Related topics

Scattering, angle of scattering, impact parameter, central force, coulomb field, coulomb forces, Rutherford atomic model, identity of atomic number and charge on the nucleus.

Principle and task

The relationship between the angle of scattering and the rate of scattering of α-particles by gold foil is examined with a semiconductor detector. This detector has a detection probability of 1 for α-particles and virtually no zero effect, so that the number of pulses agrees exactly with the number of α-particles striking the detector.

In order to obtain maximum possible counting rates, a measurement geometry is used which dates back to Chadwick (Fig. 2). It is also possible in this case to shift the foil and source in an axial direction (thus deviating from Chadwick's original apparatus), so that the angle of scattering can be varied over a wide range.

In addition to the annular diaphragm with gold foil, a second diaphragm with aluminium foil is provided in order to study the influence of the scattering material on the scattering rate.

Equipment

Annular diaphragm w. gold foil	09103.02	1
Annular diaphragm w. alumin. foil	09103.03	1
U-magnet, large	06320.00	1

Container f. nuclear phys. expts.	09103.00	1
Americium-241 source, 370 kBq	09090.11	1
Alpha detector	09100.00	1
Pre-amplifier f. alpha detector	09100.10	1
Impulse height analyser	13725.93	1
Geiger-Müller-Counter	13606.99	1
Vacuum pump, one stage	02750.93	1
Range multiplier, vacuum	11112.93	1
Moving coil instrument	11100.00	1
T-connection NW 10	02668.13	1
Hose nipple NW 10	02668.12	2
Centring and sealing ring NW 10	02668.04	3
Clamping ring NW 10	02668.03	3
Rubber tubing, vacuum, i.d. 8 mm	39288.00	2
Adapter, BNC-socket/4 mm plug pair	07542.27	1
Screened cable, BNC, l 750 mm	07542.11	3

Problems

1. The particle rates are measured at different angles of scattering between about 20° and 90°. The measurements are compared with the particle rates calculated by means of the Rutherford theory for the measurement geometry used.

2. The particle rates are measured in the case of scattering by aluminium and gold with identical angles of scattering in each case. The ratio of the two particle rates is compared with the particle rate calculated from Rutherford's scattering equation.

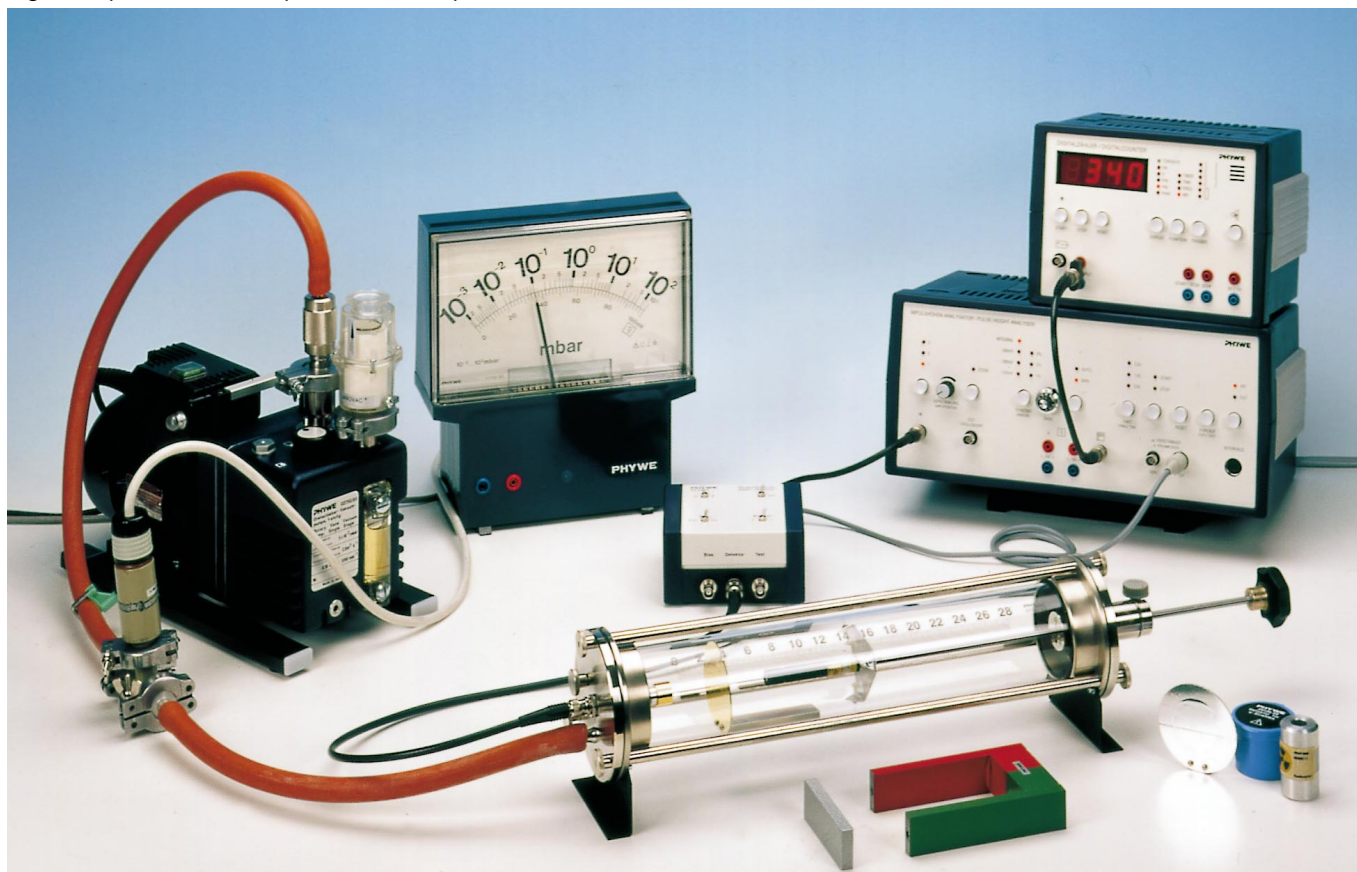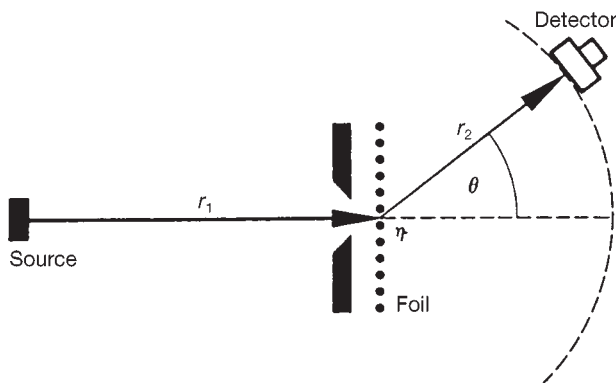Fig. 1: Experimental set-up Rutherford experiment.

Fig. 2: Chadwick's experimental set-up of the Rutherford experiment.

Set-up and procedure

Fig. 1. shows the complete experimental layout. The α-detector, the annular diaphragm with gold foil and the ^{241}Am source screwed into the movable source holder are placed in the container. The pre-amplifier is set for α-particles, Bias switch is set to internal and "–". The output is connected with the "in" of the pulse height analyzer.

The counter output is connected via a BNC-cable and an adapter BNC-socket/4 mm plug to the potential free input of the counter.

The foils are moved with the help of the magnet. Further information please refer to the operating instructions.

The pulse height analyzer remains initially switched off (no voltage applied to the α-detector).

Preparation

Evacuate the container to about 0.5 hPa. Close the vacuum hose with a hose clamp. Switch off the pump (the vacuum is sufficient when the pressure has fallen clearly below 2 hPa).

Switch on the pulse height analyzer. Select the "Integral" window on the pulse height analyzer.

Release the "Auto/Man" key (for "manual" operation).
Set to a voltage of 0.5 V with the "base" adjusting knob (0.50 scale divisions) in order to suppress the noise.

Measurements

The counting rate is determined for different distances *l* between source and detector. The distance *l* may in this case be varied between 22 cm and 4 cm, corresponding to a scattering angle range of about 20° to 90°. The measuring period selected should be sufficient to achieve a pulse count of 100.

The pulse height analyzer is then switched off, the container ventilated and opened the gold foil annular diaphragm replaced by the aluminium foil diaphragm. The procedure described above under "Preparation" is then followed and the particle rate determined at a scattering angle of about 45°. (Select a scattering angle for which a measurement is already available in the measurement series with the gold foil diaphragm).

Theory and Evaluation

Rutherford's theory on the scattering of α-particles is based on the following assumptions:

(a) The mass of the atom is concentrated in a nucleus which can be regarded as a single point relative to the dimensions of the atom as a whole.

(b) This nucleus carries a positive charge and is therefore surrounded by a coulomb field which decreases in proportion to $1/r^2$.

(c) α-particles are repelled in this coulomb field as a result of their likewise positive charge and subsequently follow hyperbolic paths.

The particle rate $\Delta n(\Theta)$ of particles scattered through the angle Θ in a solid angle $d\Omega$ is governed under the above assumptions by Rutherford's scattering equation for α-particles:

$$\Delta n(\Theta) = \; \cdot \; N \cdot d_F \cdot \frac{1}{4} \left(\frac{2\,Z\,e^2}{4\pi\varepsilon_0 \cdot 2\,E_\alpha} \right)^2 \frac{d\Omega}{sin^4 \frac{\Theta}{2}} \cdot \quad (1)$$

in which n = particle rate in the foil

$\quad N$ = atomic concentration in the foil

$\quad d_F$ = foil thickness

$\quad Z$ = nuclear charge of the scattering atoms

$\quad E_\alpha$ = energy of the α-particles

$\quad e$ = elementary charge = $1.6021 \cdot 10^{-19}$ As

$\quad \varepsilon_0$ = electric constant = $8.8524 \cdot 10^{-12}$ As/Vm

The particle rate n in the foil is obtained from the expression

$$n = \frac{Q}{4\pi r_1^2} \cdot A_F$$

in which A_F = irradiated area of the foil

$\quad Q$ = decay rate (activity) of the source

assuming the source to be a point-type emitter (see also Fig. 1).

The solid angle $d\Omega$ covered during particle counting is determined by the distance r_2 between foil and detector and by the surface area of the detector A_D (Fig. 2):

$$d\Omega = \frac{A_D}{r_2^2} \cdot$$

Incorporating these quantities in equation (1), we obtain by means of the following abbreviation:

$$S = N \cdot \frac{1}{4} \left(\frac{2\,Z\,e^2}{4\pi\,\varepsilon_0 \cdot 2\,E_\alpha} \right)^2$$

the scattering equation in the following form

$$\Delta n(\Theta) = \frac{Q \cdot A_F \cdot d_F}{4\,\pi\,r_1^2} \cdot \frac{A_D}{r_2^2} \cdot S \cdot \frac{1}{sin^4 \frac{\Theta}{2}} \cdot \quad (2)$$

For safety reasons the activity of the α-emitter used is restricted to 370 kBq. When using an arrangement as shown in Fig. 2 such low counting rates are obtained at this source strength from equation (2), that the measurements necessary to verifi-

Fig. 3: Sketch of the experimental set-up of the Rutherford experiment used in this experiment.
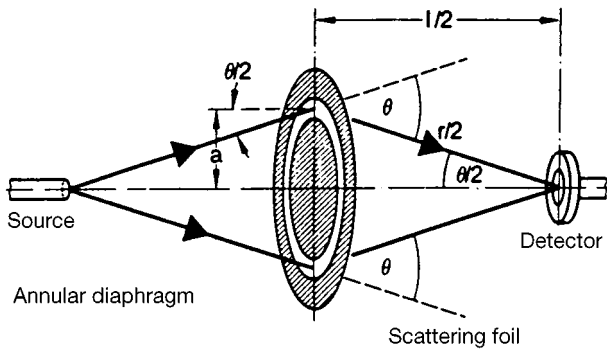
cation of the scattering equation cannot be performed during the time available for the experiment. An arrangement as shown in Fig. 3 is therefore used, corresponding to a slightly modified version of a proposal put forward by Chadwick in 1920. The detector consists of a surface barrier-layer counter which records all the α-particles striking its barrier layer and shows no zero effect. The foil forming an annular diaphragm is, like the source, arranged to be movable. The scattering angle Θ is varied by shifting these two components. The foil is always arranged centrally between source and detector, so that the scattering angle is given by following expression:

$$\Theta = 2 \arctan \frac{2\,\alpha}{l} \,.$$

The following symbol replacements are made in the scattering equation (2) for this arrangement:

– A_D by $A'_D = A_D \cos \dfrac{\Theta}{2}$

(The projectionof the detector surface in the direction of the radiation is used in this case for calculation of the solid angle covered dΩ).

– d_F is replaced by $d'_F = \dfrac{d_F}{\cos \Theta /2}$

(d'_F being the extended path travelled by the particles as a result of the oblique radiation in the foil).

– A_F is replaced by $A'_F = A_F \cdot \cos \dfrac{\Theta}{2}$

(the effective foil surface A'_F being reduced by oblique radiation).

– $r_1 = r_2$ is replaced by $r/2$

(r being the length of the path travelled by the particle from the source to the detector).

Bearing in mind also that

$$\sin \frac{\Theta}{2} = \frac{2\,\alpha}{r}$$

and that

$$\frac{1}{\sin^4 \Theta /2} = \frac{r^4}{16\,\alpha^4}$$

we obtain from the scattering equation (2) in its original form the following expression:

$$\Delta n(\Theta) = \frac{Q A_F d_F}{4\,\pi \left(\frac{r}{2}\right)^2} \cdot \frac{A_D \cos \Theta /2}{\left(\frac{r}{2}\right)^2} \cdot S \cdot \frac{r^4}{16\alpha^4} \,. \qquad (3)$$

Increasing the angle of scattering by reducing the particle path r causes an immediate fall in the counting rate in proportion to r^4 as a result of the $\sin^{-4} (\Theta/2)$ term in the Rutherford scattering equation. It also causes an increase in the particle rate in the foil in proportion to $1/r^2$ and in the solid angle covered by the detector surface in proportion to $1/r^2$, so that the influence of the $\sin^{-4} (\Theta/2)$ term is exactly compensated by the geometry of the arrengement. Only one of the three $\cos (\Theta/2)$ expressions resulting from the oblique radiation pattern can be eliminated by cancellation. Equation (3) is simplified in this way to

$$\Delta n(\Theta) = \frac{Q A_F d_F A_D}{4\,\pi\,\alpha^4} \cdot S \cdot \cos \frac{\Theta}{2} \,. \qquad (4)$$

The following values have to be introduced into this equation for calculation of the scattering rates anticipated in accordance with Rutherford's theory:

Q = 370 kBq = $2.22 \cdot 10^7$ min^{-1*}
A_F = 6 cm^2 = $6 \cdot 10^{-4}$ m^2
d_F = 1,5 µm = $1.5 \cdot 10^{-6}$ m
A_D = 50 mm^2 = $5 \cdot 10^{-5}$ m^2
α = 2 cm = $2 \cdot 10^{-2}$ m
N = $5.9 \cdot 10^{28}$ for gold
Z = 79 for gold
e = $1.60 \cdot 10^{-19}$ As
ε_0 = $8.854 \cdot 10^{-12}$ As/Vm
E_α = 4 MeV = $6.4 \cdot 10^{-13}$ J

By introduction of these values we obtain from (4)

$$\Delta n(\Theta) = 5.9 \cdot \cos \frac{\Theta}{2} \cdot \frac{1}{min} \,. \qquad (5)$$

The following simple expressionis obtained for the influence of the foil material on the scattering rate Δn:

$$\Delta n(\Theta) = S'(\Theta) \cdot d_F \cdot Z^2 \qquad (6)$$

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* Since this value can vary slightly from source to source, the exact value should be determined in a separate measurement. This is donce by determining the overall counting rate $\Delta n/\Delta t$ without the annular diaphragm at the maximum distance r between detector and source. The following expression then applies:

$$Q = \frac{\Delta n}{\Delta t} \cdot \frac{4\,\pi\,r^2}{A_D} \,.$$

by introducing into the Rutherford scattering equation (1) the following abrreviation

$$S'(\Theta) = n \cdot N \cdot \frac{1}{4} \left(\frac{2e^2}{4\,\pi\,\varepsilon_0 \cdot 2E_\alpha} \right)^2 \cdot \frac{d\Omega}{\sin^4 \Theta /2} \,.$$

A typical example of the measurement of scattering in gold at different angles is summarized in the table below, the count-

ing rates calculated from equation (5) being given in the final column for purposes of comparison.

l/cm	Θ	Measure-ment time (min)	Δn(Θ)/min⁻¹ (measured)	Δn(Θ) min⁻¹ (calculated from (5))
22	20.6°	10	11.2	5.8
16	28.1°	10	10.6	5.7
10	43.6°	15	7.4	5.5
6	67.4°	15	6.9	4.9
4	90.0°	20	5.9	4.2

When using the simplified method of calculation, the measured counting rates agree in their order of magnitude with the values calculated for the present geometry by means of Rutherford's theory. The predicted, only relatively weak dependence of the particle rates on the scattering angle Θ has been confirmed in particular.

The table also shows, however, a substantial deviation of the measured from the calculated values, the magnitude of the relative deviation being clearly dependent on the angle of scattering. Possible reasons for this discrepancy are discussed in the section "Error analysis".

Typical measurements for gold and aluminium at a scattering angle of 43.6° (l = 10 cm) are given in the table below:

Foil	Measuring time (min)	Pulse count	Δn/min⁻¹
Gold	15	111	7.4
Aluminium	60	68	1.13

$d_F \cdot Z^2$ is calculated for two scattering foils from the following table:

Foil	d_F/μm	Z	$(d_F \cdot Z^2)$/mm
Gold	1.5	79	9.36
Aluminium	8.0	13	1.35

The measured counting rates have been plotted against $d_F \cdot Z^2$ in Fig. 4. It is evident that the measurement points lie, within the limits of statistical error, exactly on a line passing through the origin of the coordinates.

Fig. 4: Counting rate for gold and aluminium as a function of $d_F Z^2$.
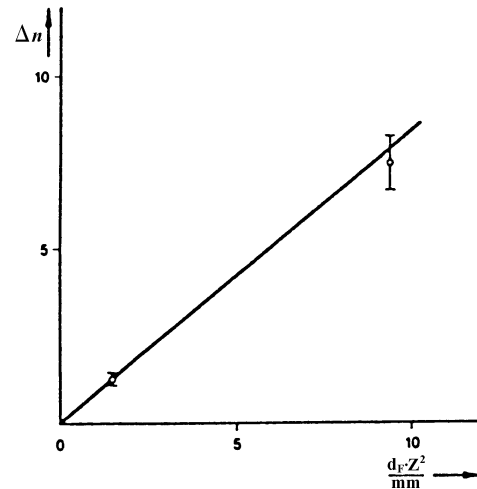
Error analysis
The particle rates calculated for different angles of scattering are clearly lower than the measured pulse rates. When analysing the error, a distinction has to be made between possible measurement errors and errors arising out of the method of calculating the rates of scattering.

Measurement errors
Counting errors may occur if serious sources of interference are present during the measurement (spark gaps etc.); the possibility of such errors is excluded in the example given here. If no such sources of interference are present and the zero effect of the detector has been checked (it should not substantially exceed 0.1/min, if the detector is operating correctly), it may be assumed that the counted pulses correspond exactly to the number of α-particles scattered in the barrier layer of the detector.

Erros in calculation of the scattering rates
A number of factors have been disregarded for simplification purposes when calculating the rates of scattering; these are largely responsible for the discrepancy between the measured and calculated values. Since however, any attempt at calculation without these simplifications would be very extensive and involved, we shall not go into this aspect in greater detail. There are, however, a number of points where the calculation could be refined:

(a) The particle rate does not fall in proportion to $1/r^2$, on particular at short distances from the source, since we are not concerned here with a point-type emitter. (The dependence of the particle rate on distance can be easily measured for the used source).

(b) α-particles are not strictly monoenergetic.

(c) The mean energy of α-particles cannot be regarded as a constant at 4 MeV for all anlges of scattering. The energy of α-

particles decreases with an increase in the angle Θ, since the energy loss in the source covering rises with an increase in oblique radiation (see Operating Instructions for the ^{241}Am source 09090.11).

(d) Screening of the deeper-lying barrier layer of the α-detector occurs at values of Θ.

(e) The probability of double scattering increases with a decrease in the angles of scattering; at low values of Θ the measured particle rat will therefore be greater than the rate calculated on the assumption that each particle has undergone a single collision with an atomic nucleus.

(f) The energy of the α-particles falls during their path through the foil as a result of collisons with electrons.

(g) The particles striking the detector are not only those which have undergone scattering through an angle Θ; the angular range $\Theta \pm \Delta\Theta$ is in fact covered.

The most effective way of refining the calculation is to take into account the true energy distribution of the α-particles and their angular dependence and also the energy loss of the α-particles along their path through the scattering foil. (See also in this connection Ramage, I. C., I. McKeown and K. W. D. Ledingham. Am. I. Phys. 1975 43 51).