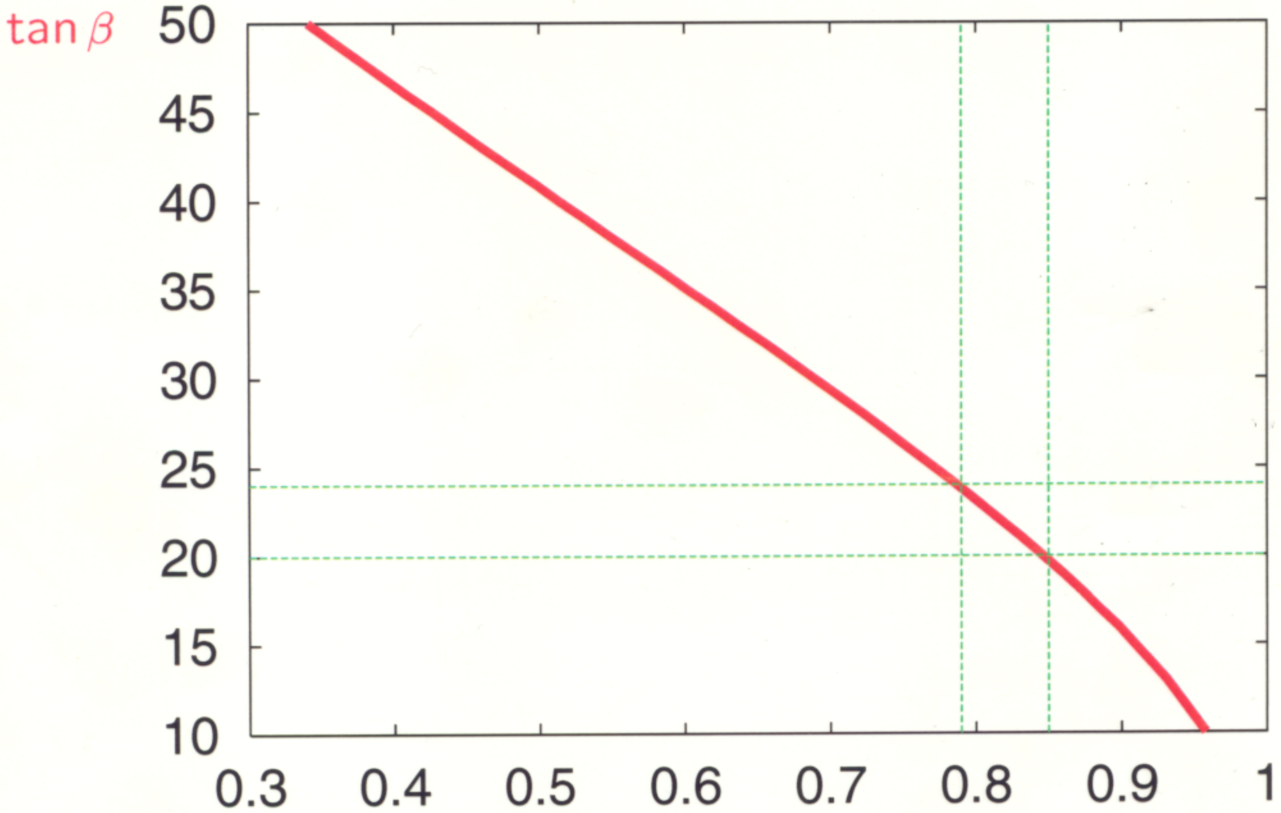


$\Rightarrow$ Precise determination of the polarization:
$P(\tilde{\tau}_1 \to \tau) = 82 \pm 3\%$!

Next step: Inversion of $P(\tilde{\tau}_1 \to \tau)$ leads to $\tan\beta$!

Inversion of $P(\tilde\tau_1 \to \tau)$

Polarization measured $P(\tilde\tau_1 \to \tau) = 82 \pm 3\%$:



$\Rightarrow$ $\tan\beta = 20 \pm 2$!

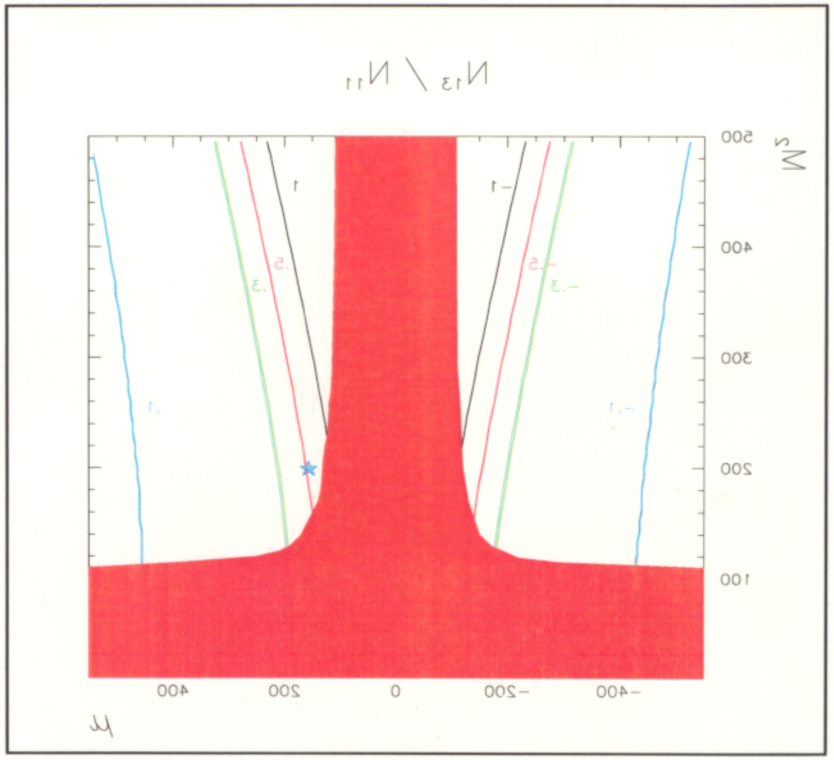
High accuracy even for high $\tan\beta$ possible!

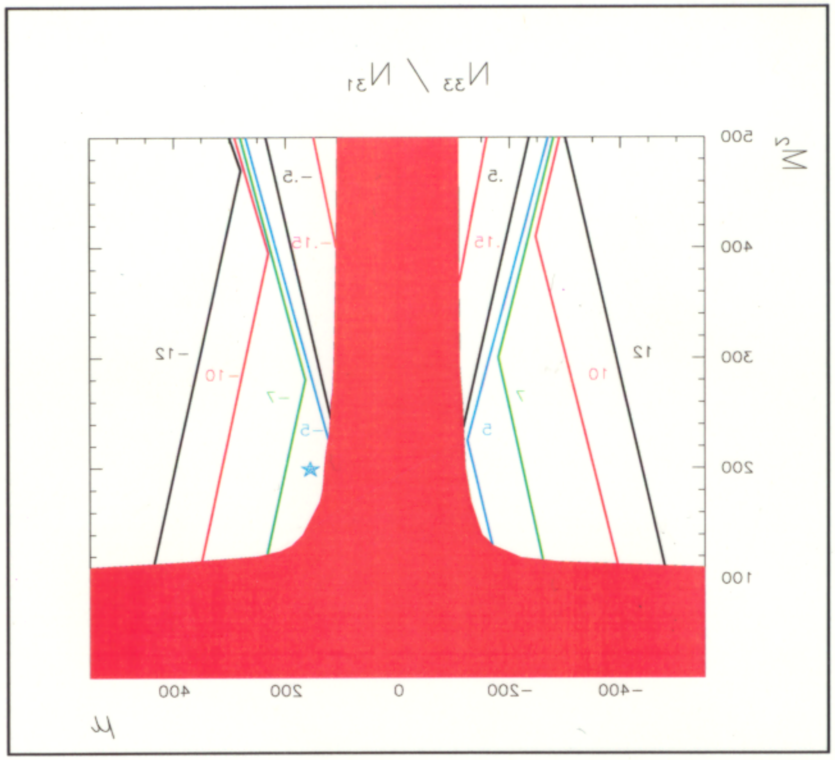
What's about the $\tilde\chi^0_1$ contributions?

$\to$ can be decoupled!

When is this method for measuring large $\tan\beta$ applicable?

The decay neutralino needs higgsino addmixture $\eta_h \sim N_{i3}/N_{i1}$:





(exclusion bounds for $m_{\tilde{\chi}^0_1} > 45$ GeV, $m_{\tilde{\chi}^\pm_1} > 103$ GeV)

In our example: $\tilde{\chi}^0_1 \sim N_{13}/N_{11} \sim 0.5$ is sufficient!

What to do if $N_{13}/N_{11} \sim 0$? ($\leftarrow$ typical mSUGRA!):

$\rightarrow$ eventually $\tilde{\tau}_1$ and $\tilde{\tau}_2$ needed to cover also heavier $\tilde{\chi}^0_i \sim N_{i3}/N_{i1} \neq 0$
$\rightarrow$ predictable a priori with already known M_1, M_2, μ!

Determination of A_τ

Final step:

Since $\tan\beta = 20 \pm 2$ already known

$\Rightarrow$ use off–diagonal terms to determine A_τ:

$$A_\tau = \frac{1}{2m_\tau}(m^2_{\tilde{\tau}_1} - m^2_{\tilde{\tau}_2})\sin(2\theta_\tau) + \mu\tan\beta$$

Measurement of $m_{\tilde{\tau}_2} = 305$ GeV:

$\rightarrow$ analogues to analyses in the TDR:

$\delta(m_{\tilde{\tau}_2}) \sim 2 - 3$ GeV expected

However, due to $m_\tau \ll (m^2_{\tilde{\tau}_1} - m^2_{\tilde{\tau}_2})$ and $\sin\theta_{\tilde{\tau}} \sim 0$:

$\Rightarrow \delta(A_\tau) \sim 2700$ GeV, ...

No chance for A_τ?

$\Rightarrow$ strongly dependent on the scenario:

if $M_L = 300 \rightarrow 200$ GeV $\rightarrow$ larger mixing angle

$\rightarrow \delta(A_\tau) \sim 400$ GeV

Analysis of the $\tilde{b}$ sector

Mixing matrix of $\tilde{b}_{1,2}$ ... as usual

$$\mathcal{M}_{\tilde{b}} = \begin{pmatrix} M_Q^2 + m_q^2 + m_z^2\cos(2\beta)(I_{3L}^q - e_q \sin^2\theta_W) & m_b(A_b - \mu\tan\beta) \\ m_b(A_b - \mu\tan\beta) & M_{\tilde{D}}^2 + m_b^2 + e_q m_z^2 \cos(2\beta)\sin^2\theta_W \end{pmatrix}$$

⇒ In principle 'same' $\tan\beta - A_b$ strategy as before

However, since $P(b)$ is not an prospective observable
study the process: $\tilde{b}_i \to \tilde{\chi}_j^{\pm} t$
→ $P(t)$ needed!

$$P_{\tilde{b}_1 \to t\tilde{\chi}_k^{\pm}} = \frac{\mathcal{G}^N f_1}{\mathcal{G}^{D_1} - \mathcal{G}^{D_2} f_2} \quad \text{with} \quad f_1 = \frac{\lambda^{\frac{1}{2}}(m_{\tilde{q}}^2, m_t^2, m_{\tilde{\chi}}^2)}{m_{\tilde{q}}^2 - m_t^2 - m_{\tilde{\chi}}^2} \quad \text{and} \quad f_2 = \frac{2 m_t m_{\tilde{\chi}}}{m_{\tilde{q}}^2 - m_t^2 - m_{\tilde{\chi}}^2}$$

$$\mathcal{G}^N = -c_h^{+2}\mu_t^2/\sin^2\beta + c_h^{-2}\mu_b^2/\cos^2\beta + 2 - (2\sqrt{2}c_h^-\mu_b/\cos\beta)\sin 2\theta_{\tilde{b}}$$
$$- (c_h^{+2}\mu_t^2/\sin^2\beta + c_h^{-2}\mu_b^2/\cos^2\beta - 2)\cos 2\theta_{\tilde{b}}$$

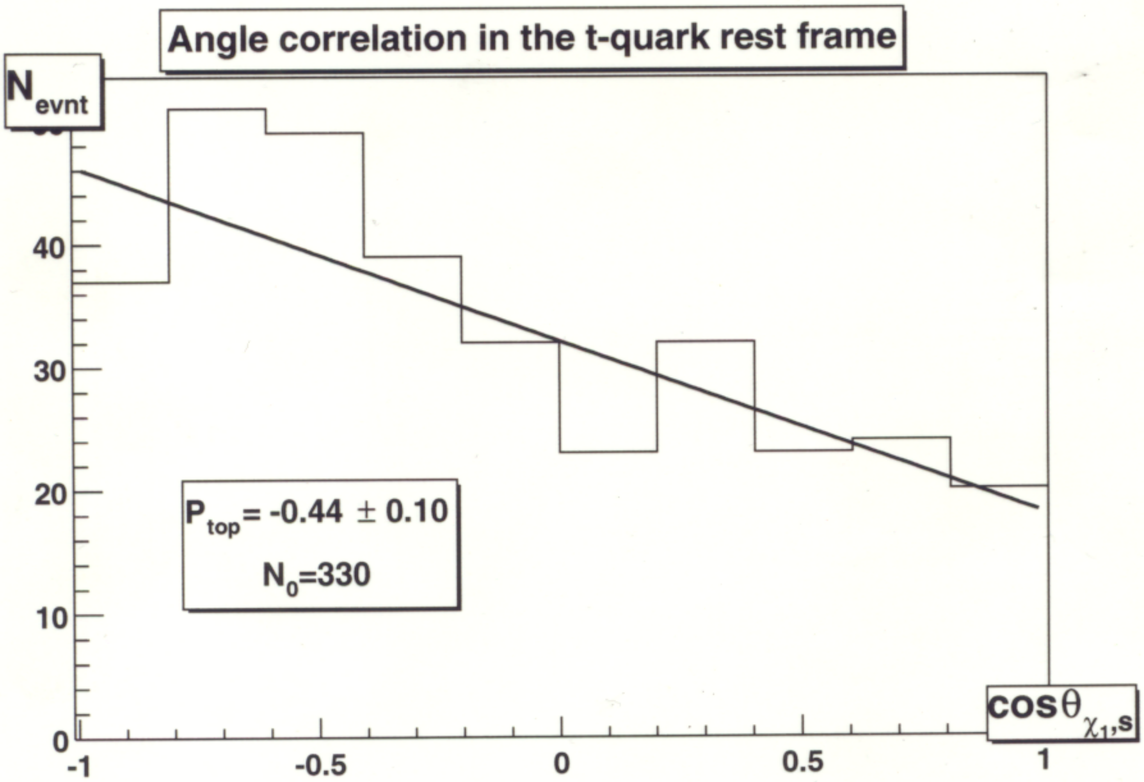
$$\mathcal{G}^{D_1}, \mathcal{G}^{D_2} = etc.$$

with $c_h^+ = V_{j2}/U_{j1}$, $c_h^- = U_{j2}/U_{j1}$: again higgsino admixture important!!!
⇒ In principle same procedure applicable

Measurement of the $P(\tilde{b}_1 \to t)$

Process: $e^+e^- \to \tilde{b}_i\tilde{b}_1 \to \tilde{b}_i\tilde{\chi}_1^+ t \to \tilde{b}_i\tilde{\chi}_1^+ bc\bar{s}$

$\Rightarrow$ determine t polarisation via angle correlations!

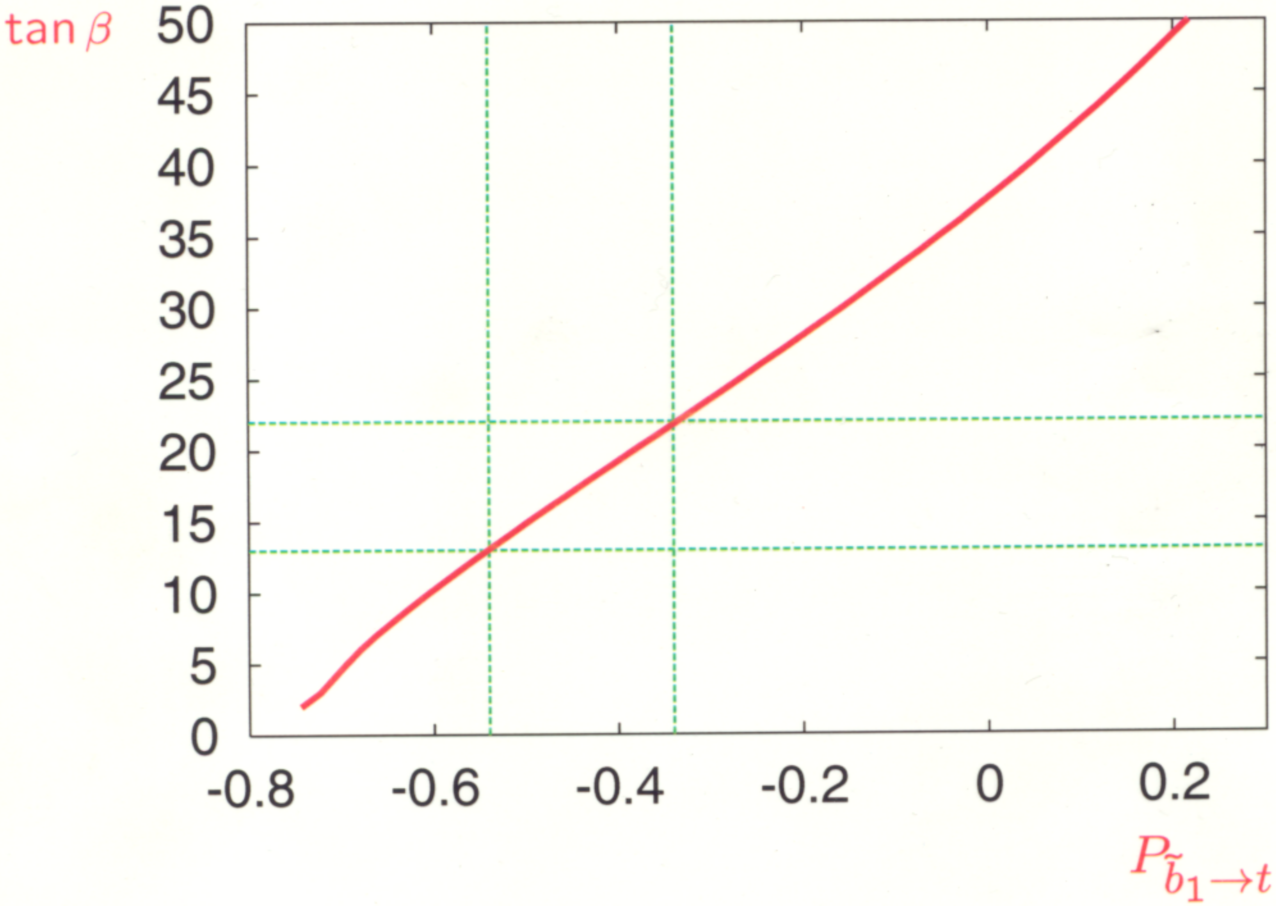


$\Rightarrow$ Determination of the polarization:
$P(\tilde{b}_1 \to t) = -44 \pm 10\%$!

$\Rightarrow$ Inversion of $P(\tilde{b}_1 \to t)$ leads to $\tan\beta$?

Inversion of $P(\tilde{b}_1 \to t)$

Polarization measured $P(\tilde{b}_1 \to t) = -44 \pm 10\%$:



$\Rightarrow$ $\tan\beta = 17.5 \pm 4.5$!

High accuracy again for high $\tan\beta$ possible!

What about A_b?

$\Rightarrow$ Same procedure as in the A_τ case

$$A_b = \frac{1}{2m_b}(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}) \sin(2\theta_b) + \mu \tan\beta$$

but higher precision: $\delta(A_b)/A_b \sim 60\%$!

(assuming $\delta(m_{\tilde{b}}) = 2$ GeV)

Analysis of the $\tilde{t}$ sector

Mixing matrix of $\tilde{t}_{1,2}$ . . . as usual

$$\mathcal{M}_{\tilde{t}} = \begin{pmatrix} M_Q^2 + m_q^2 + m_z^2 \cos(2\beta)(I_{3L}^q - e_q \sin^2\theta_W) & m_t(A_t - \mu\cot\beta) \\ m_b(A_t - \mu\cot\beta) & M_{\tilde{U}}^2 + m_t^2 + e_q m_z^2 \cos(2\beta)\sin^2\theta_W \end{pmatrix}$$

$\Rightarrow$ Again $\tan\beta - A_t$ strategy applicable? $\Rightarrow$ $\cot\beta$ dependence!
not so nice for high $\tan\beta$ . . .

Since $P(\tilde{t} \to b\tilde{\chi}^{\pm})$ does not lead to an prospective observable
$\to$ study the process: $\tilde{t}_i \to \tilde{\chi}_j^0 t$
But $Y_t \sim 1/\sin\beta$ also not very sensitive to high $\tan\beta$ . . .

$\Rightarrow$ Use $\tan\beta$ from e.g. $\tilde{b}$ sector and get A_t via mixing angle

$\Rightarrow$ $\delta(A_t)/A_t \sim 10\%$ (assuming $\delta(m_{\tilde{t}}) = 10$ GeV, $\delta\sigma/\sigma = 0.05$)

$\Rightarrow$ Use $P(t)$ for tests of mixing, Yukawa coupling etc.

Conclusions and Outlook

In principle: M_2, μ, $\tan\beta$, M_1 can be determined via only the light system!
However, $\tilde{\chi}_i^0$, $\tilde{\chi}_j^\pm$ weak dependent on $\tan\beta$:
$\rightarrow$ not suitable for determining $\tan\beta > 10$

- step–by–step procedure for $\tilde{\tau}$–$\tilde{\chi}_i^0$ applicable if $\tilde{\chi}_1^0$ has higgsino admixture
  $\rightarrow$ predictable with input from $\tilde{\chi}_{1,2}^0$, $\tilde{\chi}_1^\pm$

- Determination of $\tan\beta$ with high prec. and A_τ without an assumption on the SUSY breaking scheme!

- For $\tilde{\tau}_i \rightarrow \tau\tilde{\chi}_{2,3,4}^0$ also valid, only higgsino component needed!

- Same method for $\tan\beta$ from $P(\tilde{b}_i \rightarrow t\tilde{\chi}_k^\pm)$
  $\rightarrow$ A_b, A_t with higher precision!!!

- Input from LHC constructive?

$\rightarrow$ high $\tilde{q}$ masses?