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Related topics

Resonance, Q factor, dissipation factor, bandwidth, critical or optimum coupling, characteristic impedance, Pauli method, parallel conductance, band-pass filter, sweep.

Principle and task

The Q factor of oscillating circuits is determined from the bandwidth and by the Pauli method. In inductively coupled circuits (band-pass filters) the coupling factor is determined as a function of the coil spacing.

Equipment

Wobble-functiongenerator 2 Hz-6 MHz	11765.93
Oscilloscope, 20 MHz, 2 channels	11454.93
HF-coil, 35 turns, 75 micro-H	06915.00
HF-coil, 50 turns, 150 micro-H	06916.00
HF-coil, 75 turns, 350 micro-H	06917.00
Coil, 150 turns, short	06520.01
Variable capacitor, 500 pF	06049.10
PEK carbon resistor 1 W 5% 22 kOhm	39104.34
PEK carbon resistor 1 W 5% 47 kOhm	39104.38
PEK carbon resistor 1 W 5% 100 kOhm	39104.41
PEK carbon resistor 1 W 5% 1 mOhm	39104.52
PEK carbon resistor 1 W 5% 82 kOhm	39104.40
PEK capacitor /case 1/ 470 pF/500 V	39105.07
PEK connect.plug white 19 mm pitch	39170.00
Connection box	06030.23
G-clamp	02014.00
Meter scale, demo, I = 1000 mm	03001.00
Adapter, BNC-socket/4 mm plug pair	07542.27
Connecting cord, 250 mm, yellow	07360.02
Screened cable, BNC, I 750 mm	07542.11
Screened cable, BNC, I 1500 mm	07542.12

Problems

- 1. To determine the dissipation factor tan δ_k and the quality factor Q from the bandwidth of oscillating circuits.
- 2. To determine the dissipation factor and *Q* factor of oscillating circuits from the resonant frequency (ω_0), the capacitance $C_{\text{tot.}}$ and the parallel conductance G_p determined by the Pauli method.
- 3. To determine the coupling factor k and the bandwidth Δf of a band-pass filter as a function of the coil spacing *s*.

Set-up and procedure

- 1. Function generator:
 - choose "Sweep modus" in the "Sub Functions"
 - select apr. 400 kHz
 - Connect the "sweep Out" exit mit channel 1 of the oscilloscope

Oscilloscope:

Select the "x-y" mode

2. A "constant" current is fed to a parallel oscillating circuit consisting of a coil and a capacitor by way of the high-value resistor R_s (Fig. 2). First set the ressonance curve with the generator frequency swept and the oscilloscope on X-Y operation (H output of the generator connected to the X input). Now vary the frequency manually with the sweep switched off. Set the frequencies f_1 , f_0 and f_2 (Fig. 3) and measure them with the digital counter-timer (take several measurements and calculate an average).

Fig. 1: Experimental set-up for determining the coupling factor in a band-pass filter.



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Fig. 2: Parallel oscillating circuit with series resistor R_s.



Fig. 4: Band-pass filter circuit.



3. With sweep on, damp the oscillating circuit with additional parallel resistors R_z (additional conductances G_z). If we plot the resonance voltage U_0 reciprocally against the additional conductance G_z we obtain a straight line; the point at which the line on extrapolation intersects the *x*-axis gives the parallel conductance G_p (Pauli method, Fig. 6).

The characteristic impedance Z_k , dissipation factor tan δ_k and the Q factor Q can be calculated from the parallel conductance G_p the resonant frequency $\omega_o = 2\pi f_0$ and the oscillating circuit capacitance $C_{\text{tot.}} = C + C_{\text{ir}}$

4. Construct two identical oscillating circuits as shown in Fig. 4; they should have a fairly large spacing *s* and be tuned to the same frequency (on sweep) with the variable capacitors.

Now bring the two coils closer together (inductive coupling), and measure the voltages of the resonant frequency at the peak point or at the centre and peak points, and the frequencies f_1 , f_m , f_o as a function of the coil spacing *s* (see Fig. 7).



Fig. 3: Bandwidth of an oscillating circuit.

Theory and evaluation

For the parallel resonance circuit consisting of a dissipative inductance and a capacitance it is possible to postulate an equivalent circuit with the parallel conductances G_{pL} and G_{pC} :

In complex notation, the conductance of the parallel resonance circuit is

$$\underline{Y}(\omega) = G_{pL} + G_{pC} + j(\omega C - \frac{1}{\omega L}).$$

At resonance, the reactance components cancel each other out,

$$\omega_0 L = \frac{1}{\omega_0 C}$$

and we obtain

$$\begin{split} \underline{Y}\left(\omega_{0}\right) &= G_{\text{pL}} + G_{\text{pC}} = G_{\text{p}} \\ \text{where } G_{\text{p}} &= \frac{1}{Z_{k}} \cdot \tan \delta_{\text{k}}; \\ Z_{\text{k}} &= \frac{1}{\omega_{0}C}; \qquad \omega_{0} = 2\pi f_{0}. \end{split}$$

 Z_k is the characteristic impedance of the non-dissipative circuit, tan δ_k the dissipation factor of the circuit and $Q = \tan \delta_k^{-1}$ is the quality or Q factor.



Fig. 5: Equivalent circuit of the parallel resonance circuit.





Fig. 6: Reciprocal resonance voltage as a function of the additional conductance, used to determine G_{p} . 1. HF coil, 75 turns; 2. 150-turn coil.

- 1. The Q factor is defined in terms of the bandwidth of the oscillating circuit (cf. Fig. 3):
 - $Q = \frac{f_0}{f_2 f_1}$

For the HF coil (75 turns) and $C_{\text{tot.}} = C + C_{\text{i}} = 492$ pF, the measurements gave:

 $f_0 = 392.4 \text{ kHz}, \quad f_1 = 389.6 \text{ kHz},$ $f_2 = 394.3 \text{ kHz}, \quad Q = 83.$

For the 150-turn coil and $C_{\text{tot.}} = 492 \text{ pF}$ we measured:

 $f_0 = 234.8 \text{ kHz}, \quad f_1 = 229.2 \text{ kHz},$ $f_2 = 239.4 \text{ kHz}, \quad Q = 23.$

It should be noted that the Q factor measured applies to the whole system, so that the damping by R_s and R_i was also included along with the oscillating circuit comprising *L* and *C*.

2. When *I* is constant, the equation

 $\frac{1}{U} = \frac{1}{I} \left(G_{\rm p} + G_{\rm Z} \right)$

describes a straight line. By extrapolating from it for

$$\frac{1}{U} \rightarrow 0$$
,

 $G_{\rm p}$ can be obtained.

We must consider here the conductance values already available,

$$\frac{1}{R_{\rm s}}$$
 and $\frac{1}{R_{\rm i}}$,

so that

 $\frac{1}{11}$ is plotted against

$$G_z = \frac{1}{R_z} + \frac{1}{R_s} + \frac{1}{R_i}$$

From Fig. 6 we determined the parallel conductance $G_{\rm p} = 15 \times 10^{-6} \ \Omega^{-1}$ for the 75-turn HF coil and $C_{tot.} = 492 \ \text{pF}$. With $f_{\rm o} = 392.4 \ \text{kHz}$ we obtained $Z_{\rm k} = 824 \ \text{ohms}$, tan $\delta_{\rm k} = 12.4 \times 10^{-3} \ \text{and} \ Q = 81$.

Also from Fig. 6 we obtained the parallel conductance $G_p = 33.7 \times 10^{-6} \Omega^{-1}$ for the 150-turn coil and $C_{tot.} = 492$ pF. With $f_o = 234.8$ kHz we obtained $Z_k = 1380$ ohms, tan $\delta_k = 47 \times 10^{-3}$ and Q = 22.

3. If two identical circuits (inductive and capacitive) are coupled, we obtain a band-pass filter whose resonance behaviour greatly depends on the coupling factor *k*.

Using the measured values $U_{\rm m}$ and $U_{\rm max}$ from Fig. 7 we can calculate the coupling factor *k* in accordance with

$$k = \tan \delta_{k} \left[\frac{U_{max}}{U_{m}} \pm \sqrt{\left(\frac{U_{max}}{U_{m}} \right)^{2} - 1} \right]$$

where tan $\boldsymbol{\delta}_k$ is the dissipation factor of the single circuit.

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Fig. 7: Resonance curves for a band-pass filter at different degrees of coupling.



The stronger the coupling, the further apart are the peak point frequencies.

The bandwidth Δf of the supercrit- ical filter is determined by the frequencies at which the voltage has fallen to the voltage $U_{\rm m}$ of the centre frequency $f_{\rm m}$) and can be calculated from the peak point frequencies f_1 and f_1 in accordance with

$$\Delta f = \sqrt{2} (f_2 - f_1).$$

Note

Coupled mathematical pendulums provide a mechanical analogy to coupled oscillating circuits (the two natural frequencies of the coupled pendulums, which produce beats, correspond to the two peak point frequencies).



Fig. 8: Coupling constant as a function of the distance between the coils when the coupling is supercritical

Fig. 9: Bandwidth as a function of the coil spacing.

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