Coil in the AC circuit

LEP 4.4.04

Related topics

Inductance, Kirchhoff's laws, Maxwell's equations, AC impedance, phase displacement.

Principle and task

The coil is connected in a circuit with a voltage source of variable frequency. The impedance and phase displacements are determined as functions of frequency. Parallel and series impedances are measured.

Equipment

Coil, 300 turns	06513.01	1
Coil, 600 turns	06514.01	1
Resistor in plug-in box 50 Ohms	06056.50	1
Resistor in plug-in box 100 Ohms	06057.10	1
Resistor in plug-in box 200 Ohms	06057.20	1
Connection box	06030.23	1
Difference amplifier	11444.93	1
Function generator	13652.93	1
Digital counter, 4 decades	13600.93	1
Oscilloscope, 20 MHz, 2 channels	11454.93	1
Screened cable, BNC, l 750 mm	07542.11	2
Connecting cord, 100 mm, red	07359.01	3
Connecting cord, 500 mm, red	07361.01	5
Connecting cord, 500 mm, blue	07361.04	4

Problems

1. Determination of the impedance of a coil as a function of frequency.
2. Determination of the inductance of the coil.
3. Determination of the phase displacement between the terminal voltage and total current, as a function of the frequency in the circuit.
4. Determination of the total impedance of coils connected in parallel and in series.

Set-up and procedure

The experimental set up is as shown in Fig. 1. Since normal voltmeters and ammeters generally measure only rms (root mean square) values and take no account of phase relationships, it is prefereable to use an oscilloscope. The experiment will be carried out with sinusoidal voltages, so that to obtain rms values, the peak-to-peak values measured on the oscilloscope ($U_{\rho\text{-}\rho}$) are to be divided by $2\sqrt{2}$.

In accordance with

$$I = U/R,$$

the current can be deduced by measurement of the voltage across the resistor. The circuit shown in Fig. 2 permits the simultaneous display of the total current and the coil voltage. If, by means of the time-base switch of the oscilloscope, one half-wave of the current (180°) is brought to the full screen width (10 cm) – possibly with variable sweep rate – the phase displacement of the voltage can be read off directly in cm (18°/cm). The Y-positions of the two base-lines (GND) are made to coincide. After switching to other gain settings, the base-lines are readjusted. In order to achieve high reading accuracy, high gain settings are selected. The inputs to the difference amplifier are non-grounded.

Fig.1: Experimental set up for investigating the a. c. impedance of the coil.

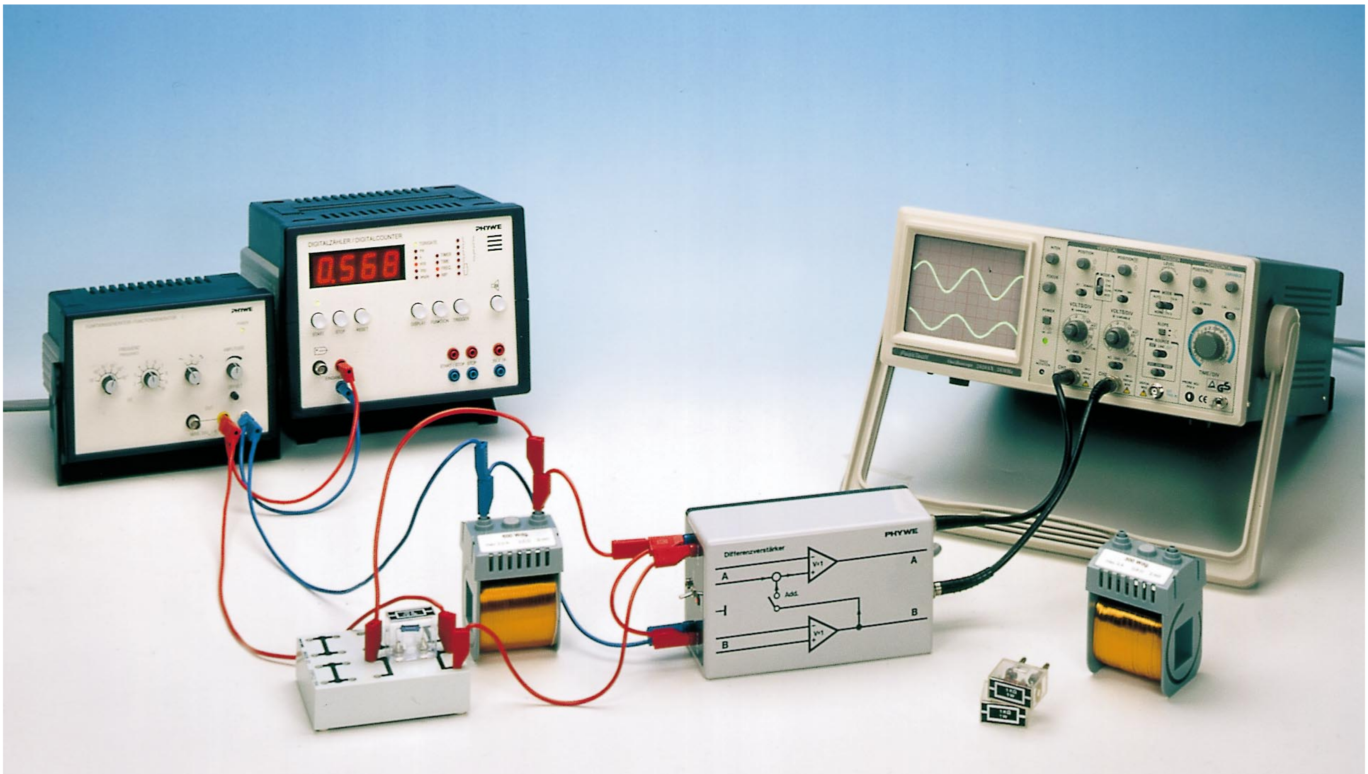


Fig. 2: Circuit for display of current and voltage with the oscilloscope.

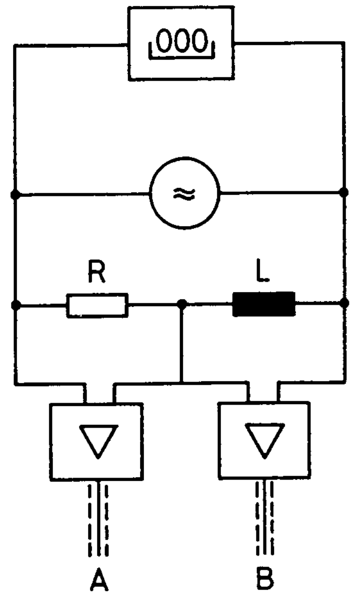
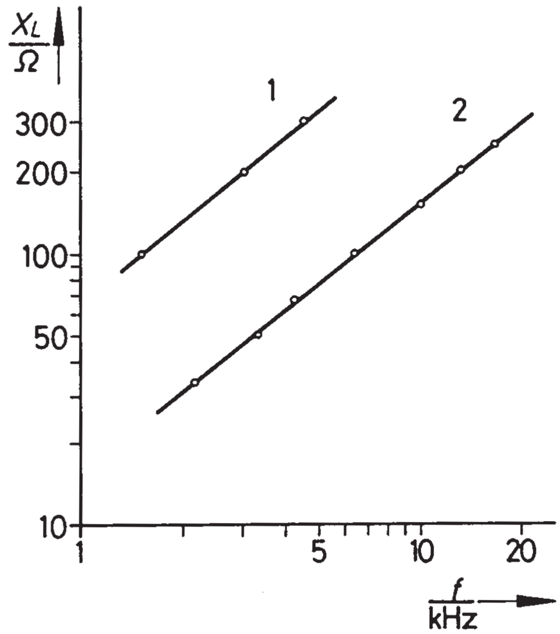


Fig. 3: Impedance of various coils as a function of the frequency.



To determine the impedance of a coil as a function of the frequency, the coil is connected in series with resistors of known value. The frequency is varied until there is the same voltage drop across the coil as across the resistor. The resitance and impedance values are then equal:

$$R_\Omega = \omega L = X_L \qquad (1)$$

The phase displacement between the terminal voltage and the total current can be measured using a similar cicuit to Fig. 2, but with channel *B* measuring the total voltage and not the voltage across the coil.

When coils are connected in parallel or in series, care should be taken to ensure that they are sufficiently far apart, since their magnetic fields influence one another.

Theory and evaluation

If a coil of inductance *L* and a resistor of resistance *R* are connected in a circuit (see Fig. 2), the sum of the voltage drops on the individual elements is equal to the terminal voltage *U*

$$U = IR + L \cdot \frac{dI}{dt}\ , \qquad (2)$$

where I is the current.

The resistors *R* are selected so that the d.c. resistance of the coil, with a value of 0.2 Ω, can be disregarded. If the alternating voltage *U* has the frequency $\omega = 2\pi f$ and the waveform

$$U = U_O \cos \omega t\ ,$$

then the solution of (2) is

$$I = I_O \cos (\omega t - \phi)$$

with the phase displacement ϕ given by

$$\text{than } \phi = \frac{\omega L}{R} \qquad (3)$$

and

$$I_O = \frac{U_O}{\sqrt{R^2 + (\omega L)^2}} \qquad (4)$$

It is customary to threat complex impedances as operators $\hat{R}_i$:

Coil $\hat{R}_L = i\omega L$,

Ohmic resistance $\hat{R} = R$.

With parallel connection,

$$\hat{R}_{tot}^{-1} = \sum_i \hat{R}_i^{-1}$$

The real impedance of a circuit is the absolute value of $\hat{R}_{tot}$ and the phase relationship, analogous to (2), is the ratio of the imaginary part to the real part of $\hat{R}_{tot}$.

From the regression line to the measured value of Fig. 3 and the exponential statement

$$Y = a \cdot X^B$$

there follows the exponent

$$B_1 = 1.02 \pm 0.01 \qquad \text{(see (1))}$$

$$B_2 = 1.01 \pm 0.01$$

With the regression line to the measured values of Fig. 3 and the linear statement

$$Y = A + B \cdot X$$

Fig. 4: Phase displacement (than ϕ) between total current and total voltage as a function of frequency.

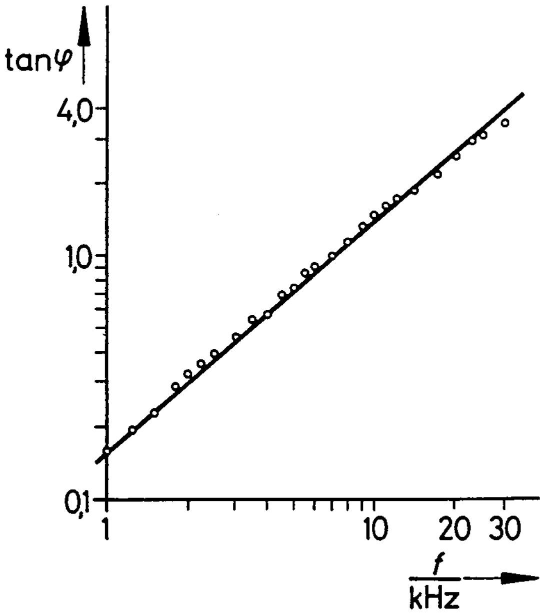
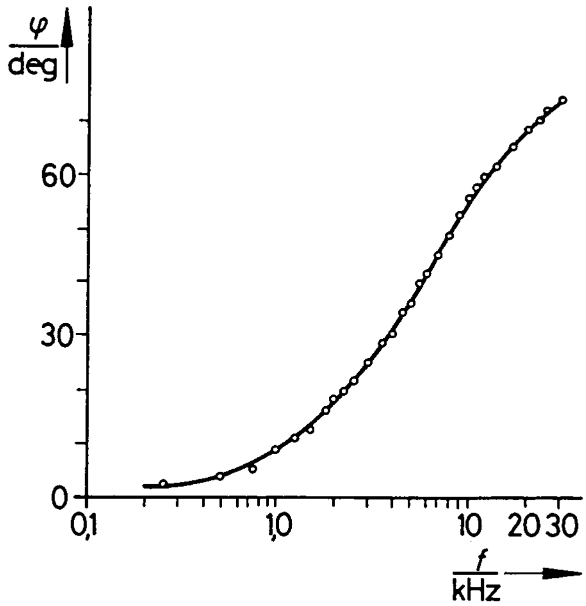


Fig. 5: Phase displacement (ϕ) between total current and total voltage as a function of frequency.



the slope

$B_1 = 0.067 \pm 0.001$ (see (1))
$B_2 = 0.015 \pm 0.001$

is obtained.

From this, with

$R = \omega L$

the inductances

$L_1 = 2.38$ mH
$L_2 = 10.4$ mH

are obtained.

From the regression line to the measured values of Fig. 4 and the exponential statement

$Y = A \cdot X^B$

the exponent

$B = 0.97 \pm 0.01$ follows (see (2))

The frequency at which the total impedance of the coils was equal to the reference of 200 Ω was determined with coils connected in parallel and in series.

Table: Total inductance of coils L_t connected in parallel (line 1) and in series (line 2).

Coil	f (200 Ω)	L_{tot}
$L_1 // L_2$	16.53 kHz	1.93 mH
$L_1 + L_2$	2.48 kHz	12.84 mH