

# K and L-absorption edges of X-rays / Moseley's law and Rydberg constant

## **Related topics**

X-ray Bremsstrahlung, characteristic radiation, Bragg equation, Bohr's atomic model, atomic energy level scheme, Moseley's Law, Rydberg constant, shielding constant.

# **Principle and task**

Various samples of elements of different atomic numbers are irradiated with X-rays of a known spectral intensity distribution. The energy of the transmitted intensity is analysed using a monocrystal analyser. The Rydberg and shielding constants are found by determining the energy at which the absorption edges occur.

## Equipment

X-ray unit. w. recorder output	09056.97	1
Counter tube, type A, BNC	09025.11	1
Chemical set for edge absorption	09056.04	1
Mortar w. pestle, 70 ml, porcelain	32603.00	1
Microspoon, special steel	33393.00	1
Screened cable, BNC, I 750 mm	07542.11	1
Connecting cord, 1000 mm, red	07363.01	2
Connecting cord, 1000 mm, blue	07363.04	2

## Problems

- 1. The intensity of the X-rays emitted from the copper anode is to be recorded as a function of the Bragg angle by means of a LiF monocrystal analyser.
- 2. The K-absorption edge is to be found for different absorber materials.

- 3. The Rydberg and shielding constants are to be calculated from the energy values of the K-absorption edges.
- 4. The L-absorption edges are to be found for different absorber materials.

# Set-up and procedure

The experiment is set up as shown in Fig. 1. First of all, the X-ray spectrum of the Cu X-ray tube should be found for the maximum anode voltage and without absorber material.

The aperture of d = 2 mm is introduced into the outlet of X-rays.

By pressing the "zero key", the counter tube and crystal holder device are brought into starting position. The crystal holders are mounted with the crystal surface set horizontally. The counter tube, with horizontal slit aperture, is mounted in such a way that the mid-notch of the counter tube closes onto the back side of the holder. Typical adjustment settings of the equipment are:

#### Pulse rate meter:

x-

Counter tube voltage	500 V
Range	(10 <sup>3</sup> –10 <sup>4</sup> ) Imp/s
Time constant	0.5 or 1.5 s
y-recorder: x-axis (ϑ-axis) y-axis (Intensity)	1 V/cm, additionally variable 0.1 V/cm, additionally variable

#### Fig. 1: Experimental set-up for determination of absorption edges.





The output of the pulse rate meter, is connected to the y-input of the recorder. The DC voltage from the X-ray unit, proportional to the angle (0.1 V/degree), is applied to the x-input. The spectra are recorded at a slow rotational speed (switch settings "V<sub>1</sub>", and "Auto") with synchronous rotation of the crystal and the counter tube.

The absorber materials should be pounded fine with the mortar and, for each material, a layer of 0.2 mm to 0.3 mm should be obtained, held betweentwo strips of Scotch tape. The layer is then fixed to the aperture of d = 5 mm. If the absorber layers are too thin, a precise edge effect cannot be observed. Intensity problems occur if the layers are too thick. It is recommended that well prepared samples of absober material are kept for repetition measurements.

Samples for *L* edge determination give rise to a higher overall absorption due to the higher atomic number, so that the sensitivity of the pulse rate meter should be increased in these cases. The time constant of the pulse rate meter can be increased to a maximum of 5.5 s to improve the signal/noise ratio.

#### Note

With automatic scanning of the angle, long time constants cause erroneous positions for the Bragg angle of the characteristic structures in the recorded spectra. Here, it is recommended, for example, that the automatic scanning is interrupted in the range of the absorption edges and that it is changed step by step by pressing the key in time intervals matched to the time constant used.

#### Theory and evaluation

X-rays wich penetrate material can lose their energy due to Compton scattering, pair formation or the photoelectric effect. The effective cross-sections of these processes depend on the wavelength of the radiation. Within the range of wavelength available only the photoelectric effect needs to be considered to any significant degree.

Fig. 2 shows schematically the curve of the transmission T as a functio of the photo-energy. According to this curve, T increases with increasing energy and then suddenly decreases at an energy characteristic for the irradiated material. This sudden increase in the absorption, i.e. the absorption edge, is caused by the onset of photo-absorption.



Fig. 2: Diagrammatic curve of transmission for edge absorption.

The binding energy  $E_n$  of an electron on the nth shell for atoms similar to hydrogen is approximately

$$|E_{n}| = \frac{m_{e} \cdot e^{4}}{8 \varepsilon_{0}^{2} h^{2}} (Z - \sigma)^{2} \cdot \frac{1}{n^{2}}$$
(1)



Fig. 3 a: Intensity of the Cu X-radiation as a function of the Bragg angle  $\vartheta$ 

Fig. 3b-h: X-ray spectra of copper with K-absorption edges for various absorbers.



Electron mass $m_{\rm e}$ = 9.1091  $\cdot 10^{-31}$  kgElementary chargee= 1.6021  $\cdot 10^{-19}$  CPlanck constanth= 6.6256  $\cdot 10^{-34}$  JsDielectric constant $\varepsilon_0$ = 8.8544  $\cdot 10^{-12}$  N<sup>-1</sup>m<sup>-2</sup>C<sup>2</sup>Atomic numberZShielding constant $\sigma$ 

With the introduction of the Rydberg constant

$$\mathsf{R} = \frac{m_e \cdot e^4}{8 \, \varepsilon_0^2 \, h^2} = 3.28989 \cdot 10^{15} \mathrm{s}^{-1} \tag{2}$$

equation (1) becomes

$$|E_{n}| = R \cdot h \left(Z - \sigma\right)^{2} \cdot \frac{1}{n^{2}}$$
(3)

For electrons within the K shell the value of the shielding constant wich depends on the atomic number is  $\sigma\approx 1$ . The intensity of the Cu X-radiation for various absorber materials is given in Fig. (3b–3h) as a function of the Bragg angle  $\vartheta$ . Fig. 3a shows the X-ray spectrum without absorber. Within the Bremsspectrum absorption edges can be clearly seen. These move with increasing atomic number in the direction of lower Bragg angles. Using the Bragg equation

$$2d\sin\vartheta = \lambda \tag{4}$$

(interplanar spacing of the LiF monocrystal analyser d = 201.4 pm, wavelength  $\lambda$ ) and the relation

$$E = h \cdot f = \frac{h \cdot c}{\lambda} \tag{5}$$

(Velocity of light  $c = 2.998 \cdot 10^8 \text{ ms}^{-1}$ )

the energy at which the absorption edge can be observed is found to be

$$E = \frac{h \cdot c}{2d \sin \vartheta} \tag{6}$$

Table 1: K-edge energies in keV

	Z	$\vartheta_{exp}$	$E_{k^{-}exp}$	$E_{k}$ -Lit.
Zn	30	18.6°	9.7	9.66
Ge	32	16.1°	11.1	11.10
As	33	15.0°	11.9	11.87
Se	34	14.3°	12.5	12.66
Br	35	13.2°	13.5	13.47
Rb	37	11.6°	15.3	15.20
Sr	38	11.0°	16.1	16.10

In Tab. 1 the edge energies found from Fig. 3 are compared with the corresponding accepted literature values for the K-energy levels.

Fig. 4: Moseley diagram of *K*-edge absorption for the determination of the Rydberg constant.



In order to find the Rydberg constant R, (3) is rearanged to

$$Z = \frac{1}{\sqrt{R} \cdot h} \cdot \sqrt{E} + \sigma \text{ (with } n = 1\text{)}$$
(7)

In Fig. 4  $\sqrt{E}$  has been plotted against Z.

From the slope 
$$a = \frac{1}{\sqrt{R \cdot h}}$$
 the following is obtained

$$R = \frac{1}{h} \cdot \frac{[\Delta (\sqrt{E})]^2}{(\Delta Z)^2} = 3.13 \cdot 10^{15} \text{s}^{-1}$$
(8)

The mean value  $\sigma\approx$  2.7 is obtained for the shielding constant.

The close agreement between the theoretical value for R and that achieved in the experiment gives retrospective justification for the assumption that K edge absorption, i.e. n = 1, is involved here.

Fig. 5 shows spectra with absorbers of higher atomic number. Two absorption edges can be seen in each case which are attributed to *L*-shells. The edge triplet of all three *L*-subshells can not be observed due to limited resolution. The absorption edge for the lower Bragg angle can be considered as the average of the closely positioned edges of the  $L_1$  and  $L_2$  levels. An evaluation of the measurement results according to (1) is no more allowed since other interactive processes must be taken into account apart from the Coulomb interaction.

However, as shown in Fig. 6, if the root of the energies which are associated to the L edges are plotted against the atomic numbers, Moseley lines are again obtained.

The experimentally determined *L*-energies are compared with the corresponding accepted literature values in Table 2.

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Fig. 5: X-ray spectra of copper with *L*-absorption edges for various absorbers.



	Table 2:	L-edge	energies	in	keV.
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	Z	E <sub>L1/2</sub> -exp	E <sub>L3</sub> -exp	E <sub>L1</sub> -lit	E <sub>L2</sub> -lit	E <sub>L3</sub> -lit
W	74	11.6	10.1	12.10	11.54	10.21
Hg	80	14.5	12.3	14.84	14.21	12.28
Pb	82	15.3	12.7	15.86	15.20	13.03
Bi	83	16.3	13.3	16.39	15.71	13.42

Fig. 6: Moseley diagram for *L*-edge absorption.



## Note

• Atomic energy values are taken from "Handbook of Chemistry and Physics", CRC Press, Inc., Florida.

Instead of the usual recording system (pulse rate meter 13622.93 and xyt-recorder 11416.97), we alternatively recommend the following:

COBRA-Interface	12100.93	1
PC-COBRA-data cable RS232	12100.01	1
Counter tube module	12106.00	1
Software COBRA		
X-ray spectrometry	14259.51	1
Personal computer		

Reference no. 25405-11