Magnetic field inside a conductor

LEP 4.3.06

Related topics

Maxwell's equations, magnetic flux, induction, current density, field strength.

Principle and task

A current which produces a magnetic field is passed through an electrolyte. This magnetic field inside the conductor is determined as a function of radius and current.

Equipment

Hollow cylinder, PLEXIGLAS	11003.10	1
Search coil, straight	11004.00	1
Power frequency generator 1 MHz	13650.93	1
LF amplifier, 220 V	13625.93	1
Digital multimeter	07134.00	2
Adapter, BNC-socket/4 mm plug pair	07542.27	1
Distributor	06024.00	1
Meter scale, demo, l = 1000mm	03001.00	1
Cursors, 1 pair	02201.00	1
Tripod base -PASS-	02002.55	1
Barrel base -PASS-	02006.55	1
Support rod -PASS-, square, l 400 mm	02026.55	1
Right angle clamp -PASS-	02040.55	1
Screened cable, BNC, l 1500 mm	07542.12	1
Connecting cord, 500 mm, yellow	07361.02	3
Connecting cord, 500 mm, blue	07361.04	3
Hydrochloric acid 1.19, 1000 ml	30214.70	1

Problems

Determination of the magnetic field inside a conductor as a function

1. of the current in the conductor,
2. of the distance from the axis of the conductor.

Set-up and procedure

The experimental set up is as shown in Fig. 1. The electrolyte (approx. 200 ml of 37 % hydrochloric acid to 4 litres of water) is poured into the hollow cylinder after it has been thoroughly mixed. The aperture must not be tightly closed, so that gases released (H_2, O_2) can escape. The various connection sockets on the hollow cylinder permit separate measurements on the electrolyte and on the jacket (hollow cylinder). Account must be taken of the fact that the magnetic field strengths to be measured lie in the µT range, i.e. the cables carrying the current - especially the return lead – also produce a magnetic field which is of the same order of magnitude.

Fig. 1: Experimental set-up fdr determining the magnetic field inside a conductor.

For the field strength measurement in the electrolyte the return lead for the current is the grid, as a current in the wall of the hollow cylinder produces no magnetic field inside the cylinder.

With this connection, there is no resultant field in the space outside the cylinder. The induced voltage U_{ind} is

$$U_{ind} = n \cdot A \ \frac{dB}{dt}$$

with the number of turns n = 1200 and the effective area A = 74.3 mm^2. Since the magnetic flux density B is produced by a sinusoidal current of frequency f or angular velocity $\omega = 2\pi f$,

$$B = B_0 \cdot \sin \omega t \ .$$

Therefore, the induced voltage is

$$U_{ind} = n \cdot A \cdot 2\pi \cdot f \cdot B_0 \sin (\omega t + \phi) \ .$$

The phase displacement ϕ is irrelevant for this measurement. Since, according to (4), the magnetic flux density is proportional to the current, the induced voltage is proportional to the current and the frequency. The current is limited by the formation of gas (electrolysis) and the frequency by the series-connected measuring instruments ($f \leq 11$ kHz). The experiment was carried out at f = 5.5 kHz and $I < 1$ A. The amplification was $1 \cdot 10^3$; position 1 is calibrated on the 10 V output.

Theory and evaluation

Maxwell's 1st equation

$$\oint_C \vec{B} \ d\vec{s} = \mu_0 \int_A \vec{j} \cdot d\vec{a} \ , \tag{1}$$

together with Maxwell's 4th equation

$$\int_A \vec{B} \cdot d\vec{a} = 0 \ , \tag{2}$$

gives the relationship between the steady electric current I flowing through the area A

$$I = \int_A \vec{j} \cdot d\vec{a} \tag{3}$$

and the magnetic field $\vec{B}$ it produces.

C is the boundary of A.

A' is any given enclosed area.

$\vec{j}$ is the electrical current density.

μ_0 is the magnetic field constant,

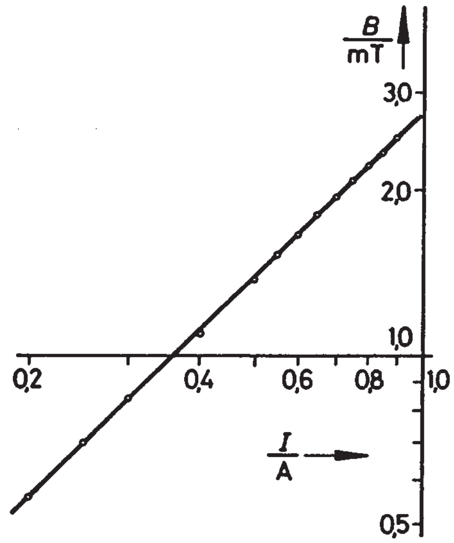
$$\mu_0 = 1.26 \cdot 10^{-6} \ \frac{\text{Vs}}{\text{Am}} \ .$$

From (1) and (2) one obtains

$$B = \underline{\mu_0} \ . \quad I \tag{4}$$

for a long straight conductor, where $|\vec{r}\,|$ is the distance of point P, at which the magnetic flux density is measured, from the axis of the conductor.

Fig. 2: Magnetic field inside a conductor as a function of the current flowing.



Since the current density $\vec{j}$ is uniform in the electrolyte, the current I flowing through the area A is expressed as a function of the current I_{tot} flowing through the whole cross-section of the electrolyte, from (3), as

$$I = I_{tot} \ \frac{r^2}{R^2} \ ,$$

so that (4) gives

$$B = \frac{\mu_0}{2\pi} \ I_{tot} \ \frac{|\vec{r}\,|}{R^2} \ . \tag{5}$$

B is measured with an induction coil. The induced voltage U is

$$U \sim B \ .$$

From the regression line to the measured values of Fig. 2, and the exponential statement

$$Y = A \cdot X^B$$

the exponent follows as

$$B = 0.989 \pm 0.003 \qquad \text{(see (5))}$$

From the regression lines to the measured values of Fig. 3 and the linear statement

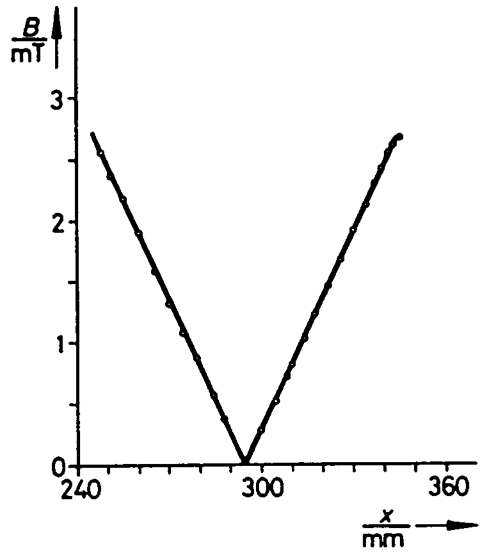
$$Y = A + B \cdot X$$

the slope follows as

B_1 = (– 0.0545 ± 0.0006) Vsm^{-3} (see (5))

and B_2 = (+ 0.0548 ± 0.0003) Vsm^{-3}

Fig. 3: Magnetic field inside a conductor as a function of the position *x*.



For the axis intercept, there follows

A_1 = 16.02 mT

A_2 = – 16.19 mT

From these, the point at which the field strength disappears is obtained as

A_1/B_1 = 294.0 mm

A_2/B_2 = 295.4 mm.