PHYWE

Internal resistance and matching in voltage source

LEP 4.1.03

Related topics

Voltage source, electromotive force (e.m.f.), terminal voltage, no-load operation, short circuit, Ohm's law, Kirchhoff's laws, power matching.

Principle and task

Both the terminal voltage of a voltage source and the current depend on the load, i. e. on the external resistance. The terminal voltage is measured as a function of the current and from it the internal resistance and no-load voltage of the voltage source are determined and the power graph plotted.

Equipment

Battery box	06030.21	1
Flat cell battery, 9 V	07496.10	1
Flat battery, 4.5 V	07496.01	1
Power supply 5 V DC/0.3 A	11076.93	1
Rheostat, 10 Ohm, 5.7 A	06110.02	1
Rheostat, 100 Ohm, 1.8 A	06114.02	1
Digital multimeter	07134.00	2
Double sockets, 1 pair, red a. black	07264.00	1
Connecting cord, 500 mm, red	07361.01	3
Connecting cord, 500 mm, blue	07361.04	2

Problems

1. To measure the terminal voltage U_t of a number of voltage source as a function of the current, varying the external resistance R_e, and to calculate the no-load voltage U_0 and the internal resistanceR_i.
   - 1.1 Slimline battery
   - 1.2 Power supply
   - 1.2.1 Alternating voltage output
   - 1.2.2 Direct voltage output
2. To measure directly the no-load voltage of the slimline battery (with no external resistance) and its internal resistance (by power matching, $R_i = R_e$).
3. To determine the power diagram from the relationship between terminal voltage and current, as illustrated by the slimline battery.

Set-up and procedure

1. Connect a variable resistor R_e to the voltage source as shown in Fig. 2. (Use the 100 Ω rheostat, or the 10 Ω rheostat for higher currents). For convenience, vary the current I in 0.1 A steps for the slimline battery, and in 0.05 A steps for power supply. Measure the terminal voltage U_t with the digital voltmeter.

Fig.1: Experimental set-up for determining the no-load voltage and internal resistance of a voltage source.

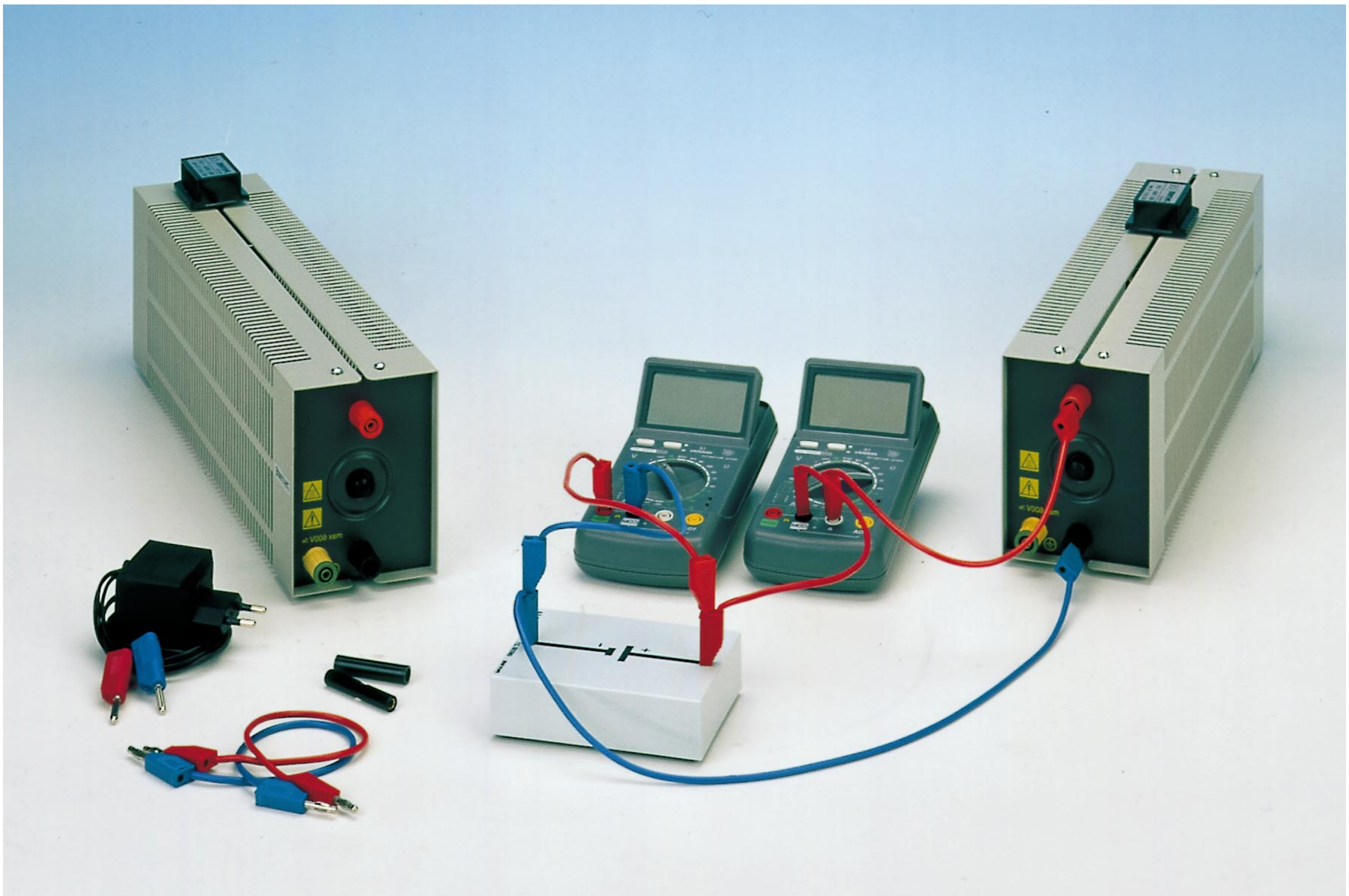
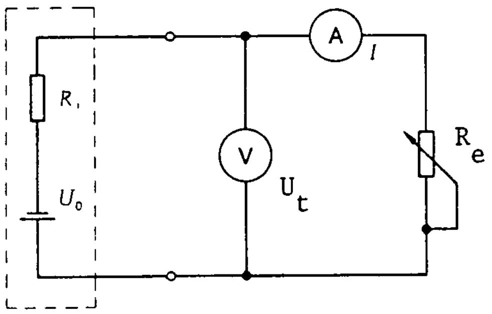


Fig. 2: Circuit for measuring terminal voltage and current.



2. First measure the no-load voltage U_0 directly, without external resistance.

Then load the voltage source (without the ammeter), with an external resistor R_e.

Set R_e so that

$$U_t = \frac{U_0}{2}$$

In this case the internal resistance is $R_i = R_e$.

Measure R_e with the resistance measuring range of the digital multimeter.

Theory and evaluation

Real voltage sources can be represented in the equivalent circuit diagram by an ideal voltage source with no-load voltage U_0 and internal resistance R_1 connected in series (Fig. 2).

If the voltage source is connected to an external resistance R_e, then according to Ohm's law a current

$$I = \frac{U_0}{R_i + R_e} \quad (1)$$

will flow.

The terminal voltage U_t is given by:

$$U_t = U_0 - R_i\, I \quad (2)$$

The regression lines with a linear portion in accordance with equation (2) give the following values:

1.1 Slimline battery

U_0 = 4.66 V

R_i = 1.47 Ω

s_{U_0} = 0.03 V

s_{R_i} = 0.02 Ω

1.2 Power supply, alternating voltage

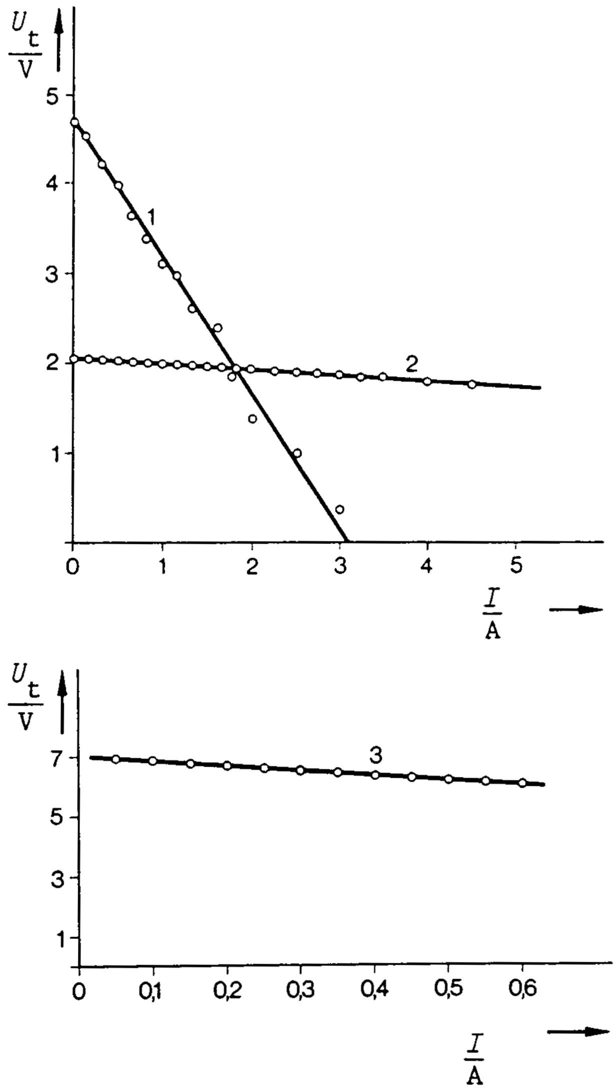
U_0 = 6.948 V

R_i = 1.55 Ω

s_{U_0} = 0.005 V

s_{R_i} = 0.01 Ω

Fig. 3: Terminal voltage as a function of the current:
1) slimline battery
2) lead-acid accumulator (not included)
3) power supply, alternating output voltage.



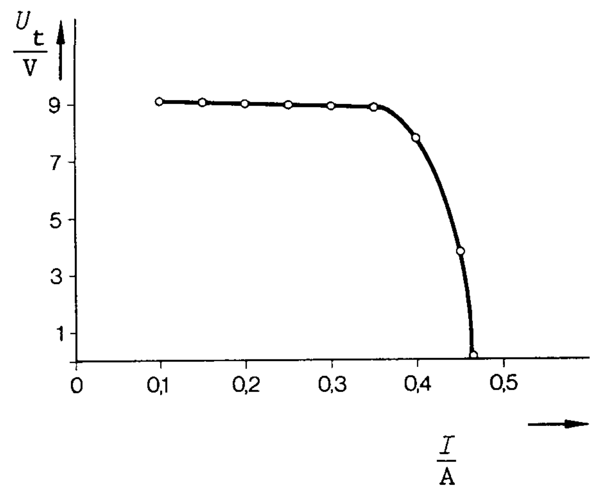
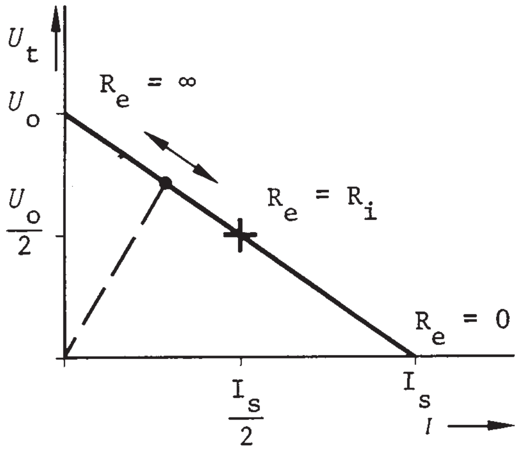


Fig. 4: Terminal voltage as a function of the current: power supply, direct voltage output 9 V, 0.3 A.

Fig. 5: Current/voltage characteristic of a voltage source with constant internal resistance R_i (U_0 = no-load voltage, I_s = short-curcuit current, R_e = external resistance;



At low currents (up to 0.3 A) the following values are obtained from the regression lines of the measurements plotted in Fig. 4 (see equation [2]):

U_0 = 9.080 V
R_i = 0.36 Ω
s_{U_0} = 0.005 V
s_{R_i} = 0.02 Ω

The curve is typical of electronically controlled power supplies: the voltage stabilisation causes a low internal resistance (at low currents); the current limiter makes the internal resistance rise suddenly so that a given value is not exceeded.

2. Ideally is a linear relationship between the terminal voltage U_t and the current I (equation 2, Fig. 5).

The external resistance R_e determines the ratio of terminal voltage to current at the working point.

$$R_e = \frac{U_t}{I} \tag{3}$$

We destinguish three types of load:

No-load: (voltage matching)

$$R_e = \infty$$

No current flows and there is no voltage drop over R_i

$$U_t = U_0$$

Short-circuit (current matching)

$$R_e = 0$$

The voltage drops by U_0 across the internal resistance, so that $U_t = 0$.

The short-circuit current that flows is:

$$I_s = \frac{U_0}{R_i} \tag{4}$$

Power matching

(Resistance matching)

$$R_e = R_i$$

In this case

$$U_t = \frac{U_0}{2}, \quad I = \frac{I_s}{2}.$$

The measurements taken with the slimline battery gave

U_0 = 4.69 V ± 0.01 V

R_i = 1.50 Ω ± 0.01 Ω

3. The power diagram of a voltage source shows the mutually opposed relationships between terminal voltage and current, and the derived power output, as a function of the external load. For this purpose, the measured values are normalised with the corresponding characteristic quantities of the voltage source.

From equations (1) and (4) we obtain

$$\frac{I}{I_s} = \frac{1}{1 + \frac{R_e}{R_i}} \tag{5}$$

Equation (2) gives

$$\frac{U_t}{U_0} = 1 - \frac{1}{1 + \frac{R_e}{R_i}} \tag{6}$$

or, from (5):

$$\frac{U_t}{U_0} + \frac{I}{I_s} = 1 \tag{7}$$

The power absorbed by resistor R_e is

$$P = I^2 \cdot R_e \tag{8}$$

It is normalised with the power absorbed by R_i in the event of a short-circuit

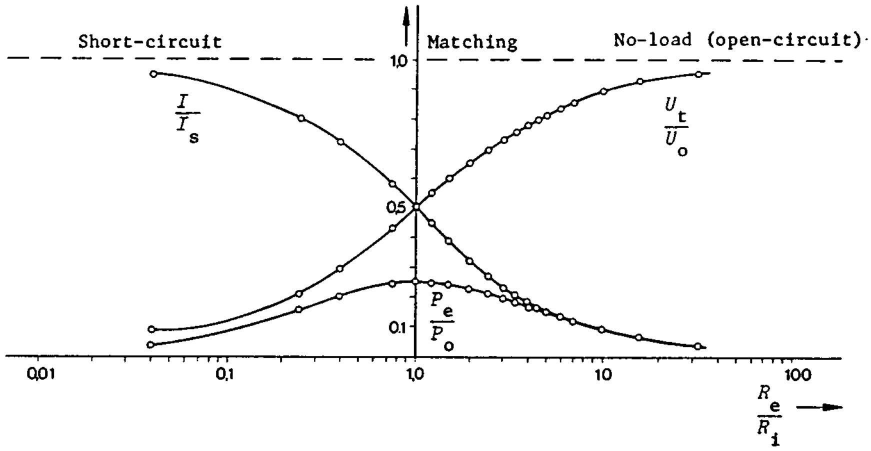
$$P_0 = \frac{U_0^2}{R_i} \tag{9}$$

Therefore,

$$\frac{P}{P_0} = \frac{U_t}{U_0} \cdot \frac{I}{I_s} = \frac{R_e/R_i}{(1+R_e/R_i)^2} \tag{10}$$

Fig. 6 shows the relationship between equations (5), (6) and (10), using the values (para. 1.1) measured with the slimline battery. The size of the external resistor R_e was calculated using equation (3). The value of R_e thus includes the internal reistance of the ammeter. The power absorbed by R_e is at its maximum value when $R_e = R_i$.

Fig. 6: Power diagram of a voltage source.



Note

Aditional experiments can be performed with a standard car batterie supplied locally.