Beam energy measurement at linear colliders using spin precession

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Contents

Methods of energy measurement, required accuraces

At storage rings the energy is calibrated by the method of resonance depolarizaion developed in Budker INP. At LEP M_z has been measured with accuracy of 2.5×10^{-5}. At VEPP-4 J/Ψ mass accuracy 3×10^{-6} has been achieved.

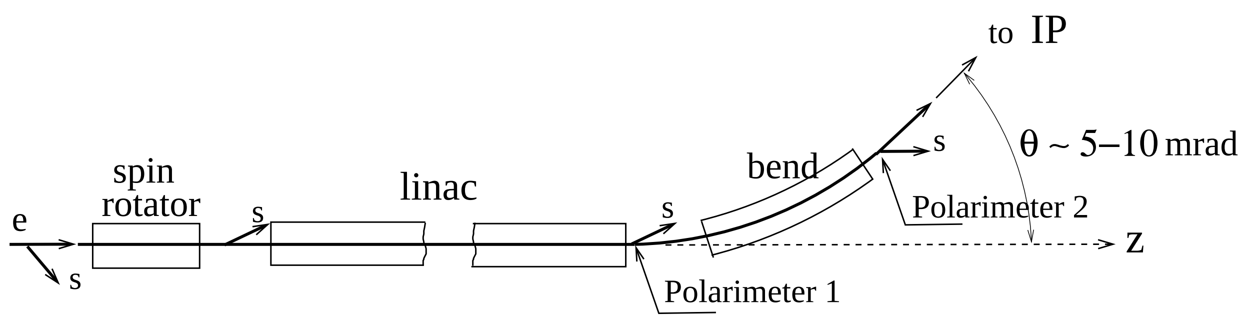
At LC, the required knowledge of the energy
10^{-4} for t-quark,
3×10^{-5} for WW,
10^{-6} for Z.

For LC three methods are considered:

- magnetic spectrometer (10^{-4} for 100 nm BPM res.)
- Moller(Babha) scattering (10^{-5} is expected, but may be much worse due to plasma focusing (V.T))
- radiative return to Z.

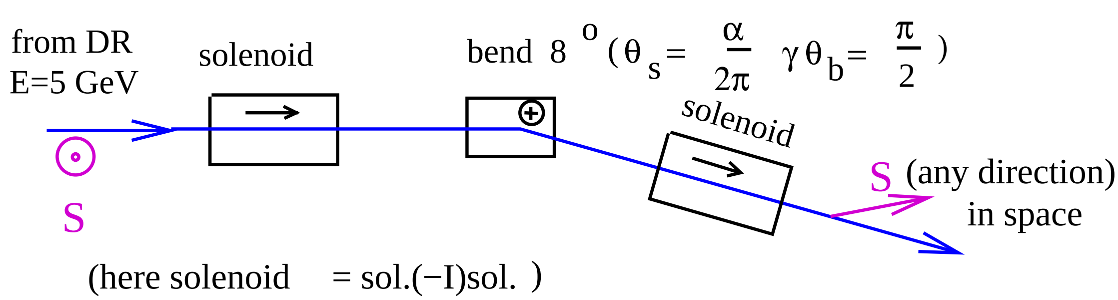
Below a new method is considered based on spin precesion in big-bend regions at LC.

Principle of the method



Spin rotator at TESLA

(top view)



Electrons at LC have a high degree of longitudinal polarization.

Spin rotator can change (arbitrary) the direction of spin at the entrance of LC.

All LC have a big band with the angle about 10 mrad.

Using the Compton scattering one can measure the longitudinal polarization, the absolute accuracy is $< O(1\%)$, but relative accuracy can be much better.

The angle of spin in respect to direction of motion (θ_s) changes proportional to the bending angle (θ_b):

$$\theta_s = \frac{\mu'}{\mu_0}\gamma\theta_b \approx \frac{\alpha\gamma}{2\pi}\theta_b$$

For $E = 1$ TeV and $\theta_b \sim 10$ mrad the spin rotation angle $\theta_s = 23.2$ rad.

The bending angle can be measured with a very high accuracy, then by measuring θ_s one can determine the beam energy

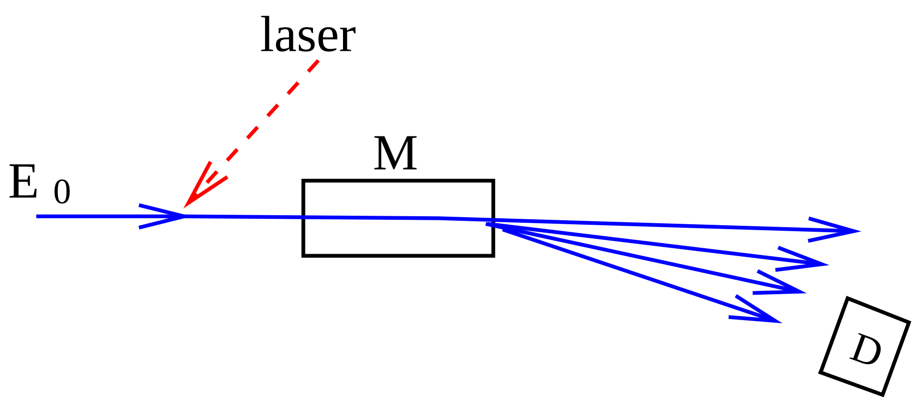
The accuracy

$$\frac{\Delta E}{E} = \frac{\Delta\theta_s}{\theta_s} = \frac{2\pi\Delta\theta_s}{\alpha\gamma\theta_b} \sim \frac{0.43}{E(\text{TeV})\theta_b(\text{mrad})}\Delta\theta_s$$

Measurement of the spin angle

Longitudinal polarization of electrons is measured by scattering of laser photons on the electrons. After scattering of 1 eV laser photon the 500 GeV electron loses up to 90 % of its energy, namely these low energy electrons are detected for measurement of the polarization

Compton polarimeter



The energy spectrum of the scattered electrons is defined by the Compton cross section

$$\frac{d\sigma}{dy} = \frac{d\sigma_u}{dy}[1 + \mathcal{P}_\gamma \mathcal{P}_e \ F(y)], \qquad y = \frac{E_0 - E_e}{E_0}.$$

The unpolarized Compton cross section

$$\frac{d\sigma_u}{dy} = \frac{2\sigma_0}{x}\left[\frac{1}{1-y} + 1 - y - 4r(1-r)\right],$$

$$F(y) = \frac{rx(1-2r)(2-y)}{1/(1-y) + 1 - y - 4r(1-r)},$$

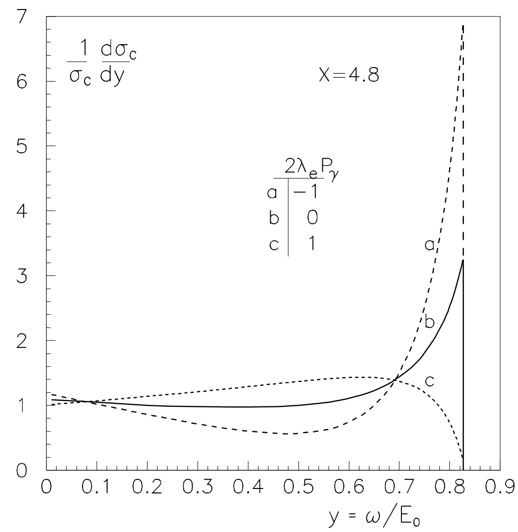
$$\sigma_0 = \pi r_e^2 = \pi\left(\frac{e^2}{mc^2}\right)^2 = 2.5 \times 10^{-25}\text{cm}^2,$$

$$E_{e,\,\text{min}} = \frac{1}{x+1}E_0 \quad x \approx \frac{4E_0\omega_0}{m^2c^4} = 19\left[\frac{E_0}{\text{TeV}}\right]\left[\frac{\mu\text{m}}{\lambda}\right],$$

where $\mathcal{P}_e = 2\lambda_e$ is the longitudinal electron polarization (doubled mean electron helicity) and $\mathcal{P}_\gamma$ is the photon helicity, ω_0 is the laser photon energy, λ the wavelength.

For example, at $E_0 = 250$ GeV and $\lambda = 1\ \mu m$, $x \approx 4.8$, the minimum electron is about $0.18E_0$.

The scattered photon spectrum



P_c is the helicity of laser photons
$2\lambda_e = P_e$ is the longitudinal polarization of electrons

The counting rate (or just the signal in the polarimeter in the case of analog device which is better suited for our task) is very sensitive to the product of laser and electron helicities

$$\dot{N} \propto (1 - \mathcal{P}_\gamma \mathcal{P}_e) + \mathcal{O}(0.1 - 0.2).$$

$\mathcal{P}_e \; = P_e \cos\theta$, where P_e is the absolute value of the polarization degree, θ the angle between the electron spin and momentum.

The number of events in the polarimeter for a certain time

$$N = A\cos\theta + B$$

This gives

$$\cos\theta = \frac{2N - (N_{\text{max}} - N_{\text{min}})}{N_{\text{max}} + N_{\text{min}}}$$

and the precession angle

$$\theta_s = \theta_2 - \theta_1.$$

Statistical accuracy

Assuming that both $|\sin\theta_i|$ are chosen to be large enough (at any energy it is possible to make both $|\sin\theta_i| > 0.7$) and $N_{\text{min}}, N_{\text{max}}$ and N are measured, the statistical accuracy of the precession angle

$$\sigma(\theta_s) < \frac{5}{\sqrt{N}},$$

where N is the number of events in each polarimeter for the total time of measurement.

If the Compton scattering probability is 10^{-7} (that is easy) and 30% of scattered electrons with minimum energies are detected, then the counting rate for TESLA $2 \cdot 10^{10} \times 14\,\text{kHz} \times 10^{-7} \times 0.3 = 10^7$ per second. The statistical accuracy of θ_s for 10 minutes run is 6×10^{-5}.

To decrease systematic errors one has to make some additional measurements (see below), that makes the measurement time longer roughly by factor of 3. The accuracy of the energy measurement for 1/2 hour run and $\theta_b = 10$ mrad

$$\frac{\Delta E}{E} \sim \frac{2.5}{E[\text{TeV}]} \times 10^{-6}.$$

If one spend only 1% of the time for the energy calibrations the overall statistical accuracy for 10^7 sec will be *much better* than 10^{-5} for any LC energy and bending angles larger than several mrads.

It seems, the statistical accuracy is not a limiting factor, the accuracy will be determined by systematic errors.

Procedure of the energy measurement

Systematic errors depend essentially on a procedure of measurements . It should account for the following requirements:

- for the energy calibration polarized electrons and circularly polarized laser photons are used, but the result should not depend on accuracy of knowledge of their polarizations;
- the measurement procedure includes some spin manipulations using the spin rotator, the accuracy of such manipulation should not contribute to the result;
- change of the spin rotator parameters may lead to some variations of the electron beam sizes, position in the polarimeter and backgrounds, influence of these effects should be minimized.

Below we describe several procedures which can considerably reduce possible systematic errors.

Measurement of $N_{\text{max}}, N_{\text{min}}$

The maximum and minimum signals in the polarimeter correspond to $\theta = 0$ or $\theta = \pi$. To measure N_{max} one can use a priory knowledge of the accelerator properties and put the spin in the forward direction with some accuracy $\delta\theta$. Our goal is to measure the signal with the accuracy at the level of 10^{-5}. This needs $\delta\theta < 5 \times 10^{-3}$. It is difficult to guarantee such accuracy, it is better to avoid this problem.

The procedure. In the first measurement instead of $\theta = 0$ the spin has some small unknown angles θ_x and θ_y, then the counting rate

$$N_{\text{max,1}} \approx A + B \cos(\sqrt{\theta_x^2 + \theta_y^2}) \approx A + B(1 - \theta_x^2/2 - \theta_y^2/2)$$

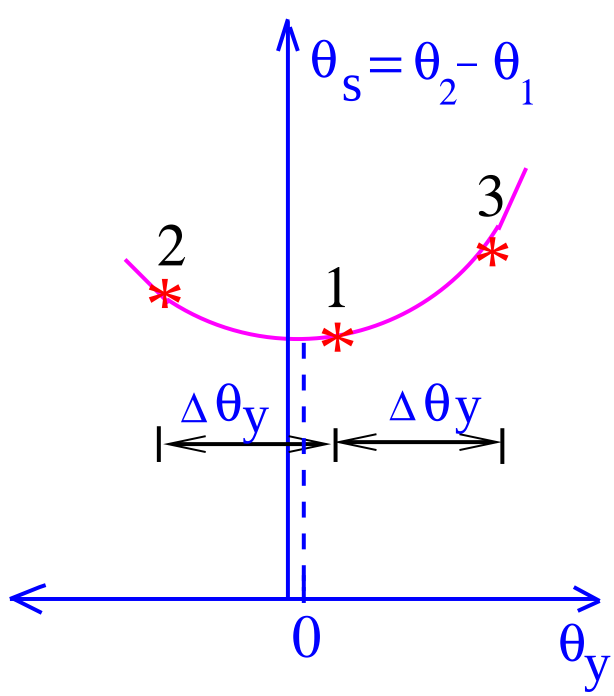
To exclude the uncertainty one can make some fixed *known* variations of θ_x and θ_y on about 10^{-2} rads based on knowledge of the accelerator parameters.

The equation has 4 unknown variables: A, B, θ_x, θ_y, to find them one needs 3 additional measurements.

Solving the system of four linear equations one can find θ_x, θ_y, and after that make the final correction which put the spin in horizontal plane with negligibly small accuracy and collect larger statistics to determine N_{max}. The minimum value of the signal, N_{min}, is found is in similar way making variations around $\theta = \pi$.

Positioning of spin to the bending plane

For precise measurement of the precession angle the spin should be kept in the bending plane. Initially, one can put the spin to this plane with an accuracy given by knowledge of the system. The residual unknown angle θ_y can be excluded in a simple way. It is clear that the *measured* precession angle is a symmetrical function of θ_y and therefore depends on this small angle in a parabolic way. Let us make three measurement of the precession angle at θ_y (unknown) and $\theta_y \pm \Delta\theta_y$. These three measurement give three values of the precession angle $\theta_s(1)$, $\theta_s(2)$, $\theta_s(3)$ which correspond to three equidistant values of θ_y. After fitting the results by parabola one obtains the maximum (or may be the minimum, depending on the horizontal angles) value of θ_s which corresponds to the the position of the spin vector in the bending plane.



Variation of electron beam sizes and positions in polarimeters.

In existing designs of the spin rotators these variation are compensated, but some residual effects can remain.

Variation of laser-electron luminosity

$$\frac{\Delta L}{L} = \left(\frac{\sigma_{y,e}}{\sigma_{y,L}}\right)^2 \frac{\Delta\sigma_{y,e}}{\sigma_{y,e}} + \left(\frac{2\sigma_{x,e}}{\sigma_{z,L}\theta}\right)^2 \frac{\Delta\sigma_{x,e}}{\sigma_{x,e}}.$$

Electron beam sizes at maximum LC energies (but not at the interaction point) are of the order of $\sigma_{z,e} = 100 - 300$ μm, $\sigma_{x,e} \sim 10\,\mu$m, $\sigma_{y,e} \sim 1\,\mu$m. To reduce the sensitivity the electron beam sizes by a factor of 1000 one should take

$$\sigma_{y,L} = \sigma_{x,L} \approx 30\sigma_{y,e} \sim 30\ \mu\text{m}, \quad \sigma_{z,L}\theta \approx 30 \times 2\sigma_{x,e} \sim 600\,\mu\text{m}.$$

$\sigma_{z,L} \sim 0.5$ cm and $\theta \sim 0.1$ is OK.

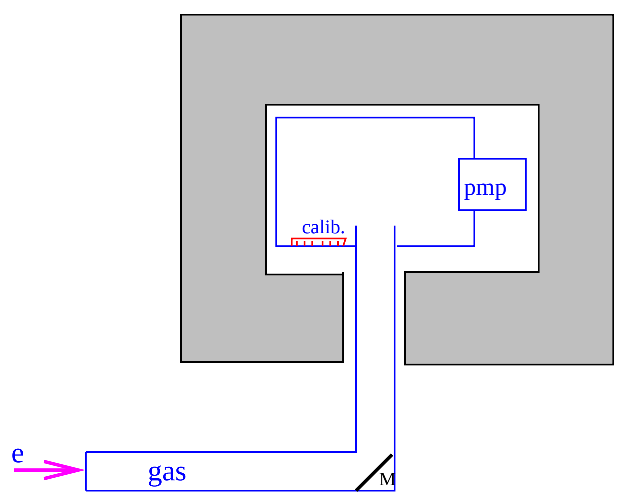
There required laser flash energy

$$A \approx \omega_0 \frac{4\pi\sigma_{x,e}\sigma_{y,e}(30)^2 k}{\sigma_c}.$$

For example, for $\lambda = 1$ μm ($\omega_0 = 1.24$ eV), $\sigma_{x,e} = 10$ μm, $\sigma_{y,e} = 1$ μm, $k = 10^{-7}$ and $\sigma_c = 1.7 \times 10^{-25}$ cm^2 (for $E_0 = 250$ GeV) we get $A = 1.3 \times 10^{-4}$ J. The average laser power at 20 kHz collision rate is 2.5 W (no problem).

Detector

As a detector of the Compton scattered electrons one can use the gas Cherenkov detector successfully performed in the Compton polarimeter at SLC. It detects only particles traveling in a forward direction and is blind for wide angle background. The expected number of particles in the detector from one electron bunch is about 1000. Cherenkov light is detected by several photomultipliers.



To correct nonlinearities in the detector one can use several calibration light sources which can work in any combination covering all dynamic range.

For accurate subtraction of variable backgrounds (constant background is not a problem) one can use events without laser flashes. Main source of background is bremsstrahlung on the gas. Its rate is smaller than from Compton scattering and does not present a problem.

Measurement of bending angle

We assumed that the bending angle can be measured with negligibly small accuracy. Indeed, beam position monitors can measure the electron beam position with submicron accuracy. In this way one can measure the direction of motion.

Measurements of the angle between two lines separated by several hundreds meters in air is not a simple problem, hopefully it can be done, there is no physics limitation at this level. For example, gyroscopes (with correction to Earth rotation) provide much better accuracy than we need.

Systematic errors

Some possible sources of systematic errors were discussed before. Realistic estimation can be done only after the experiment. Measurement of signals (averaged over many pulses) on the level 10^{-4} does not look unrealistic. The statistical accuracy can be several times better and allows to see some possible systematic errors.

If systematics are on the level 10^{-4}, the accuracy of the energy calibration is about

$$\frac{\sigma_E}{E} \sim \frac{0.5 \times 10^{-4}}{\theta_b[\text{mrad}]E[\text{TeV}]}$$

Measurement of the magnetic field vs spin precession

There is a good question to be asked: may be it is easier to measure magnetic field in all bending magnets instead of measurement of the spin precession angle?

Yes, it is more straightforward way. However, we discuss the method which potentially allows the accuracy of the energy measurement at LC of about 10^{-5}. Bending magnets in the big-bends should be weak enough, $B \sim 10^3$ G, to preserve small energy spread and emittances. Who can guarantee 10^{-2} G accuracy of the magnetic field when the Earth field is about 1 G?

Some remark on the energy measurement by Moller scattering

In this method electrons are scattered on electrons of a gas target, the energy is measured using angles and energies of both final electrons in a small angle detector. For LEP-2 the estimated precision was about 2 MeV, limited by Fermi motion of electron in the target.

Here I would like to pay attention on one effects which was missed in all previous considerations. It is a plasma focusing of electrons. The electron beam ionizes the gas target, free electron quickly leave the beam volume while ions begin to focus electrons. This effect destroys the beam quality because the kick is *much larger* than the vertical angles in the beam. The energy resolution also will be considerably larger than the above-mentioned limit.

Conclusion

The method of beam energy measurement at linear colliders using spin precession has been considered. The accuracy on the level a few 10^{-5} looks possible.

see more details in hep-ex/0302036