Balmer series / Determination of Rydberg's constant

Related topics

Diffraction image of a diffraction grating; visible spectral range; single electron atom; Bohr's atomic model; Lyman, Paschen, Brackett and Pfund Series; energy level; Planck's constant; binding energy.

Principle and task

The spectral lines of hydrogen and mercury are examined by means of a diffraction grating. The known spectral lines of Hg are used to determine the grating constant. The wave lengths of the visible lines of the Balmer series of H are measured.

Equipment

Spectrum tube, hydrogen	06665.00	1
Spectrum tube, mercury	06664.00	1
Holders for spectral tubes, 1 pair	06674.00	1
Cover tube for spectral tubes	06675.00	1
Connecting cord, 50 KV, 1000 mm	07367.00	2
Object holder, 5×5 cm	08041.00	1
Diffraction grating, 600 lines/mm	08546.00	1
High voltage supply unit, 0-10 kV	13670.93	1
Insulating support	06020.00	2
Tripod base -PASS-	02002.55	1
Barrel base -PASS-	02006.55	1
Support rod -PASS-, square, l 400 mm	02026.55	1
Right angle clamp -PASS-	02040.55	3
Stand tube	02060.00	1
Meter scale, demo, l = 1000 mm	03001.00	1
Cursors, 1 pair	02201.00	1
Measuring tape, l = 2 m	09936.00	1

Problems

1. Determination of the diffraction grating constant by means of the Hg spectrum.
2. Determination of the visible lines of the Balmer series in the H spectrum, of Rydberg's constant and of the energy levels.

Set-up and procedure

The experimental set-up is shown in Fig. 1. Hydrogen or mercury spectral tubes connected to the high voltage power supply unit are used as a source of radiation. The power supply is adjusted to about 5 kV. The scale is attached directly behind the spectral tube in order to minimize parallax errors. The diffraction grating should be set up at about 50 cm and at the same height as the spectral tube. The grating must be aligned so as to be parallel to the scale.

The luminous capillary tube is observed through the grating. The room is darkened to the point where it is still possible to read the scale. The distance 2 *l* between spectral lines of the same color in the right and left first order spectra are read without moving one's head. The distance d between the scale and the grating is also measured.

Three lines are clearly visible in the Hg spectrum. The grating constant *g* is determined by means of the wavelengths given in Table 1. Rydberg's constant, and thus the energy levels in hydrogen, are determined from the measured wavelengths by means of Balmer's formula.

Fig. 1: Experimental set-up to determine the spectral lines of the hydrogen atom.

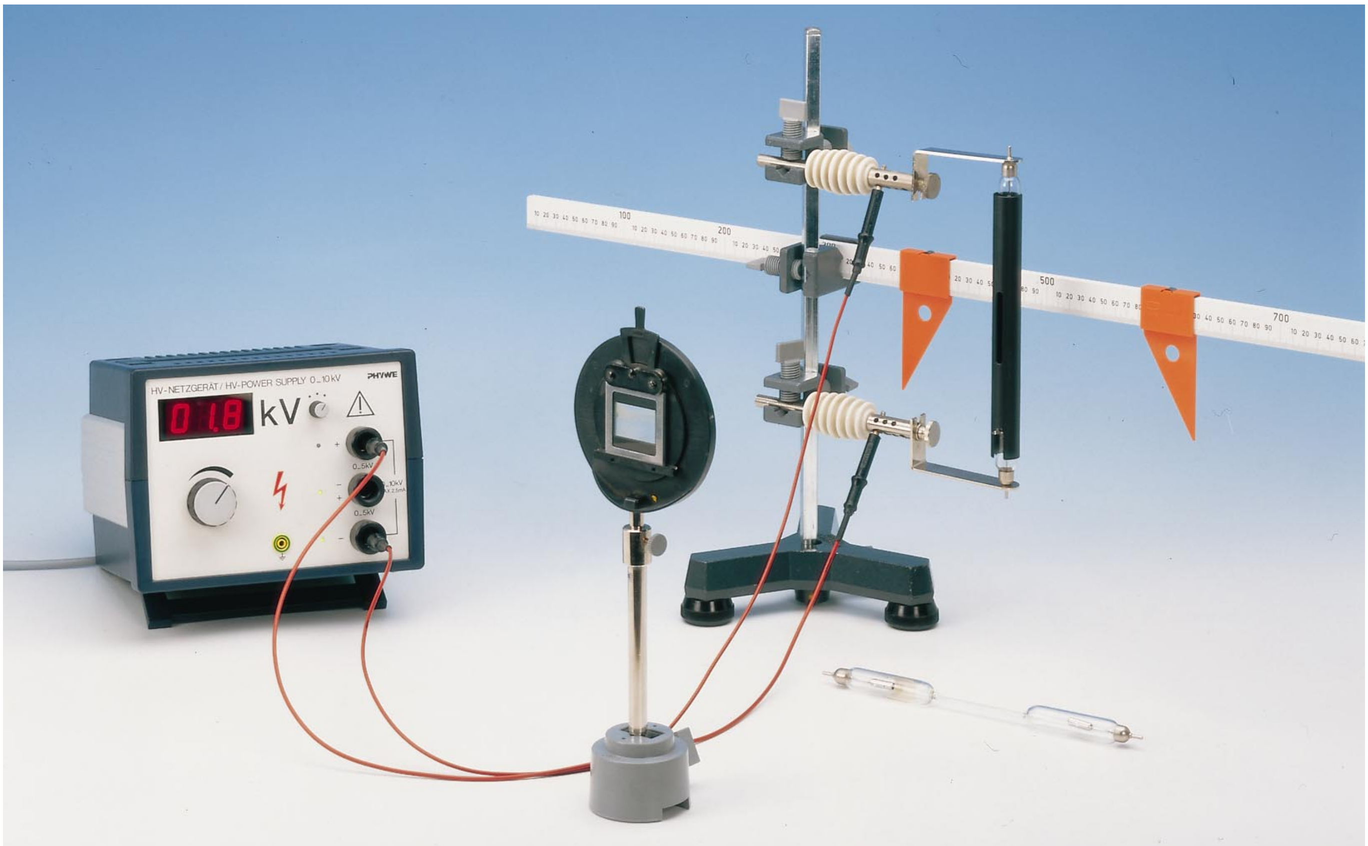
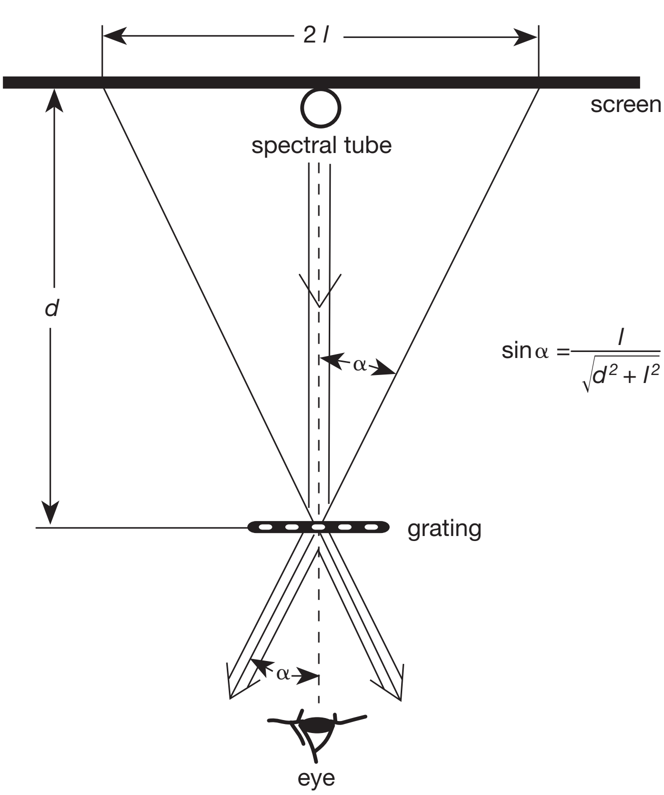


Fig. 2: Diffraction at the grating.



Theory and evaluation

1. Diffraction grating

If light of wavelength λ impinges on a grating with constant g, it is diffracted. Intensity peaks occur when the angle of diffraction α fulfills the following condition:

$$n \cdot \lambda = g \cdot \sin\alpha \; ; \quad n = 0, 1, 2, ... \qquad (1)$$

Light is collected by the eye on the retina, therefore the light source is seen in the color of the observed spectral line on the scale in the prolongation of the light beams.

For the diffraction of the n^{th} order, the following relation is deduced from the geometrical structure (Fig. 2):

$$n \cdot \lambda = g \cdot \frac{l}{\sqrt{d^2 + l^2}} \qquad (2)$$

In the examples given in Table 1, the average obtained for the three measurements of the grating constant is g = 1.672 µm.

Tab. 1: Determination of the grating constant from the wavelengths of the Hg spectrum

Color	λ / nm	$2l$ / mm	g / µm
yellow	578.0	330	1.680
green	546.1	311	1.672
blue	434.8	244	1.661

2. Hydrogen spectrum

Due to collision ionization, H_2 is converted to atomic hydrogen in the spectral tube. Electrons from the H atoms are exited to higher energy levels through collisions with electrons. When they return to lower energy levels, the atoms emit light of frequency f given by the energy difference of the concerned states:

$$\Delta E = h \cdot f \qquad (3)$$

where h is Planck's constant.

Applying Bohr's atomic model, the energy E_n of a permitted electron orbit is given by:

$$E_n = -\frac{1}{8} \frac{e^4 m_e}{\varepsilon_0^2 h^2} \frac{1}{n^2} \quad n = 1,2,3\ldots \qquad (4)$$

where $\varepsilon_0 = 8.8542 \cdot 10^{-34}$ As/Vm is the electric field constant, $e = 1.6021 \cdot 10^{-19}$ C is the electronic charge and $m_e = 9.1091 \cdot 10^{-31}$ kg is the mass of the electron at rest. The emitted light can therefore have the following frequencies:

$$f_{nm} = \frac{1}{8} \frac{e^4 m_e}{\varepsilon_0^2 h^3} \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \quad n, m = 1,2,3\ldots \qquad (5)$$

If the wave number $N = \lambda^{-1}$ is used instead of the frequency f, substituting $c = \lambda \cdot f$ one obtains:

$$N = Ry_{th} \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \qquad (6)$$

$$\text{where } Ry_{th} = \frac{1}{8} \frac{e^4 m_e}{\varepsilon_0^2 h^3 c} = 1.097 \cdot 10^7 \, m^{-1}$$

Here Ry_{th} is Rydberg's constant, which follows from Bohr's atomic model.

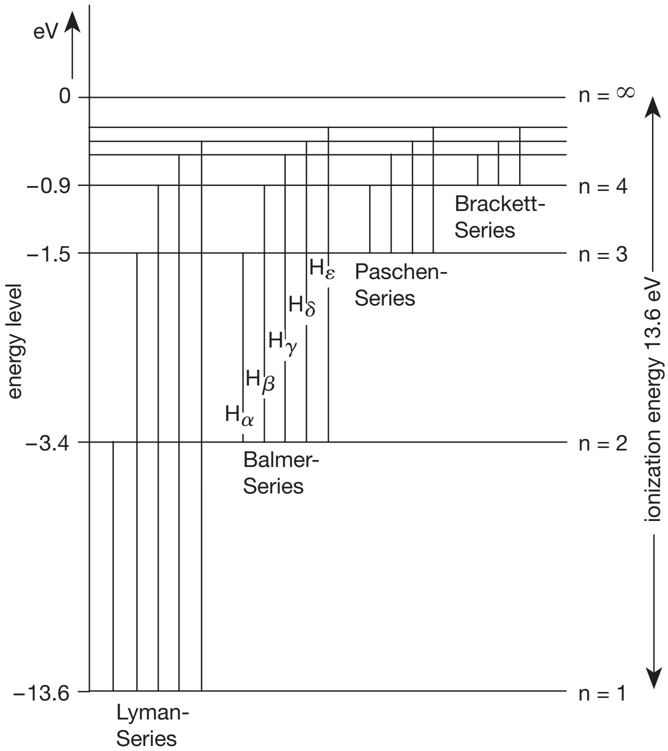


Fig. 3: Energy level diagram of the H atom.

n = 1 : Lyman series
Spectral range: ultraviolet
n = 2 : Balmer series
Spectral range: ultraviolet till red
n = 3 : Paschen series
Spectral range: infrared
n = 4 : Bracket series
Spectral range: infrared
n = 5 : Pfund series
Spectral range: infrared

Fig. 3 shows the energy level diagram and the spectral series of the H atom. For $m \rightarrow \infty$, one obtains the limits of the series; the associated energy is thus the ionization energy (or the binding energy) for an electron in the n^{th} permitted orbit. The binding energy can be calculated by means of the equation:

$$E_n = -Ry_{th} \cdot h \cdot c \, \frac{1}{n^2}$$

where $c = 2.99795 \cdot 10^8$ m/s and $h = 6.6256 \cdot 10^{-34}$ J s = $4.13567 \cdot 10^{-15}$ eV s. The ground state is found to be 13.6 eV.

Tab. 2: Examples of measurements for the H spectrum (Balmer series) Distance d = 500 mm

Line	$2l$	λ_{exp}	λ_{lit}	Ry_{exp}
H_α	384 mm	656 nm	656.28 nm	$1.097 \cdot 10^7$ m^{-1}
H_β	275 mm	489 nm	486.13 nm	$1.093 \cdot 10^7$ m^{-1}
H_γ	243 mm	436 nm	434.05 nm	$1.092 \cdot 10^7$ m^{-1}
H_δ	–	–	410.17 nm	–

average: $Ry_{exp} = 1.094 \cdot 10^7 \, m^{-1}$

Notices

- Next to the atomic hydrogen spectrum, the molecular H_2 band spectrum may be observed if the room is sufficiently darkened. The numerous lines, which are very close to each other, are due to the oscillations of the molecule.
- The H_δ line is situated on the border of the visible spectral range and is too weak to be observed by simple methods.
- The treatment of more complex atoms requires quantum mechanics. In this case, the energies of the states are determined by the eigenvalues of the hamiltonian of the atom. For atoms similar to hydrogen, calculations yield the same results as Bohr's atomic model.