Inductance of solenoids

LEP 4.4.03

Related topics

Law of inductance, Lenz's law, self-inductance, solenoids, transformer, oscillatory circuit, resonance, damped oscillation, logarithmic decrement, Q factor.

Principle and task

A square wave voltage of low frequency is applied to oscillatory circuits comprising coils and capacitors to produce free, damped oscillations. The values of inductance are calculated from the natural frequencies measured, the capacitance being known.

Equipment

Induction coils, 1 set	11006.88	1
consisting of		
Induction coil, 300 turns, dia. 40 mm	11006.01	1
Induction coil, 300 turns, dia. 32 mm	11006.02	1
Induction coil, 300 turns, dia. 25 mm	11006.03	1
Induction coil, 200 turns, dia. 40 mm	11006.04	1
Induction coil, 100 turns, dia. 40 mm	11006.05	1
Induction coil, 150 turns, dia. 25 mm	11006.06	1
Induction coil, 75 turns, dia. 25 mm	11006.07	1
Coil, 1200 turns	06515.01	1
Oscilloscope, 20 MHz, 2 channels	11454.93	1
Function generator	13652.93	1
PEK capacitor /case 1/ 1nF/ 500 V	39105.10	1
PEK capacitor /case 1/ 470 pF/500 V	39105.07	1
Vernier caliper	03010.00	1
Measuring tape, l = 2 m	09936.00	1
Adapter, BNC-plug/socket 4 mm	07542.26	1
Connection box	06030.23	1
Connecting cord, 250 mm, yellow	07360.02	4
Connecting cord, 500 mm, yellow	07361.02	2

Problems

To connect coils of different dimensions (length, radius, number of turns) with a known capacitance C to form an oscillatory circuit. From the measurements of the natural frequencies, to calculate the inductances of the coils and determine the relationships between

1. inductance and number of turns
2. inductance and length
3. inductance and radius.

Set-up and procedure

Set up the experiment as shown in Fig. 1 + 2.

A square wave voltage of low frequency ($f \approx 500$ Hz) is applied to the excitation coil L. The sudden change in the magnetic field induces a voltage in coil L and creates a free damped oscillation in the LC oscillatory circuit, the frequency f_0 of which is measured with the oscilloscope.

Coils of different lengths l, diameters $2r$ and number of turns N are available (Fig. 3). The diameters and lengths are measured with the vernier caliper and the measuring tape, and the numbers of turns are given.

Fig. 1: Experimental set-up for determining inductance from the resonant frequency of an oscillatory circuit.

Fig. 2: Set-up for determining inductance *L.*

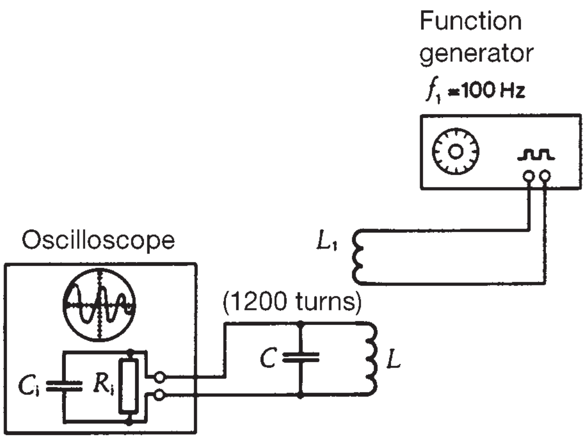


Fig. 3: Table of coil data

No.	N	$\frac{2r}{\text{mm}}$	$\frac{l}{\text{mm}}$	Cat. No.
1	300	40	160	11006.01
2	300	32	160	11006.02
3	300	26	160	11006.03
4	200	40	105	11006.04
5	100	40	53	11006.05
6	150	26	160	11006.06
7	75	26	160	11006.07

Three measurements, each with different values of capacitance C (C_1 = 1 nF, C_2 470 pF, and $C_3 = C_1 + C_2$ in parallel), should be taken with each coil.

The following coils provide the relationships between inductance and radius, length and number of turns that we are investigating:

1.) 3, 6, 7 $\rightarrow L \quad = f(N)$
2.) 1, 4, 5 $\rightarrow L/N^2 \quad = f(l)$
3.) 1, 2, 3 $\rightarrow L \quad = f(r)$

As a difference in length also means a difference in the number of turns, the relationship between inductance and number of turns found in Problem 1 must also be used to solve Problem 2.

Notes

The distance between L_1 and L should be as large as possible so that the effect of the excitation coil on the resonant frequency can be disregarded. There should be no iron components in the immediate vicinity of the coils.

The tolerance of the oscilloscope time-base is given as 4%. If a higher degree of accuracy is required, the time-base can be calibrated for all measuring ranges with the function generator and a frequency counter prior to these.experiments.

Theory and evaluation

If a current of strength I flows through a cylindrical coil (solenoid) of length *l,* cross sectional area $A = \pi \cdot r^2$, and number of turns N, a magnetic field is set up in the coil. When $l >> r$ the magnetic field is uniform and the field strength H is easy to calculate:

$$H = I \cdot \frac{N}{l} \tag{1}$$

The magnetic flux through the coil is given by

$$\Phi = \mu_o \cdot \mu \cdot H \cdot A \tag{2}$$

where μ_o is the magnetic field constant and μ the absolute permeability of the surrounding medium.

When this flux changes, it induces a voltage between the ends of the coil,

$$\begin{aligned} U_{ind.} &= -N \cdot \dot{\Phi} \\ &= -N \cdot \mu_o \cdot \mu \cdot A \cdot \frac{N}{l} \cdot \dot{I} \\ &= -L \cdot \dot{I} \end{aligned} \tag{3}$$

where

$$L = \mu_o \cdot \mu \cdot \pi \cdot \frac{N^2 \cdot r^2}{l} \tag{4}$$

is the coefficient of self-induction (inductance) of the coil.

Inductivity Equation (4) for the inductance applies only to very long coils $l >> r$, with a uniform magnetic field in accordance with (1).

In practice, the inductance of coils with $l > r$ can be calculated with greater accuracy by an approximation formula (Kohlrausch, Praktische Physik (Practical Physics), Vol. 2):

$$L = 2.1 \cdot 10^{-6} \cdot N^2 \cdot r \cdot \left(\frac{r}{l}\right)^{3/4}$$

$$\text{for } 0 < \frac{r}{l} < 1 \tag{5}$$

In the experiment, the inductance of various coils is calculated from the natural frequency of an oscillating circuit.

$$\omega_0 \quad \frac{1}{\sqrt{LC_{tot.}}} \tag{6}$$

$C_{tot.}$ is the sum of the capacitance the known capacitor and the input capacitance C_i of the oscilloscope (see Fig. 2).

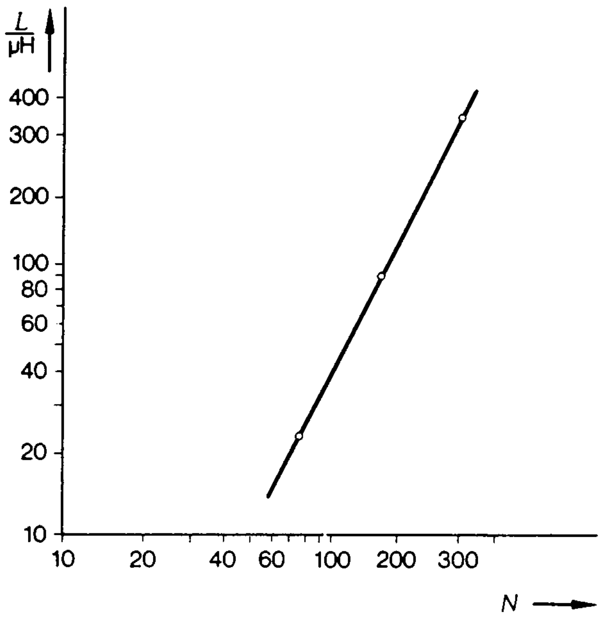
The internal resistance R_i of the oscilloscope exercises a damping effect on the oscillatory circuit and causes a negligible shift (approx. 1%) in the resonance frequency.

The inductance is therefore represented by

$$L = \frac{1}{4\,\pi^2\, f_o^2\, C_{tot.}} \tag{7}$$

where $C_{tot.} = C + C_i$ and $f_0 = \frac{\omega_o}{2\pi}$

1. Fig. 4: Inductances of the coils as a function of the number of turns, at constant length and constant radius.



Applying the expression

$$L = A \cdot N^{B}$$

to the regression line from the measured values in Fig. 4 gives the exponent

B = 1.966 (see (4) and (5))

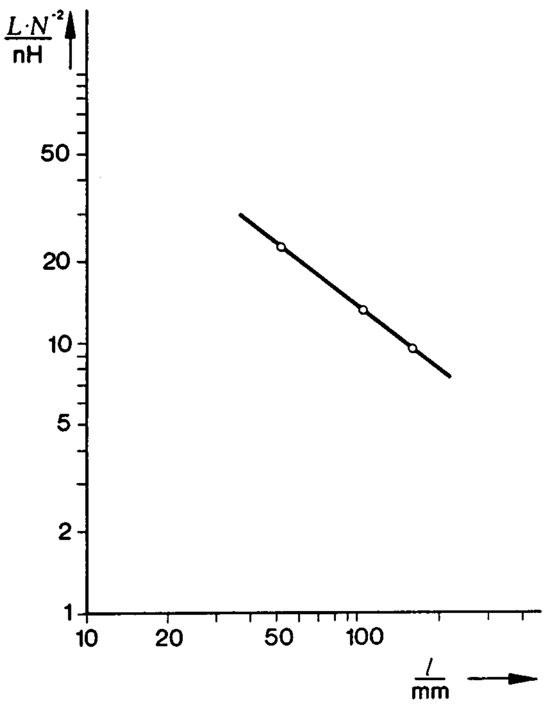
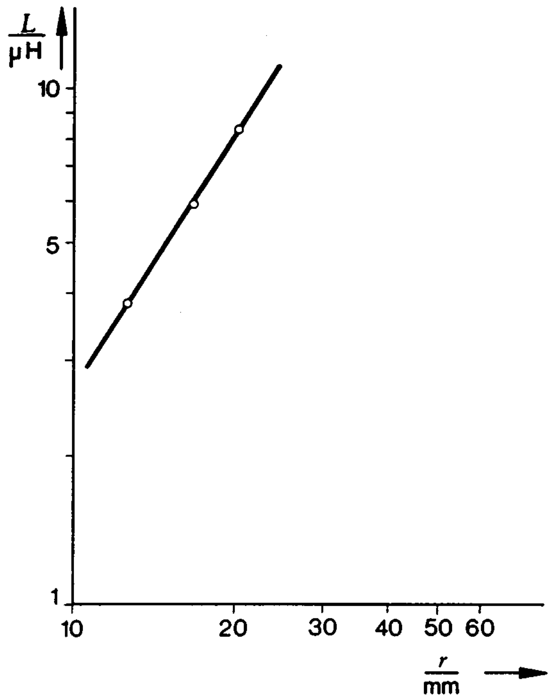


Fig. 5: Inductance per turn as a function of the length of coil, at constant radius.

3. Fig. 6: Inductance of the coils as a function of the radius, at constant length and number of turns.



with a standard error

$$s_B = 0.001.$$

2. Now that we know that $L \sim N^2$, we can demonstrate the relationship between inductance and the length of the coil.

Applying the expression

$$L = A \cdot l^{B}$$

to the regression line from the measured values in Fig. 5 gives the exponent

B = − 0.76 ± 0.04. (see (5))

Applying the expression

$$L = A \cdot r^{B}$$

to the regression line from the measured values in Fig. 6 gives the exponent

B = 1.76 ± 0.07. (see (5))

(5) is thus verified within the limits of error.