Matter in the universe

J.W. van Holten

1 The nature and origin of matter

When considering the properties and forms of appearance of matter, two simple questions arise:
1. What is the universe made of?
2. Is matter always and everywhere the same?
At the present time, the answer to the first question is largely unknown. We do know, of course, what our world is made of, including we ourselves: protons, neutrons and electrons, bound together by nuclear and electromagnetic forces into atoms and molecules. Large aggregates of these particles exist in the form of planets, stars and galaxies, bound and shaped by the gravitational force.

When we look up into the sky, we see thousands of stars. Telescopes on earth and on board of satellites have revealed that the stars in our galaxy actually have to be counted in hundreds of billions (10^{11}), and that the number of galaxies in the universe have to be counted in hundreds of millions (10^8). Based on such counts, a rough estimate of the number of nucleons (protons and neutrons) in the visible universe is of the order of 10^{78}. As the universe is apparently electrically neutral, a similar number of electrons must be present, but electrons being much lighter than nucleons, they contribute much less to the mass and energy density of the universe.

Although the total number of nucleons is very large, it represents a small average density of less than 1 nucleon per 10 m^3 in the universe at large. The mass of a proton being

$$m_p = 1.673 \times 10^{-27}\,\mathrm{kg} \; = 0.938\,\mathrm{GeV/c^2},$$

this corresponds to an energy density of about 0.1 $\mathrm{GeV/m^3}$, give or take a factor of 2. However, there are good grounds to believe that the actual energy density in the universe is of the order of 30 to 50 times larger:

$$\varepsilon_{universe} \approx 5\,\mathrm{GeV/m^3}.$$

This energy is *not* present in the form of radiation or neutrinos. In fact, the unknown components of the energy density do not produce light or other types of electromagnetic radiation. Therefore it is refered to as *dark matter*, or *dark energy*; its nature is the subject of intense scientific speculation.

The answer to the second question is easier: matter is not eternal and immutable, and the nature of the material content of the universe has changed considerably in the course of time. In fact, the universe itself is not static, but has evolved from a very compact hot and dense state some 13 billion years ago to the large, rather empty and cold place we presently observe it to be. During this cooling presumably a number of phase transitions have occurred, but also the balance between different components has shifted.

In following, we briefly sketch the evolution of the universe. Next, we discuss the known and observed components of matter and radiation it contains; and finally we turn to the question of dark matter and energy, explaining why it is conjectured to exist and what it could consist of.

2 The evolution of the universe

Various kinds of observational evidence show, that over very large distance scales (volumes much larger than galaxies or even clusters of galaxies) the universe is homogeneous and isotropic. The most impressive evidence in this respect comes from the measurement of the cosmic background radiation, a thermal black body radiation with a temperature of $T = 2.728$ K, which fills the universe homogeneously. Fluctuations in the temperature in various directions of the sky are observed to be smaller than about 1 part in 10^5. Therefore, ignoring 'local' structures in the visible universe, for describing its evolution we take the universe to look the same to any observer, independend of his position or the direction he looks in. This is sometimes called the *cosmological principle*.

Actually we have to be a little bit more careful. As the universe is not static, and as relativity implies that simultaneity of events depends on the frame of reference of the observer, the more precise statement is, that there exist a special reference frame in which the slices of equal time (three dimensional spaces of simultaneous events) represent a homogeneous and isotropic universe. We will take this prefered frame as our reference system. It has the special feature, that on average galaxies are in fixed positions with respect to this frame, except that all distances in all directions can change together by a scale factor depending on time. Such changes do not upset the homogeneity or isotropy of the universe. Thus we assign every point in the universe a unique and fixed coordinate vector $\mathbf{r}$, with the actual distance dR between nearby points at any time given by a universal scale factor $a(t)$ such that

$$dR(t) = a(t)|d\mathbf{r}|. \tag{1}$$

It remains to specify what is meant precisely by the intrinsic co-ordinate distance $|d\mathbf{r}|$. This depends on the geometry of the 3-dimensional space. General relativity tells us, that specifying this geometry is equivalent to specifying a global gravitational field. With matter and radiation being distributed homogeneously in space, the gravitational field, and as such the three-dimensional geometry, must be homogeneous and isotropic as well. It follows, that the only geometries that are allowed are geometries of constant spatial curvature. Of course, the curvature can change with time, but in our special reference frame it can not change from one point to the next at equal time. We can therefore distinguish three type of geometries: universes with constant positive curvature (like a 3-dimensional sphere), flat universes, and universes with a constant negative curvature (like a 3-dimensional hyperboloid). For each of these cases we can introduce a metric such that the distance between nearby point is given by

$$|d\mathbf{r}|^2 = \frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{2}$$

where $k = +1$ describes a spherical universe, $k = 0$ a flat universe, and $k = -1$ a hyperbolic universe. Therefore, as far as the geometry is concerned, the complete history of the universe is described by specifying the curvature constant k and the universal scale factor $a(t)$.

To compute the behaviour of the scale factor as a function of time in principle one has to apply Einstein's theory of general relativity. However, the equations governing the evolution of a homogeneous and isotropic universe can also be guessed by using simple physical arguments. As matter and radiation, like space itself, are supposed to behave as a continuous istropic and homogeneous medium, we model matter and radiation by a perfect fluid, characterized completely in terms of its density, pressure and temperature. The energy density ε, related to an equivalent mass density ρ by $\varepsilon = \rho c^2$, as well as the pressure p and the temperature T, are functions of time only.

We can now first establish a relation between the scale factor a and the mass density ρ from standard mechanical energy conservation. Establish a coordinate frame, and consider a mass element m (a typical galaxy, say) at a distance $R = ar$ from the origin, with r fixed. As the universe expands or contracts, this element has a kinetic energy

$$T = \frac{m}{2}\,\dot{R}^2 = \frac{mr^2}{2}\,\dot{a}^2, \tag{3}$$

as measured by an observer in the origin, provided the observed velocity is small compared to the velocity of light. W.r.t. the same observer, it also has a potential energy, the gravitational energy arising from the attractive influence of the total mass inside the sphere of radius R around the origin:

$$V = -\frac{GmM}{R} = -\frac{4\pi G m \rho a^2}{3r}. \tag{4}$$

Here we use the newtonian gravitational potential, which is correct in the limit of non-relativistic motion for a matter fluid consisting of elements with non-zero rest mass. In such a quasi-static situation the total energy of the mass element is conserved: $T + V = E$, a constant. In terms of the scale factor this becomes

$$\dot{a}^2 - \frac{8\pi G\rho}{3}\,a^2 = -kc^2, \tag{5}$$

with $k = -2E/(mc^2r^2)$. Now as there is no prefered point in the universe (the cosmological priniciple) the constant k must be the same for any mass element in the universe. Hence we can always choose units such that this constant takes one of the values $k = (+1, 0, -1)$. Observe that for $k = +1$ the system has negative total energy and the mass element will not be able to escape to infinity; for $k = -1$ the total energy is positive, and the mass element is able to escape to infinity; finally, for $k = 0$ we have the limiting case where the mass element is barely able to escape to infinity, but its velocity approaches zero when doing so.

Even though eq.(5) has been derived under special assumptions and in the non-relativistic approximation, it turns out that it holds also in the relativistic case and in the presence of radiation as well as ordinary matter. The physical implications of the various values of k carry over as well: for $k = +1$ the universe is closed: it reaches a maximal size and then recollapses; in contrast, for $k \geq 0$ the universe will expand forever.

Another equation for the scale factor can be derived from the first law of thermodynamics. It states, that the expansion of a homogeneous and isotropic universe is an adiabatic process, maintaining thermal equilibrium:

$$T\frac{ds}{dt} = \frac{du}{dt} + p\frac{dv}{dt} = 0, \tag{6}$$

where s is the entropy per comoving volume element of the fluid, u the thermodynamic energy per fluid element, and $v = v_0 a^3$ the comoving volume of the fluid element. The energy of the fluid element with energy density $\varepsilon = \rho c^2$ is $u = \varepsilon v = \varepsilon v_0 a^3$. Then eq.(6) becomes

$$\frac{d}{dt}\left(\varepsilon a^3\right) + p\,\frac{da^3}{dt} = 0. \tag{7}$$

Again it turns out that this equation is in full agreement with, and can in fact be derived from, Einstein's equations.

Equations (5) and (7) provide two equations between three variables: a, ρ and p. In order to determine a complete solution, we need a third equation, the equation of state for the material fluid relating p and ρ or ε. On dimensional grounds one easily sees that it must be of the type

$$p = \eta\varepsilon = \eta\rho c^2, \tag{8}$$

with η a dimensionless number. Its value depends on the nature of the fluid: different mixtures of matter and radiation give different values of η. Three values describe particularly simple situations, which are relevant in later discussions:

$$\eta = \begin{cases} \frac{1}{3}, & \text{for pure radiation;} \\ 0, & \text{for cold non-relativistic matter;} \\ -1, & \text{for gravitational vacuum energy.} \end{cases} \tag{9}$$

Indeed, for a relativistic gas, like a gas of photons or neutrinos, thermodynamics gives $\varepsilon = 3p = 3nkT$, whereas for non-relativistic matter at $T = 0$ the pressure vanishes; more generally, at low temperatures the pressure is generically very small compared to the rest energy density of a gas consisting of particles with a finite rest mass.

As follows from eqs.(7) and (9), the gravitational energy of the vacuum corresponds to a situation in which the pressure and energy density are constant and opposite: $p = -\varepsilon =$ constant. This constant can be turned into another constant with the dimension of curvature, i.e. $1/(\text{lenght})^2$, by taking

$$\Lambda = \frac{32\pi G p}{c^4}. \tag{10}$$

The constant Λ is known in general relativity as the *cosmological constant*. Its geometrical meaning is, that it represents a constant space-time curvature, i.e. a four-dimensional curvature, as opposed to k, which describes only spatial (three-dimensional) curvature.

The equation of state (8) and the first law of thermodynamics (7) can be combined into a direct relation between energy density and the scale factor:

$$\frac{d}{dt}\left(\varepsilon a^3\right) + p\frac{da^3}{dt} = \frac{d\varepsilon}{dt}a^3 + 3(1+\eta)\varepsilon a^2\frac{da}{dt} = 0, \tag{11}$$

which is equivalent to the equation

$$\frac{d}{dt}\left(\varepsilon a^{3(1+\eta)}\right) = 0 \quad \Rightarrow \quad \varepsilon a^{3(1+\eta)} = \text{constant}. \tag{12}$$

If we let the subscript 0 refer to some reference time t_0, then it follows that the values of ε and a at any earlier (or later) time are related by

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\rho}{\rho_0} = \left(\frac{a_0}{a}\right)^{3(1+\eta)}. \tag{13}$$

Substitution of this result into the first evolution equation (5) allows us to solve for the evolution of simple model universes. We illustrate this for the case $k = 0$. Then we have:

$$\dot{a}^2 = \frac{8\pi G\rho}{3}\, a^2, \qquad \rho = \rho_0\left(\frac{a_0}{a}\right)^{3(1+\eta)}. \tag{14}$$

For a universe filled with relativistic matter and radiation, $\eta = 1/3$ and we obtain

$$\dot{a}^2 = \frac{8\pi G\rho_0 a_0^4}{3a^2}, \tag{15}$$

with the expanding solution

$$a(t) = a_0\sqrt{1 + \kappa_0(t - t_0)}, \qquad \kappa_0^2 = \frac{32\pi G\rho_0}{3}. \tag{16}$$

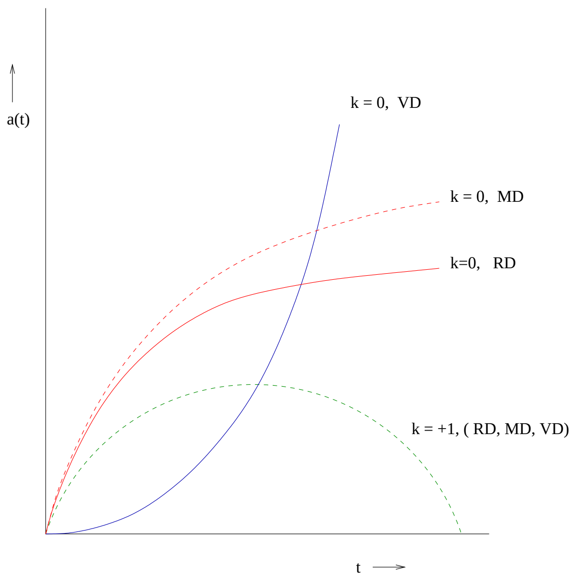

Fig. 1: Evolution of the universe for various values of the parameters (η, k).

If we take the reference time t_0 to be far in the past, when the energy density in radiation was large, we can neglect the unit term under the square root, and find for large times $a \simeq a_0\sqrt{\kappa_0(t - t_0)}$. The expansion velocity then behaves like

$$\dot{a} = \frac{a_0}{2}\sqrt{\frac{\kappa_0}{t - t_0}} \quad \Rightarrow \quad \frac{\dot{a}}{a} = \frac{1}{2(t - t_0)}. \tag{17}$$

The solution is shown in fig. 1, the curve labeled RD and $k = 0$. For a universe filled with cold matter $\eta = 0$, and

$$\dot{a}^2 = \frac{8\pi G\rho_0 a_0^3}{3a}. \tag{18}$$

In this case the expanding solution is

$$a(t) = a_0\left(1 + \frac{2\kappa_0}{3}(t - t_0)\right)^{2/3}. \tag{19}$$

Again, if t_0 is an early time at which the density and hence κ_0 is large, then at late times $a \simeq a_0(2\kappa_0(t - t_0)/3)^{2/3}$, and

$$\dot{a} = \frac{2a_0}{3}\left(\frac{2\kappa_0}{3}\right)^{2/3}(t - t_0)^{-1/3} \quad \Rightarrow \quad \frac{\dot{a}}{a} = \frac{2}{3(t - t_0)}. \tag{20}$$

This solution is also shown in fig. 1, the curve labeled MD and $k = 0$. As illustrated by the curves, in both cases $\dot{a} \to 0$ at late times.

Finally we consider the case of an evolution dominated by vacuum energy, i.e. a cosmological constant. Then $\rho = \rho_0$, a constant, and we have exponential expansion

$$a(t) = a_0 e^{H_0(t-t_0)}, \qquad H_0 = \kappa_0/2. \tag{21}$$

This is illustrated in fig. 1 by the curve labeled VD and $k = 0$. In every case we have chosen the expanding solution, because it fits observations. Of course, solutions describing an imploding universe exist as well, but we do not consider them here.

3 The cosmological parameters

The first evidence for the expansion of the universe was provided by the measurement of the redshift of galaxies by Hubble in the 1920's. The redshift of a light source in the universe is defined by

$$z = \frac{\lambda_0 - \lambda_e}{\lambda_e} = \frac{\lambda_0}{\lambda_e} - 1. \tag{22}$$

Here λ_e is the wavelength of the emitted light, whereas λ_0 is the wavelength of the observed light. Hubble showed, that statistically there was a linear relation between the distance and the redshift of galaxies:

$$z = \alpha R, \tag{23}$$

at least for galaxies not too far away (the only ones Hubble could observe). For such galaxies the time of emission t_e could be estimated from

$$R = c(t_0 - t_e), \tag{24}$$

provided the distance R was known by some other method. The linear relation (23) can easily be explained by the model of the expanding universe. Indeed, the wavelength scales with the scale factor: $\lambda(t) \propto a(t)$. Therefore

$$z = \frac{\lambda_0}{\lambda_e} - 1 = \frac{a_0}{a_e} - 1. \tag{25}$$

If light emitted at time t_e reaches us at time t_0, the scale factor has changed by an amount we can write in a Taylor series expansion as

$$a_e = a_0(1 + H_0(t_e - t_0) + ...) \tag{26}$$

As Hubble's galaxies were not too far away (on a cosmological scale) and correspondingly $t_e - t_0$ was not too large on the scale on which the universe evolves, these terms suffice to fit Hubble's observations. Combining eqs.(23), (25) and (26) one finds to a first approximation

$$z = H_0(t_0 - t_e) = \frac{H_0}{c} R \quad \Leftrightarrow \quad H_0 = c\alpha. \tag{27}$$

H_0 is called Hubble's constant. Of course, the actual motion of galaxies is not only due to cosmic expansion, but also due to proper motion w.r.t. the prefered frame of the cosmic microwave background. Therefore for any particular galaxy z includes the effect of the Doppler shift arising from this proper motion. However, *on average* galaxies are at rest w.r.t. the cosmic reference frame, and therefore Hubble's law holds as a statistical relation. The present best value of H_0 from observations is

$$H_0 = h_0 \times 100\,\mathrm{km\ s^{-1}\ Mpc^{-1}}, \qquad h_0 = 0.7 \pm 0.07. \tag{28}$$

For large redshifts the linear law is modified, because one can no longer rely on the linear approximation (26) for the expansion parameter. To discuss the evolution of the universe over longer time scales it is customary to introduce a time-dependend *Hubble parameter* $H(t)$ defined as

$$H = \frac{\dot{a}}{a}. \tag{29}$$

It has the dimension of an inverse time. Indeed, $\tau = 1/H$ is a first approximation to the age of the universe at any given time. The value (28) gives for an approximate present age of the universe

$$\tau_0 = \frac{1}{H_0} = 1.4 \times 10^{10}\,\text{years}. \tag{30}$$

Eq.(14) shows, that in a flat universe the Hubble parameter is directly related to the density by

$$\rho = \rho_c = \frac{3H^2}{8\pi G}. \tag{31}$$

This density is called the *critical density.* Note, that as $H(t)$ is a function of time, the critical density is a function of time as well.

The critical density can be used to introduce a dimensionless density parameter

$$\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G\rho}{3H^2}. \tag{32}$$

This allows us to rewrite the evolution equation (5) in the dimensionless form

$$\Omega = 1 + \frac{kc^2}{H^2}. \tag{33}$$

Obviously, for a flat universe $k = 0$ and $\Omega = 1$, whereas for a spherical universe $k = +1$ and $\Omega > 1$, and for a hyperbolic universe $k = -1$ and $\Omega < 1$.

From the present value of the Hubble parameter we can calculate the present value of the critical density:

$$\rho_{c0} = \frac{3H_0^2}{8\pi G} = 5.3 \left(\frac{h_0}{0.7}\right)^2 \text{ GeV m}^{-3} \text{ c}^{-2}. \tag{34}$$

The amount of baryonic matter in the universe is estimated to represent a density of the order $\Omega_{bar} = 0.015$ — 0.030. Then the actual density is

$$\rho_{bar,0} = \Omega_{bar}\, \rho_{c0} \approx 0.078 — 0.16 \,\text{GeV m}^{-3} \text{ c}^{-2}, \tag{35}$$

of the order of one nucleon per 10 cubic meters.

4 Thermal evolution of the universe

Thermal equilibrium can be maintained in the universe whenever the rate of thermal relaxation is fast compared to the expansion rate of the universe. In practice, this means whenever the thermal relaxation time is short compared to the Hubble time:

$$\tau_{thermal} < \frac{1}{H}. \tag{36}$$

If this is not the case and the universe expands very rapidly compared to thermal equilibration processes, this can lead to a situation in which the mean free path of some particles is becoming longer than the linear size of the observable universe. In this situation particles cease to interact at all and decouple from the rest of the matter in the universe. Only gravity then drives their evolution.

First consider the case of thermal equilibrium. For cold matter the total number of particles is conserved (no particle creation or annihilation takes place), and can we use the ideal gas laws

$$u = \varepsilon a^3 = \bar{n}mc^2 + \frac{3}{2}\bar{n}kT, \qquad pv = pa^3 v_0 = n_0 kT, \tag{37}$$

with $\bar{n} = n_0/v_0$ a constant. Combining these relations with the second evolution law (7) we find

$$\frac{3}{2}\frac{dT}{dt} + \frac{T}{a^3}\frac{da^3}{dt} = 0, \tag{38}$$

with the solution

$$a^2 T = \text{constant}. \tag{39}$$

Thus every time the scale factor grows by a factor 2, the temperature goes down by a factor 4.

For relativistic matter and radiation, the energy density depends on the temperature according to the Stefan-Boltzmann law:

$$\varepsilon(T) = \alpha T^4, \tag{40}$$

where the constant of proportionality depends on the nature of the radiation: if the radiation is a mixture of particles of type i, each at the same temperature, then each component in the gas contributes to the energy density with its own constant of proportionality

$$\varepsilon = \sum_i \varepsilon_i = \sum_i \alpha_i T^4 = \alpha T^4 \quad \Rightarrow \quad \alpha = \sum_i \alpha_i. \tag{41}$$

Statistical mechanics tells us, that the constant is actually the same for each type of relativistic boson:

$$\alpha_{boson} = 0.76 \times 10^{-15}\ \mathrm{J\ m^{-3}\ K^{-4}}, \tag{42}$$

and also for each type of relativistic fermion:

$$\alpha_{fermion} = \frac{7}{8}\alpha_{boson} = 0.67 \times 10^{-15}\ \mathrm{J\ m^{-3}\ K^{-4}}. \tag{43}$$

Therefore

$$\alpha = \left(n_B + \frac{7}{8} n_F\right) \alpha_{boson}, \tag{44}$$

where n_B and n_F denote the number of species of bosons and fermions, respectively.

After this digression we return to eq.(40). In combination with eq.(13) we infer that

$$\varepsilon a^4 = \alpha (aT)^4 = \text{constant} \quad \Rightarrow \quad aT = \text{constant}. \tag{45}$$

Thus relativistic matter cools slower than non-relativistic matter: the radiation temperature only drops by a factor 2 when the scale factor grows by a factor 2.

For the case of decoupled matter or radiation, one can show a simple result: if the matter or radiation is in thermal equilibrium before the decoupling, then it evolves after decoupling as if it remains in thermal equilibrium. This means, that cold matter, characterized by a Maxwell-Boltzmann distribution, maintains a Maxwell-Boltzmann distribution with a temperature changing inversely proportional to the square of the scale factor: $T_{mat} \propto a^{-2}$; relativistic matter can be shown to follow a Bose-Einstein or Fermi-Dirac distribution with temperature inversely proportional to the scale factor itself: $T_{rad} \propto a^{-1}$.

5 The cosmic background radiation

The cosmic microwave background follows a pure planckian black-body spectrum, with a temperature of 2.73 K. As the universe is completely transparent to this radiation, i.e. the microwave photons can travel without scattering, the radiation is thermally decoupled from the rest of the matter in the universe. This has not always been the case: when the universe was much smaller and hotter, all matter was ionized and photons interacted continuously with the charged particles (nucleons and electrons) in the hot plasma.

The transition from an opaque to a transparent universe marks the time of decoupling of the photon gas in the universe. It took place somewhat below the ionisation temperature of hydrogen, at about 3×10^3 K, corresponding to about 0.3 eV. Now as the photon gas is a relativistic boson gas, its temperature is related to the scale factor by relation (45). A drop in temperature from 3000 K to 2.73 K corresponds to a change in scale factor by

$$\frac{a_0}{a_{dec}} \approx \frac{3000}{2.73} \approx 1100. \tag{46}$$

As this is very much larger than 1, eq.(25) implies that the redshift since that time was $z = 1100$, corresponding to the transition from (nearly) visible light at decoupling time to microwave frequencies at the present time.

Recent very accurate studies of the background radiation have revealed very small fluctuations in the temperature of the radiation from different directions of the sky, of the order of 10^{-5} or less. These temperature fluctuations are thought to arise from gravity-induced density fluctuations in the hot plasma in the early universe; one can think of such fluctuations as sound waves, propagating through the plasma. Photons get trapped in denser regions, which will have a slightly higher temperature than the more diluted regions. The effect on the microwave background is maximal when the half-period of the wave fits the Hubble time at the moment of decoupling. The wavelength of such waves can be computed, and converted to an angular size. Comparing these with the typical angular size of observed temperature fluctuations one can perform a triangulation and thereby get a handle on the geometry of space. From such measurements one infers that the spatial geometry of the universe is actually flat: $k = 0$. It follows, that the density of the universe is critical: $\Omega = 1$ and $\rho_0 \approx 5.3$ GeV m^{-3} c^{-2}, as given by (34).

With this information we can also calculate the age of the universe at the time of decoupling. For a radiation dominated (hot and relativistic) universe with $k = 0$, we can use eq.(16) with t_0 the time of decoupling, and $t = 0$ the time of the initial singularity, at which $a(0) = 0$:

$$0 = a_0\sqrt{1 - \kappa_0 t_0} \quad \Rightarrow \quad t_0 = \frac{1}{\kappa_0} = \sqrt{\frac{3}{32\pi G\rho_0}}. \tag{47}$$

Now at decoupling time

$$\rho_0 c^2 = \alpha T_0^4, \qquad \alpha = (n_B + \frac{7}{8}\, n_F)\, \alpha_{boson} = \frac{29}{8}\, \alpha_{boson}. \tag{48}$$

The 29/8 comes from assuming contributions of one species of massless bosons (photons) and 3 species of massless fermions (neutrinos). The decoupling temperature T_0 is 3000 K, as mentioned above. Putting all the numbers into eq.(47) gives an age estimate of

$$t_0 \simeq 3 \times 10^5 \, \text{yrs} \tag{49}$$

at decoupling time.

6 The neutrino background

If we extrapolate to even earlier times, we eventually encounter energy densities and temperatures at which not only the electromagnetic, but also the weak intereactions were effective in maintaining thermal equilibrium. The weak interactions allow transitions transfering weak isospin, like

$$p^+ + e^- \leftrightarrow n + \nu, \qquad n + \bar{e}^+ \leftrightarrow p^+ + \bar{\nu}, \qquad n \rightarrow p^+ + e^- + \bar{\nu}. \tag{50}$$

Another reaction important in establishing ethermal quilibrium between free electrons and neutrinos is $e^- + \bar{e}^+ \leftrightarrow \nu + \bar{\nu}$. The interactions (50) are effective in maintaining equilibrium between leptons and nucleons at temperatures above 1 MeV ($\sim 10^{10}$ K). The corresponding redshift is about $z \sim 3 \times 10^9$, and the universe was about 1 second old.

Therefore we take as our starting point a situation in which nucleons, electrons and positrons, neutrinos and photons were in thermal equilibrium at a temperature of $T_1 \sim \mathcal{O}(1\,\text{MeV})$. At that time the weak interactions ceased to be effective, and neutrinos decoupled. Afterwards their distribution remained thermal, with a characteristic temperature given by $T_\nu(t) = a_1 T_1 / a(t)$, for $t > 1$ second.

As for the remaining interacting gas of electrons, positrons and photons, it maintained thermal equilibrium at the same temperature as the effective neutrino temperature, until the temperature dropped below a few hundred keV and electrons and positrons started to recombine. At that time virtually all positrons were annihilated, but in the process a lot of extra photons were created. This made the temperature of the photons rise. As a result, after the $\bar{e}^+ e^-$-annihilation the photon temperature no longer equalled that of the neutrinos.

Thermodynamics allows one to calculate the temperature difference, on the assumption that the process proceeded adiabatically. Using

$$Ts = u + pv, \tag{51}$$

with $u = 3pv = \varepsilon v$ and $\varepsilon = \alpha T^4$, we find

$$\frac{s}{v_0} = \frac{4}{3} \frac{\varepsilon v}{T v_0} = \frac{4}{3} \alpha (aT)^3. \tag{52}$$

During the transition the entropy does not change, and neither does the temperature of the decoupled neutrinos. Before the transition we have one species of massless bosons (photons) and 2 species of massless fermions (e^-, $\bar{e}^+$) interacting to maintain thermal equilibrium at the same temperature T_ν:

$$n_B + \frac{7}{8} n_F = \frac{11}{4} \quad \Rightarrow \quad \frac{s_1}{v_0} = \frac{4}{3} \cdot \frac{11}{4} \cdot \alpha_{boson} (aT_\nu)^3. \tag{53}$$

After the annihilation we have only one species of massless bosons, no longer in interaction with a relativistic fermion gas (only a small dilution of non-relativistic protons and electrons). The entropy after the transition is therefore calculated to be

$$\frac{s_2}{v_0} = \frac{4}{3} \alpha_{boson} (aT_\gamma)^3, \tag{54}$$

where T_γ is the new photon temperature. Equating the two expressions (53) and (54) then gives

$$T_\gamma = \left(\frac{11}{4} \right)^{1/3} T_\nu. \tag{55}$$

Note, that the precise temperature at which the reheating of the photon gas happens is not important, as photon and neutrino temperatures scale in the same way with $a(t)$, the size of the universe. At the present time, at which the photon temperature has dropped to $T_{0\gamma} = 2.73$ K, we therefore predict a background neutrino gas with a temperature of

$$T_{0\nu} = \left(\frac{4}{11} \right)^{1/3} 2.73 = 1.95\,\mathrm{K}. \tag{56}$$

At these extremely low energies these neutrinos will be very hard to detect. If neutrinos have a mass, then at low temperatures they become non-relativistic; for example, neutrinos with a mass of the order of $m_\nu \simeq 10^{-3}$ eV, become non-relativistic at temperatures below ~ 10 K.

7 Primordial hydrogen and helium

After the decoupling of the neutrinos and the weak interactions, the number of protons and neutrons is fixed, except for the decay of free neutrons described by the last of eqs.(50). However, this process is very slow compared to the age of the universe at that time ($\tau_n = 887$ s), so plenty of neutrons remained around to form stable light nuclei at the next step in the evolution of matter.

The initial ratio of neutrons to protons is determined by thermal equilibrium (in the non-relativistic regime for nucleons) through the Boltzmann factor

$$\frac{n_n}{n_p} = e^{-\Delta m / kT_1}. \tag{57}$$

Here $\Delta m = m_n - m_p = 1.3$ MeV$/c^2$, and T_1 is decoupling temperature; its value can be estimated to be $kT_1 \approx 0.75$ MeV. Now the total number of nucleons is $n_N = n_p + n_n =$ constant, by conservation of baryon number; hence the fraction of nucleons in the form of neutrons is

$$X_n(t_1) = \frac{n_n}{n_N} = \frac{1}{1 + e^{\Delta m/kT_1}} = 0.15. \tag{58}$$

The number of neutrons then starts to decay exponentially, whilst the total number of nucleons remains constant. At a later time t the fraction of free neutrons is then reduced to

$$X_n(t) = X_n(t_1)\, e^{-(t-t_1)/\tau_n}. \tag{59}$$

All neutrons would have disappeared this way if it were not for the fact that when the universe cooled and the density of nucleons was still high, neutrons formed stable bound states with protons: deuterium (D = ^{2}H), tritium (^{3}H), helium-3 (^{3}He) and especially α-particles (^{4}He). Clearly, deuterium was formed first, in 2-body reactions $p + n \rightarrow D + \gamma$, under an energy release of 2.2 MeV. However, the binding energy of α-particles is 24 MeV, which is so much higher than that of deuterium, or tritium or ^{3}He, that within a very short time —at least to a first approximation— all neutrons end up in ^{4}He by secondary reactions.

In order for the synthesis of deuterium to proceed at an appreciable rate, each neutron should encounter a proton at least once during the time the universe exists. This implies, that the number density of nucleons at the temperature where the reaction takes place should be related to the cross section σ_{pn} by

$$n\langle \sigma_{pn} v \rangle_T \approx \frac{1}{t}. \tag{60}$$

The quantity enclosed in angular brackets is the average value at temperature T of the volume swept out by the nucleons per unit of time. For a cross section of $\sigma_{pn} \sim 10^{-2}$ millibarn (10^{-33} m^2), this condition is satisfied at a density of about 10^{24} nucleons/m^3 and a temperature of about $T_{nuc} = 6 \times 10^8$ K, or 60 keV. At that temperature the age of the universe was about 200 seconds. By that time the fraction of nucleons in neutrons was reduced to

$$X_n(t_2 = 200\text{s}) = 0.12. \tag{61}$$

All these neutrons ended up in α-particles, containing 2 protons and 2 neutrons each. This implies that the number of neutrons in the universe equals half the number of primordial α-particles formed; for the number densities this implies $n_n = 2n_\alpha$. Then the fraction of the mass densities of baryons in primordial helium was

$$\frac{\rho_\alpha}{\rho_N} = \frac{n_\alpha m_\alpha}{n_N m_N} = \frac{4n_\alpha}{n_N} = \frac{2n_n}{n_N} = 2X_n(t_2) = 0.24. \tag{62}$$

This prediction agrees very well with the amount of primordial ^{4}He that is observed today.

8 The matter density in the universe

The number density of nucleons to get successful synthesis of light elements with the correct ratio of primordial ^{4}He over hydrogen as given above is $n_N = 10^{24}/\mathrm{m}^3$ at a time the universe was 200 seconds old. Extrapolated to the present epoch, this is reduced to about $n_{N0} = 0.1/\mathrm{m}^3$. This is in agreement with the more sophisticated estimates including non-luminous baryonic matter, on which the estimates (35), and corresponding $\Omega_{bar} \simeq 0.02$, were based.

In addition to the energy of ordinary matter composed of baryons and leptons, there is energy in radiation. However, the energy density in the cosmic microwave background as given by the Stefan-Boltzmann law amounts to

$$\varepsilon_{cmb} = \alpha_{boson} T_{cmb}^4 = 0.26 \times 10^{-3}\,\mathrm{GeV\ m}^{-3}, \tag{63}$$

implying that $\Omega_{cmb} = 0.5 \times 10^{-4} \ll \Omega_{bar}$. It follows, that the microwave background no longer plays a role in determining the large-scale evolution of the universe.

Nevertheless, an analysis of the microwave background gives strong support to the hypothesis that the universe is critical: $\Omega = 1$. These arguments indicate, that most of the energy-matter density in the universe must be non-baryonic. There are quite a number of observations supporting the existence of dark matter. These include:

1. *Rotation curves of galaxies.* Keplers third law states, that for circular orbits the square of the period τ of a planetary orbit is proportional to the cube of its radius R:

$$\tau^2 = \frac{4\pi^2}{GM}\, R^3, \tag{64}$$

where M is the central mass (mass of the sun) whose gravitational field binds the planet in its orbit. A similar result should also hold for stars at the edges of galaxies, assuming that the central mass is the galactic mass concentrated in the luminous bulk of the galaxy. Measurements of such stellar velocities show, that this is not the case: orbital periods of stars do not increase with distance fast enough. This implies that a considerable part of the galactic mass is found far out, including the region near the edge of the luminous disk. Whether the non-luminous matter which accounts for the missing mass is in the form of dust, planets, black holes or massive neutral particles, floating in a halo around galaxies, is unknown. However, as nucleosynthesis puts an upper limit on the baryon density of the universe, not all missing mass can be in the form of dark baryonic matter.

2. *Dynamics of galactic clusters.* One can model the motion of galaxies in galactic clusters by assuming their masses proportional to their visual size, and their interactions resulting from newtonian gravity as determined by the distibution of masses inside the cluster. From such simulations one concludes not only that galaxies seem to have more mass than expected on the basis of their luminosity,

but also that there seems to be a considerable amount of mass distributed in between the visible galaxies.
3. *Gravitational lensing.* Light is deflected in gravitational field. As a result, large masses (like galaxies or clusters of galaxies) can distort the images of light sources further away. Many instances of such gravitational lensing effects are known. More recently, gravitational lensing effects have been observed in regions where there seems to be no visible object acting as source of the gravitational bending field. Whether these are 'dark galaxies' or some other massive objects is not known.

If one accepts that the mass-energy density of the universe is critical: $\Omega = 1$, the question arises what the nature of the dark matter or dark energy is. Many conjectures have been made. For example, if neutrinos are massive, they can contribute significantly to the energy density. At the time of neutrino decoupling, they were highly relativistic, and their energy density was given by the fermionic Stefan-Boltzmann equation:

$$\varepsilon_\nu = \frac{7}{8}\, \alpha_{boson}\, T_\nu^4. \tag{65}$$

If they remained relativistic up to the present era, they would now have a temperature of 1.95 K, and the neutrino number density would be about $10^9/\mathrm{m}^3$. If these particles would be massive, their mass would contribute an amount $10^9 \times m_\nu c^2$ to the energy density. This would equal the critical density if

$$m_\nu \simeq 5\, \mathrm{eV/c^2}. \tag{66}$$

Put differently: neutrino masses cannot exceed this order of magnitude for the universe to keep on expanding forever. Evidence for non-zero neutrino masses has been established in recent times, but the order of magnitude is more likely to be in the range of 10^{-3} eV. This falls short by quite a large amount to reach the critical density of the universe.

Other conjectured candidates for dark matter are neutral massive particles predicted by supersymmetric extensions of the standard model. In these extensions each known particle (quark, lepton or gauge boson) is accompanied by a heavy partner of the same charge but different spin: quarks and leptons are required to have spino-0 scalar partners, vector bosons have spin-1/2 fermion partners. Also the graviton, the quantum of the gravitational field, is conjectured to have a spin-3/2 particle, the gravitino. Some of these particles, if they exist, could be present as dark matter in the universe.

Finally, there is also the possibility that $\Omega = 1$ not because of some particular form of matter, but because of the presence of a cosmological constant. If the cosmological constant exists, the expansion of the universe should be accelerating, rather than slowing down. Some evidence for this has recently been claimed on the basis of measurements of the brightness of supernovae in the early universe. The possibility of a non-zero cosmological constant has therefore regained some acceptability, and would have important consequences for our understanding of the evolution of the universe.