Maxwellian velocity distribution

LEP 3.2.03

Related topics

Kinetic theory of gases, temperature, gas, molecules, model kinetic energy, average velocity, velocity distribution.

Principle and task

By means of the model apparatus for kinetic theory of gases the motion of gas molecules is simulated and the velocity is determined by registration of the throw distance of the glass balls. This velocity distribution is compared to the theoretical MAXWELL-BOLTZMANN equation.

Equipment

Kinetic gas theory apparatus	09060.00	1
Receiver with recording chamber	09061.00	1
Power supply 0-12V DC/6V, 12V AC	13505.93	1
Digital stroboscope	21809.93	1
Stopwatch, digital, 1/100 sec.	03071.01	1
Tripod base -PASS-	02002.55	2
Connecting cord, 750 mm, red	07362.01	1
Connecting cord, 750 mm, blue	07362.04	1
Test tube, 160×16 mm, 100 pcs	37656.10	1
Test tube rack f. 12 tubes, wood	37686.00	1

Problems

1. Measure the velocity distribution of the “model gas”.
2. Compare the result to theoretical behaviour as described by the MAXWELL-BOLTZMANN distribution.
3. Discuss the results.

Set-up and procedure

The experimental set-up is as shown in Figure 1.

According to the operating instructions for the determination of particle velocities the apparatus is equipped with the corresponding receiver with recording chamber.

At first determine the average weight of one glass ball by weighing out a known number (100) of balls to avoid the time consuming counting of glass balls during the experiments. In a next experiment the average number of glass balls pushed out from the apparatus during 1 minute is determined. Therefore fill the apparatus with 400 glass balls and set it to the following conditions:

– height of the upper piston: 6 cm
– frequency of the oscillator: 50 s^{-1} (controlled by the voltage and the stroboscope).

Fig. 1: Experimental set-up: Maxwellian velocity distribution.

Then open the outlet for 1 minute and determine the number of pushed out balls by weighing. Afterwards the apparatus is refilled with these balls and the same experiment is repeated. For preparing the simulation experiment four test tubes were filled each with the determined average number of lossed balls per minute. Then make the following settings at the apparatus:

– height of the upper piston: 6 cm
– height difterence between outlet and receiver: 8 cm
– number of balls: 400
– frequency of oscillator: 50 s^{-1}

When the frequency is stable open the outlet for 5 minutes. Every one minute fill up the apparatus with the balls in a test tube to maintain a constant "particle density". Afterwards the test tubes are refilled.

This experiment is repeated four times.

The number of glass balls in each of the 24 compartments of the receiver is determined by weighing.

Theory and evaluation

The molecules of an ideal gas posseses per definition only kinetic energy which is defined by

$$\overline{E}_k = \frac{m}{2} \cdot \overline{c}^{\,2} \qquad (1)$$

$\overline{E}_k$ Average kinetic energy
m Mass of the molecule
$\overline{c}$ Average velocity of the molecule

Based on the kinetic theory the pressure of an ideal gas can be described by

$$p = \frac{1}{3} \cdot \rho \cdot \overline{c}^{\,2} \qquad (2)$$

p Pressure
ρ Density

Combining of the equations (1), (2) and the equation of state for ideal gases

$$p \cdot V_{mol} = R \cdot T \qquad (3)$$

V_{mol} Molar volume
R Gas constant
T Absolute temperature

leads to the following expression for c:

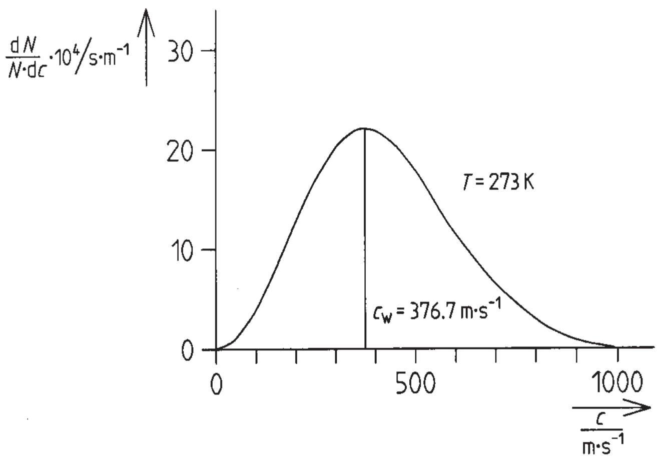
$$\overline{c} = \left(\frac{3 \cdot R \cdot T}{M}\right)^{1/2}$$

$$\overline{c} = \left(\frac{3 \cdot k \cdot T}{m}\right)^{1/2} \qquad (4)$$

k BOLTZMANN constant

This means that the average kinetic energy is directly proportional to the absolute temperature of the gas which is the interpretation of temperature on the molecular level.

Fig. 2: Distribution of molecule velocities of oxygen at 273 K.



The direct determination of the velocity of a certain molecule is impossible because it changes incessantly caused by collisions with other molecules. For a great number of molecules one can derive a distribution function for molecule velocities by means of statistical methods. This was done by MAXWELL and BOLTZMANN with the following result:

$$\frac{dN}{N} = \sqrt{\frac{2}{\pi}} \cdot \left(\frac{m}{k \cdot T}\right)^{3/2} \cdot c^2 \cdot e^{-\left(\frac{m \cdot c^2}{2kT}\right)} \cdot dc \qquad (5)$$

This equation describes the probability that the velocity of a molecule is within the interval (c,c+dc). As an example the corresponding distribution function for oxygen at 273 K is shown in Figure 2.

For the velocity at the maximum c_w of the curve (velocity with highest probability) the following relation can be derived:

$$c_w = \left(\frac{2 \cdot k \cdot T}{m}\right)^{1/2} \qquad (6)$$

Introducing of equation (6) into equation (5) leads to

$$\frac{dN}{N} = \frac{4}{\sqrt{\pi}} \cdot \left(\frac{1}{c_w^2}\right)^{3/2} \cdot c^2 \cdot e^{-\left(\frac{c^2}{c_w^2}\right)} \cdot dc \qquad (7)$$

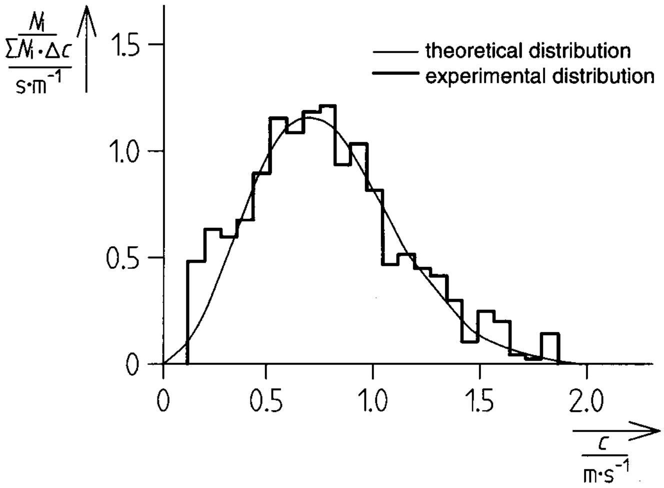
Note that $c_w \neq \overline{c}$
and $c_w : \overline{c} : \sqrt{\overline{c^2}} = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3} = 1 : 1.13 : 1.22$.

Concerning the model experiment with glass balls the velocity of the balls can be determined by means of the throw distance s:

$$c = s \cdot \left(\frac{g}{2 \cdot h}\right)^{1/2} = K \cdot s \qquad (8)$$

g Acceleration at the earth surface (= 9.81 ms^{-1})
h Height difference between outlet and receiver

Fig. 3: Experimental and theoretical velocity distribution in the model experiment



Now the experimental results (number of balls per throw distance interval) can be displayed graphically in the form

$$\frac{1}{\Sigma N_i} \cdot \frac{N_i}{\Delta c} = f(c) \qquad (9)$$

N_i Number of balls in the interval i, i = 1…23

Δc Velocity interval corresponding to Δs = 1 cm (0.078 ms^{-1})

as shown in Figure 3.

The theoretical distribution function can be evaluated by means of equation (7) using the velocity at the maximum of the experimental distribution as c_w The result for the example in Figure 3 is also shown in the diagram. The agreement between the two curves is reasonably good taking into account the model character of the experiment.