Determination of the diffraction intensity at slit and double slit systems

LEP 2.3.05

Related topics

Huygens principle, interference, Fraunhofer and Fresnel-diffraction, coherence, laser.

Principle and task

Slit and double slit systems are illuminated with laser light. The corresponding diffraction patterns are measured by means of a photodiode which can be shifted, as a function of location and intensity.

Equipment

Laser, He-Ne 1.0 mw, 220 V AC	08181.93	1
Universal measuring amplifier	13626.93	1
Optical profile bench l = 60 cm	08283.00	1
Base f.opt.profile-bench, adjust.	08284.00	2
Slide mount f. opt. pr.-bench, h 80 mm	08286.02	4
Slide mount, lateral. adjust., cal.	08082.03	1
Lens holder	08012.00	2
Object holder, 5×5 cm	08041.00	1
Lens, mounted, f +20 mm	08018.01	1
Lens, mounted, f +100 mm	08021.01	1
Photoelement f. opt. base plt.	08734.00	1
Diaphragm, 3 single slits	08522.00	1
Diaphragm, 4 double slits	08523.00	1
PEK carbon resistor 1 W 5 % 2.2 kOhm	39104.23	1
Multi-range meter A	07028.01	1
Connecting cord, 750 mm, red	07362.01	1
Connecting cord, 750 mm, blue	07362.04	1

Problems

1. Determination of the intensity distribution of the diffraction patterns due to two slits of different widths.
   The corresponding width of the slit is determined by means of the relative positions of intensity values of the extremes. Furthermore, intensity relations of the peaks are evaluated.
2. Determination of location and intensity of the extreme values of the diffraction patterns due to two double slits with the same widths, but different distances between the slits. Widths of slits and distances between the slits must be determined as well as the intensity relations of the peaks.

Set-up and procedure

The experimental set-up is shown in fig. 1. A broadened and parallel laser beam, obtained with the lenses f = 20 mm and f = 100 mm, must impinge centrally on the photocell. The photocell is situated at the centre of its shifting range. The slit diaphragm is then set onto the photocell and the diaphragm with a simple slit or the one with two slits is set into the diaphragm support. It must be made sure that the slit which is to be investigated is placed centred and perpendicularly to the beam. If experiments are being carried out with two slits, these must receive the same luminous intensity.

The laser and the measurement amplifier should be warmed up for about 15 minutes before work starts, so as to avoid bothersome intensity fluctuations during measurements. The photodiode with a 2.2 kΩ resistor in parallel is connected to

Caution: Never look directly into a non attenuated laser beam

Fig. 1: Experimental set-up for the investigation of the diffraction intensity of slits and double slits.
(Component locations on the optical bench: laser = 2.5 cm; lens f/20 mm = 14 cm; lens f/100 mm = 27 cm; slits/double slits = 32.5 cm; slide mount lateral adjustm, calibr. = 139.5 cm).

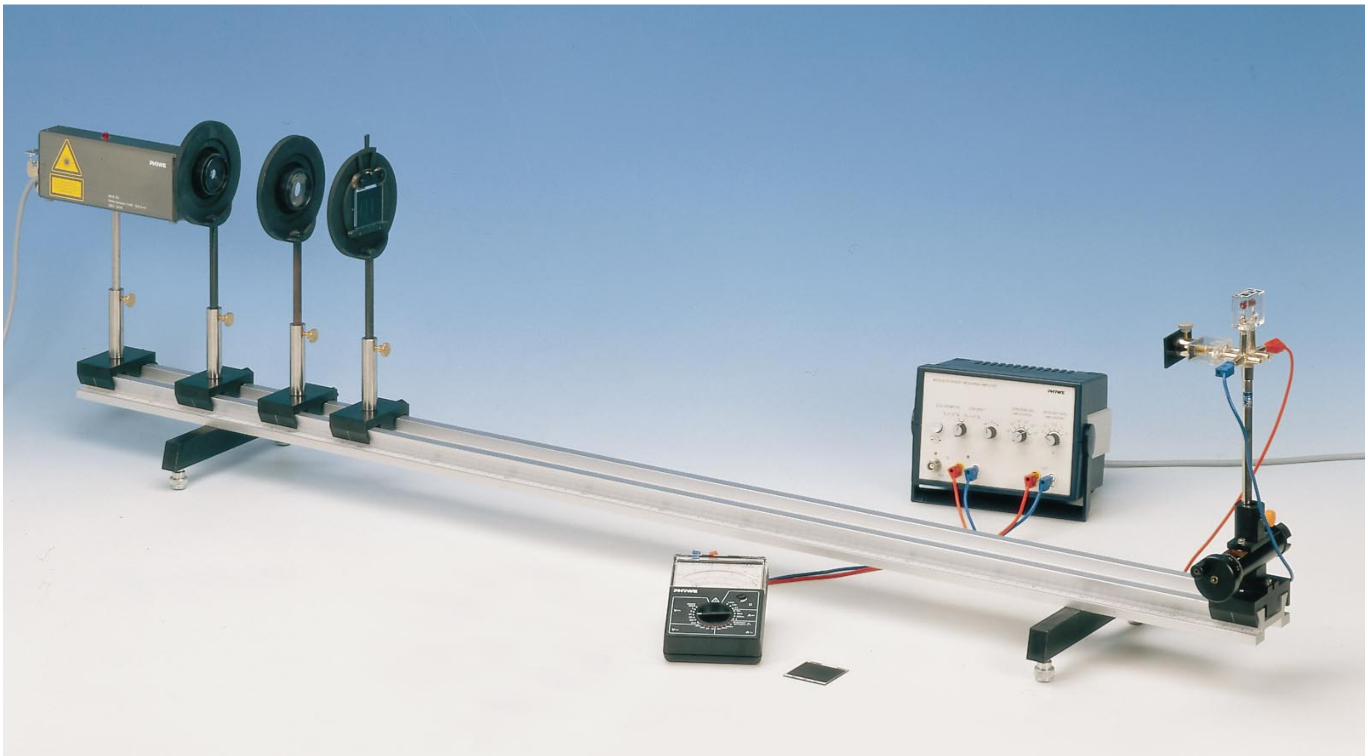
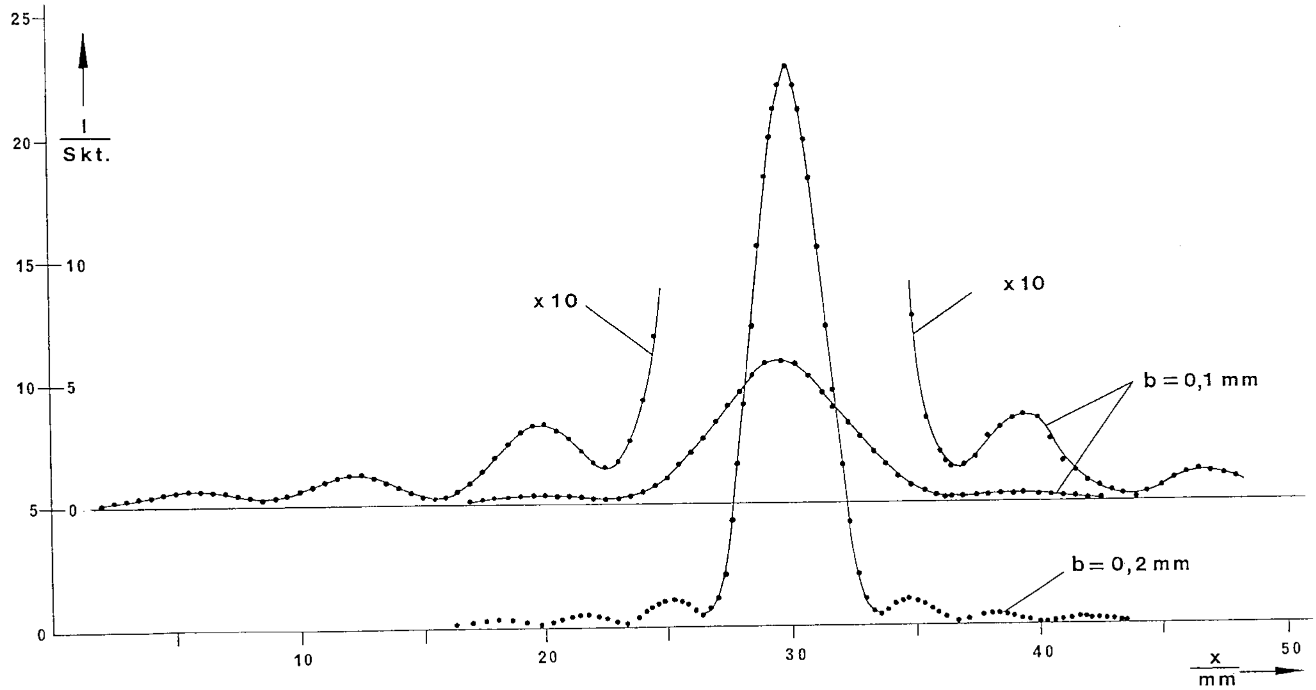


Fig. 2: Diffraction intensity I as a function of location x for the single slit b_1 = 0.1 mm and b_2 = 0.2 mm. The x axis of the graph for b_1 = 0.1 mm is shifted upwards. The intensity of the areas next to the central peak is represented enlarged by a factor of 10. (Distance between slit and photodiode L = 107 cm; λ = 632.8 nm).



the 10^4 Ω-input of the measurement amplifier (amplification factor 10^3–10^5). When the amplification factor is changed, the zero of the measurement amplifier must be controlled with covered photodiode. For the single slits b_1 = 0.1 mm and b_2 = 0.2 mm, the positions of peaks and minima must be determined exactly, whilst the intensities between them are determined by shifting the photodiode by steps of 0.3 mm to 0.5 mm.

In the case of double slit systems, only the positions of extremes and the intensities of peaks must be determined.

Theory and evaluation

If monochromatic light of wavelength λ impinges on a system of parallel and equidistant slits, the luminous intensity I of the beams diffracted in the direction φ is given through:

$$I(\varphi) \propto b^2 \frac{\sin^2\left(\frac{\pi}{\lambda}\cdot b\cdot\sin\varphi\right)}{\left(\frac{\pi}{\lambda}\cdot b\cdot\sin\varphi\right)^2}\cdot\frac{\sin^2\left(\frac{p\pi}{\lambda}\cdot g\cdot\sin\varphi\right)}{\sin^2\left(\frac{\pi}{\lambda}\cdot g\cdot\sin\varphi\right)} \quad (1)$$

(b = width of the slit, g = distance between slits, p = number of slits)

The first factor in (1) gives the intensity distribution for a single slit, the second factor describes the modification of the diffracted intensity due to the combined effect of p slits.
For a double slit (p = 2), (1) yields:

$$I(\varphi) \propto b^2 \frac{\sin^2\left(\frac{\pi}{\lambda}\cdot b\cdot\sin\varphi\right)}{\left(\frac{\pi}{\lambda}\cdot b\cdot\sin\varphi\right)^2}\cdot 4\cos^2\left(\frac{\pi}{\lambda}\cdot g\cdot\sin\varphi\right) \quad (2)$$

If one considers only a single slit (1st class interference), this gives minimum intensity when the numerator of the first factor becomes zero. This always is the case when the following applies:

$$\sin\varphi_k = \frac{k\cdot\lambda}{b};\ (k = 1,2,3,\ldots) \quad (3)$$

For the secondary 1st class peaks one obtains approximately:

$$\sin\varphi_{k^*} = \frac{2k^*+1}{2}\cdot\frac{\lambda}{b};\ (k^* = 1,2,3,\ldots) \quad (4)$$

The particular case of the central 1st class peak for φ = 0 is not seized by (4).

If several slits act together (2nd class interference), supplementary minima appear, whose location is determined by the points at which the second factor becomes zero.
If p = 2, (2) yields:

$$4\cos^2\left(\frac{\pi}{\lambda}\cdot g\cdot\sin\varphi\right) = 0 \quad (5)$$

This expression becomes zero if

$$\sin\varphi_h = \frac{2h+1}{2}\cdot\frac{\lambda}{g};\ (h = 0,1,2,3,\ldots) \quad (6)$$

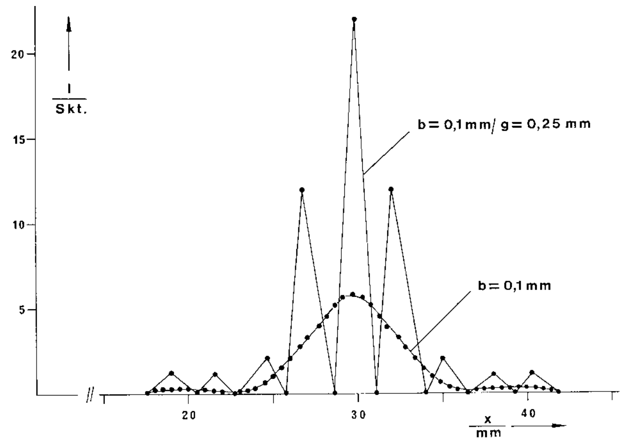
Supplementary 2nd class peaks appear when

$$4\cos^2\left(\frac{\pi}{\lambda}\cdot g\cdot\sin\varphi\right) = \pm 1 \quad (7)$$

This condition is fulfilled if

$$\sin\varphi_{h^*} = h^*\cdot\frac{\lambda}{g};\ (h^* = 0,1,2,\ldots) \quad (8)$$

Fig. 3: Diffraction intensity I as a function of location x for the single slit b_1 = 0.1 mm and for double slit g_1 = 0.25 mm/b_1 = 0.1 mm). For the double slit, only the locations and intensities of the extreme values were determined.



According to (1), the following applies for the intensities of the 1st class peaks:

for sin φ = 0 the following holds: $I_0 \propto b^2$ (principal maximum) (9)
for sin φ = 3/2 λ/b the following holds: $I_1 \propto 0.045\ I_0$ (1st order peak)
for sin φ = 5/2 λ/b the following holds: $I_2 \propto 0.016\ I_0$ (2nd order peak)
for sin φ = 7/2 λ/b the following holds: $I_3 \propto 0.0083\ I_0$ (3rd order peak)

From (1) and (7), it follows that the intensity of the main 2nd class peak is four times that of the intensity obtained with the corresponding single slit. Furthermore, the peaks of the single slit are modulated by 2nd class secondary peaks.

Single slit diffraction

Fig. 2 shows the intensity distributions for diffraction by the slits with widths b_1 = 0.1 mm and b_2 = 0.2 mm.

Table 1 gives the slit widths calculated with the assistance of (3) and (4), as well as the intensity relations which must be compared to (9).

Table 1

Slit b_1	Slit b_2
b_1 = 0.0982 mm; $\Delta b_1/b_1$ = ±3.5%	b_2 = 0.1985 mm; $\Delta b_2/b_2$ = ±0.5%
I_1/I_0 = 3.2 Skt./58 Skt. = 0.055	I_1/I_0 = 10 Skt./228 Skt. = 0.044
I_2/I_0 = 1.05 Skt./58 Skt. = 0.018	I_2/I_0 = 4 Skt./228 Skt. = 0.018
I_3/I_0 = 0.46 Skt./58 Sk.t = 0.008	I_3/I_0 = 2.1 Skt./228 Skt. = 0.009

Comparing the intensities I_0 of the central peaks for both simple slits, one obtains: I_{02}/I_{01} = 228 Skt / 58 Skt = 3.93. This value agrees satisfactorily with the requirement $I_{02}/I_{01} = b_2^2/b_1^2 = 4$.

Fig 2 also shows that the central peak is twice as broad as the other secondary peaks, and that if the width of the slit is doubled, the width of the central peak is reduced by a factor of 2.

Double slit diffraction

Fig. 3 shows the intensity distribution for diffraction by the double slit g_1 = 0.25 mm/b_1 = 0.1 mm. In this case, only extreme values were measured and linked by straight lines, as the experimental determination of the intermediary intensities is too complicated.

Evaluation yields the following values: g_1 = (0.251 ± 0.003) mm; b_1 = (0.0965 ± 0.0025) mm.

The distance between two minima of the double slit is λ/g, but the width of the single slit central peak is 2λ/b, so that the central 1st class peak is interspersed by 2λ/b : λ/g = 2g/b = 5 2nd class peaks. Only two 2nd class secondary peaks fall into the first 1st class secondary peak, because the zeros for the single and the double slit no longer coincide, but the zeros of the single slit are preserved in any case.

Comparison of the intensities I_0 and I_0^* of the central peak from single slit and double slit yields:

I_0^*/I_0 = 220 Skt / 58 Skt = 3.8. This relation agrees satisfactorily with the expected value $I_0^* = p^2\ I_0 = 4\ I_0$ (p = 2 in the case of the double slit).

Investigation of the diffraction pattern (not illustrated) of the double slit g_2 = 0.5mm / b_1 = 0.1mm shows that there are now 10-2nd class peaks in the central single slit peak, and that correspondingly 5 complete 2nd class peaks are found in the secondary peak of the single slit, in opposition to the previous double slit, because in this case, the minima of single and double slits coincide.