# Working definition of voltage

LEP
4.1.05

## Related topics

Maxwell's equations, electrostatics, electric field, voltage, inductive charges.

## Principle and task

Electric charge is displaced within the field of a plate capacitor using an electrically conducting transport plate. Voltage is directly measured by means of performed work and of the transmitting charge.

## Equipment

| | | |
|---|---|---|
| Balance LGN 310, on rod | 11081.01 | 1 |
| Apparatus f. defin. of the volt | 11209.88 | 1 |
| consisting of | | |
| Hooks, 3 pcs | 02274.04 | 1 |
| Plate capacitor, 283×283 mm | 06233.02 | 1 |
| Lever bar with sliding mass | 11209.01 | 1 |
| Charge transfer plate | 11209.02 | 1 |
| Capacitor plate with spacers | 11209.03 | 1 |
| Universal measuring amplifier | 13626.93 | 1 |
| High voltage supply unit, 0-10 kV | 13670.93 | 1 |
| PEK capacitor/case 1/0.1 mmF/500 V | 39105.18 | 1 |
| Tripod base -PASS- | 02002.55 | 2 |
| Support rod -PASS-, square, l 250 mm | 02025.55 | 1 |
| Support rod -PASS-, square, l 630 mm | 02027.55 | 1 |
| Right angle clamp -PASS- | 02040.55 | 3 |
| High-value resistor, 10 MOhm | 07160.00 | 1 |
| Multi-range meter A | 07028.01 | 1 |
| Connecting cord, 100 mm, green-yell | 07359.15 | 1 |
| Connecting cord, 500 mm, red | 07361.01 | 1 |
| Connecting cord, 500 mm, blue | 07361.04 | 1 |
| Connecting cord, 50 KV, 500 mm | 07366.00 | 1 |
| Screened cable, BNC, l 750 mm | 07542.11 | 1 |
| Adapter, BNC socket - 4 mm plug | 07542.20 | 1 |
| Connector, T type, BNC | 07542.21 | 1 |
| Adapter, BNC-plug/socket 4 mm. | 07542.26 | 1 |

## Problems

1. A system is set up which consists of two capacitor plates and a smaller metallic plate which can oscillate between the plates. A direct voltage is applied to the capacitor, which is sufficient to keep the oscillating movement permanently going.
2. The transported amount of charge is measured as a function of performed mechanical work.
3. It must be demonstrated that the ratio work/charge corresponds to the voltage applied to the plate capacitor, within the limits of measurement precision.

## Set-up and carrying out

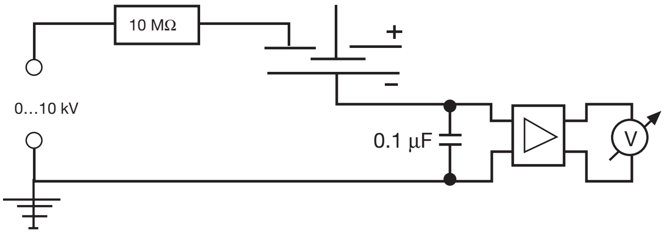
The experimental set-up is shown in fig. 1.

The lower capacitor plate is set horizontally using the adjusting feet of the tripod support. The shaft of the transport plate, which hangs in the plate capacitor, is passed through the hole in the upper capacitor plate and is hanged by the hook to the end of the lever rod.

The pivoting shaft of the lever rod is attached to the long support rod by means of a double clamp. The height of the rota-

Fig. 1: Measurement set-up: Working definition of voltage.

Fig. 2: Wiring diagram.




tion axis must be adjusted in such a way that the transport plate is exactly in the middle of the plate capacitor when the lever rod is set to its horizontal equilibrium position by means of the counter-weight. In this case, the notch in the shaft of the transport plate is at the same height as the upper capacitor plate.

The balance is now attached to the horizontal short support rod. It acts vertically on the lever rod, over the pin situated near the axis. The height of the balance is adjusted so that the transport plate is free in the middle plane of the capacitor, whilst the balance beam is held on the mark of the transport plateís initial position. The smaller torque acting on the lever arm on the capacitor side is compensated by displacing the counter-weight on the opposite lever arm. The stabilising sliding weight of the lever rod is far from the axis. If adjustment is correct, in the resting position of the filed motor, the transport plate is situated in the middle between the capacitor plates, whilst the balance beam points to the mark.

The electric wiring diagram is shown in figure 2.

Both the lower connector of the high voltage power supply and the lower capacitor plate are grounded. The upper capacitor plate is connected to the upper connector over the 10 MΩ protective resistor. The measurement amplifier is set to high input resistance, to amplification factor 1 and to time constant 0, whilst the range selector switch of the voltmeter is set to 10 V.

The experiment is carried out in two parts:

1. For the first part, a voltage of 4–10 kV is applied to the plate capacitor. After having been set in movement, the transport plate oscillates between the capacitor plates, continuously transferring the amount of charge $Q_0$ per transport run.

The velocity of the field motor can be reduced through displacement of the sliding weights on the balance beam. The transported charge $Q$ is determined over the voltage increase $U$ measured at the capacitor $C$ = 100 nF, according to $Q = C \cdot U$ (the electric current always flows from the upper capacitor plate to the measurement amplifier). Voltages higher than 10 V are avoided by discharging the 100 nF capacitor from time to time.

2. During the second part of the experiment, the average electrostatic force $\vec{F}_{ave}$ which acts on the transport plate is determined. For this, the transport plate is charged through contact with the upper capacitor plate and brought manually into the central position (the balance beam points to the mark). The absolute value of the gravitational force $F = m \cdot g/5$ ($g$ = 9,81 m/s$^2$ taking the shorter lever arm into account) for which the transport plate remains in the centre, is the average electrostatic force $\vec{F}_{ave}$ acting on the transport plate.

**Theory and evaluation**

Electrostatic processes in vacuum (and with a good degree of approximation in air) are completely described by Maxwell's equations:

$$div\,\vec{E} = \frac{\rho}{\epsilon_0} \qquad (1)$$

$$rot\,\vec{E} = 0 \qquad (2)$$

where $\rho = dQ/dV$ is the charge density, $\epsilon_0$ = 8,8542 · 10 As/(Vm) the induction constant and E the electric field intensity. It is defined as the quotient of an electric force $F$ acting on an arbitrary charged body and its charge $Q$:

$$\vec{E} = \frac{\vec{F}}{Q} \qquad (3)$$

Applying Gauss-Ostrogradski's theorem to the first equation,

$$\oint\!\!\oint_A \vec{E}\, d\vec{A} = \frac{Q}{\epsilon_0} \qquad (4)$$

the closed surface integral of the electric field is equal to the charge surrounded by the surface, multiplied by a constant factor.

Using equation (4), for example the homogeneous field of the plate capacitor can be determined:

$$\vec{E} = \frac{Q}{\epsilon_0} = \epsilon_0 \frac{A}{d_0} \frac{U}{\epsilon_0 A} = Ud \qquad (5)$$

where $A$ is the surface of the capacitor and $\pm Q$ are the charges on the capacitor plates. In equation (5), the following relation between charge $Q$, applied voltage $U$ and the capacity

$$C = \epsilon_0 \frac{A}{d_0}$$

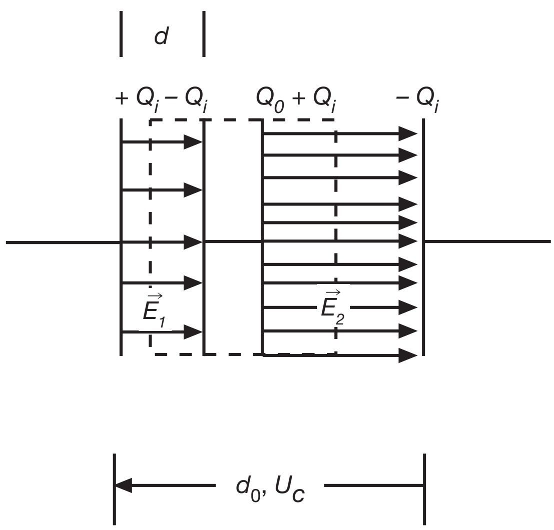
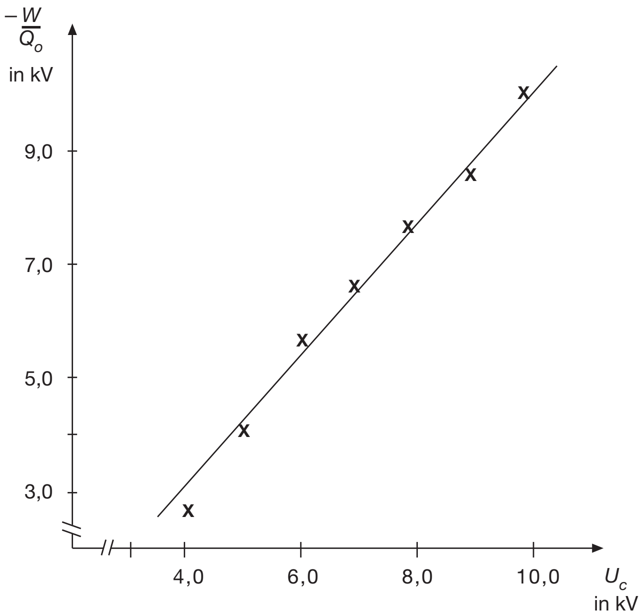



Fig. 3: Electric field and charges at the capacitor plates and at the transport plate. The dotted lines indicate the volume of integration.

Fig. 4: Quotient between performed work and transported charge as a function of plate capacitor voltage $U$.




of the plate capacitor, where $d_0$ is the distance between the plates, was used:

$$C = C \cdot U = \epsilon_0 \frac{A}{d_0} \cdot U \quad (6)$$

In this experimental set-up, a transport plate is situated between the capacitor plates. During transport, induction charges are generated on the surfaces of the transport plate, so that the system may be considered as consisting of two plate capacitors in series (cf. fig. 3). If the transport plate with surface $A_0$ is in contact with a capacitor plate, the charge $Q_0$ on the plate is calculated according to:

$$Q_0 = \epsilon_0 \frac{A_0}{d} \cdot U$$

When the plate are separated, this charge remains on the transport plate.
Furthermore, two opposed induction charges of the same magnitude, $Q_i$, are displaced on both sides of the transport plate. If the surface of integration is laid adequately, as indicated in Fig. 3, the following relation is obtained for the electric fields from the symmetry of the array and from equation (4):

$$-E_1 \cdot A_0 + E_2 \cdot A_0 = Q/\epsilon_0 \quad (7)$$

The electric fields generated by the charges exert a force on the charges. It must be taken into account that the electric field is in turn weakened by the charges, until it becomes zero inside the transport plate. In the average, only half the field acts effectively on the charges. The electrostatic force acting on the transport plate thus becomes:

$$\left|\vec{F}_{e1}\right| = \left|\vec{F}_2 - \vec{F}_1\right| = 0.5 \cdot (E_2(Q_i + Q_0) - E_1 \cdot Q_i)$$

Using equations (5) and (7), the following relation results:

$$\left|\vec{F}_{e1}\right| = \frac{Q_0}{2} \cdot \left(\frac{U_1}{d} + \frac{Q_i + Q_0}{\epsilon_0 A}\right) \quad (8)$$

In order to determine the induction charges $\pm Q_i$, the linear relation of the single voltages between the capacitor plates and the transport plate ($U_1$, $U_2$), and total voltage $U$, is used. Together with equation (6), the following relation is obtained for the latter:

$$U_c = U_1 + U_2 = \frac{1}{\epsilon_0 A} (d_0(Q_i + Q_0) - d \cdot Q_0)$$

and for the induction charge:

$$Q_i = Q_0 \frac{d}{d_0} \quad (9)$$

The electrostatic force on the transport plate is thus:

$$\left|\vec{F}_{e1}\right| = \frac{Q_0 U_c}{d_0} \left(\frac{d}{d_0} + \frac{1}{2}\right) \quad (10)$$

Work performed during a transport:

$$W = \int_0^{d_0} \vec{F}\, d\vec{r} = \frac{Q_0 U_c}{d_0} \left[\frac{1}{2}\left(\frac{d^2}{d_0} + d\right)\right]_0^{d_0} = -Q_0 \cdot U_c \quad (11)$$

is thus the same as if a probe charge $Q_0$ was transported over the same distance in the constant field $E = U_c/d_0$, which would prevail between the capacitor plates if no transport plate was between them: the initial and final states of both transport processes are identical, the performed work is independent of the path and of the induced charges.
The average force $F_{ave}$, which according to equation (10) is present exactly in the middle between the capacitor plates, thus allows to reach conclusions concerning the performed work. This allows to determine the applied voltage according to equation (11):

$$U = \int_0^{d_0} \vec{F}\, d\vec{r} = \int_0^{d_0} \vec{E}\, d\vec{r} = -\frac{d_0 \cdot F_{ave}}{Q_0} \quad (12)$$

This is exactly the definition of the voltage between the capacitor plates.

The following table gives a summary of typical measurement values (example):

| $U_c$ in kV | $U_e$ in V | n | $m$ in mg | $Q$ in nAs | $W$ in mJ | $-W/Q$ in kV |
|---|---|---|---|---|---|---|
| 4.0 | 9.1 | 6 | 5.5 | 79.2 | 0.220 | 2.7 |
| 5.0 | 9.8 | 5 | 10.5 | 100 | 0.412 | 4.1 |
| 6.0 | 9.2 | 4 | 17.0 | 117.5 | 0.667 | 5.7 |
| 7.0 | 8.0 | 3 | 23.0 | 136.3 | 0.903 | 6.6 |
| 8.0 | 9.4 | 3 | 31.0 | 160.1 | 1.216 | 7.6 |
| 9.0 | 7.1 | 2 | 39.5 | 181.4 | 1.550 | 8.5 |
| 10.0 | 7.9 | 2 | 50.0 | 197.9 | 1.962 | 9.9 |

In Fig. 4, the linear relation between the applied voltage and the quotient of work to charge is plotted. Systematically too low values for the measured work are due to friction losses and to the acceleration work of the transport plate together with the connected lever system.
This experiment is meant to show that voltage can be measured directly on the base of work and charge and not only over currents, as is usual in practice.