Dispersion and resolving power of the prism

Related topics

Maxwell relationship, dispersion, polarizability, refractive index, prism, Rowland grating, spectrometer-goniometer.

Principle and task

The refractive indices of liquids, crown glass and flint glass are determined as a function of the wavelength by refraction of light through the prism at minimum deviation. The resolving power of the glass prisms is determined from the dispersion curve.

Equipment

Spectrometer/goniom. w. vernier		35635.02	1
Lamp holder, pico 9, f. spectr.lamps		08119.00	1
Spectral lamp Hg 100, pico 9 base		08120.14	1
Power supply for spectral lamps		13662.93	1
Prism, 60 degrees, h 30 mm, crown		08231.00	1
Hollow prism		08240.00	1
Diffraction grating, 600 lines/mm		08546.00	1
Glycerol	250 ml	30084.25	1
Methanol	500 ml	30142.50	1
Cyclohexene for synth.	500 ml	31236.50	1
Wash bottle, plastic	250 ml	33930.00	1
Bench clamp, -PASS-		02010.00	1
Stand tube		02060.00	1

Problems

1. To adjust the spectrometer-goniometer.
2. To determine the refractive index of various liquids in a hollow prism.
3. To determine the refractive index of various glass prism.
4. To determine the wavelengths of the mercury spectral lines.
5. To demonstrate the relationship between refractive index and wavelength (dispersion curve).
6. To calculate the resolving power of the glass prisms from the slope of the dispersion curves.

Set-up and procedure

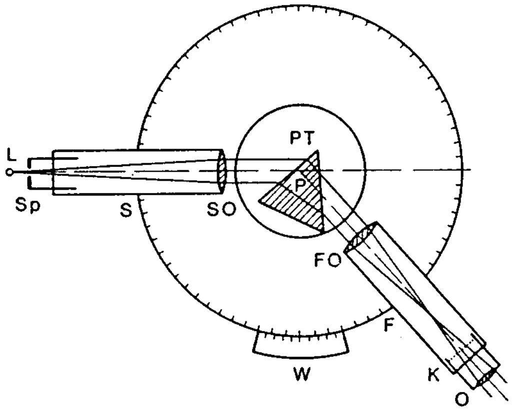
The experiment is set up as shown in Fig. 1.
The spectrometer-goniometer and the grating should be adjusted in accordance with the operating instructions. When the adjustment is correct, a parallel beam of light will pass through the prism (Fig. 2).

The aperture, or slit, is projected into the plane of the cross-wires with the telescope set to infinity and observed with the eyepiece which is used as a magnifier.

The prism is then set to the minimum deviation and the angular position ϕ_1 of the telescope read off on the vernier for each

Fig. 1: Experimental set-up for determining dispersion in liquids.

Fig. 2: Set-up and path of rays in the spectrometer.



(L = light source, Sp = slit in drawtube, S = collimator, SO = collimator lens, PT = prism table with adjusting screws, P = prism, FO = telescope lens, F = telescope, O = eyepiece, K = cross-wires, W = graduated circle with vernier).

spectral line. The prism is then turned so that the light falls on the adjacent surface and is deviated to the opposite side. The angle ϕ_2 is no read off for each spectral line, at minimum deviation.

A ruled grating which is secured in a holder perpendicular to the collimator axis, and takes the place of the prism, is used to determine the wavelengths of the mercury spectral lines. The angles of first-order diffracted lines are measured to the right and left of the undeviated image of the slit.

The spectral lamp reaches its maximum lumiosity after approx. 5 minutes' warm-up time. When setting up the lamp, ensure that air can circulate unimpeded through the ventilation slots on the lamp housing.

Theory and evaluation

The refractive index of a medium is linked to the relative permittivity ε_r by the Maxwell relationship

$$n = \sqrt{\varepsilon_r \mu_r} \qquad (1)$$

For most substances the permeability

$$\mu_r = 1.$$

According to Clausius and Mossotti, the following relationship exists between the relative permittivity and the molecular polarizability α of a medium:

$$\alpha = \frac{3\,\varepsilon_o}{N} \cdot \frac{\varepsilon - 1}{\varepsilon + 2} \qquad (2)$$

where N is the concentration of the polarizable molecules and ε_o is the electric field constant.

The polarizability depends on the frequency $w = 2\pi\,\nu$ of the incident light. The following is approximately true, beyond the natural frequency $w_o = 2\pi\,\nu_o$ of an atom or molecule:

$$\alpha = \frac{e^2}{m} \cdot \frac{1}{w_o^2 - w^2} \qquad (3)$$

where e is the elementary charge and m is the mass of an electron.

When (1) and (3) are substituted in (2) we obtain

$$\frac{n^2 - 1}{n^2 + 2} \cdot \frac{e^2 \cdot N}{3\,\varepsilon_o\,m}\,\frac{1}{w_o^2 - w^2} \qquad (4)$$

Although equation (4) only takes one natural frequency into account, this formula adequately describes the decrease in the refractive index as the wavelength increases, outside the range of natural frequencies.

The wavelength of the spectral lines are determined with a diffraction grating which is placed in the path of the rays, instead of the prism. Fo a wavelength λ, the grating constant G and angle ϕ at which the first order diffraction pattern appears, the following applies:

$$\lambda = G \cdot \sin\phi$$

λ is determined from the average of several measurements:

λ_{red} = 627.3 nm
λ_{yellow} = 579.8 nm
λ_{green} = 547.7 nm
$\lambda_{turquoise}$ = 493.9 nm
λ_{blue} = 438.5 nm
λ_{violet} = 405.1 nm.

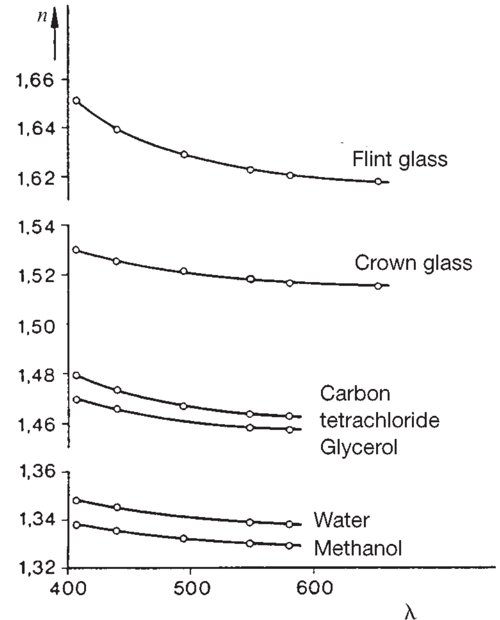
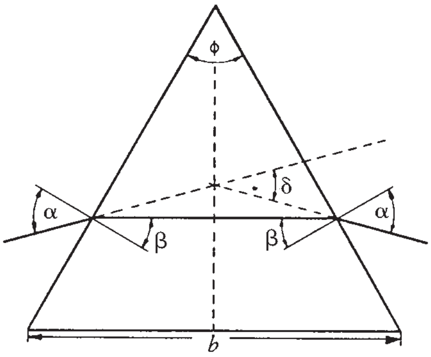


Fig. 3: Dispersion curves of various substances.

Fig. 4: Refraction by the prism when the path of a ray is symmetrical.



If a ray of light passes symmetrically through a prism (Fig. 4), minimum deviation δ occurs.

If α is the angle of incidence, β the angle of reflection, and θ the angle of the prism then

$$\sin\alpha = n \cdot \sin\beta \qquad (4)$$

$$\beta = \frac{\theta}{2} \quad \text{and} \quad \delta = 2\alpha - \theta \qquad (5)$$

From these we obtain

$$n = \frac{\sin\frac{\theta+\delta}{2}}{\sin\frac{\theta}{2}} \qquad (6)$$

The angle of minimum deviation δ is obtained from the difference between the angles ϕ_1 and ϕ_2 measured at the two different prism position (Fig. 5):

$$\delta = \frac{\varphi_1 - \varphi_2}{2}$$

The dispersion curve (Fig. 3) is determined from the angles measured for the various mercury spectral lines.

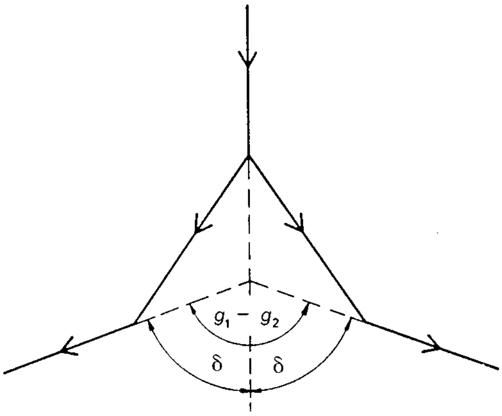


Fig. 5: Measurement of the angle of minimum deviation.

The performance of a spectrometer is characterised by its 'resolving power'. Two wavelengths λ and λ + dλ are still perceived as separate spectral lines when the principal maximum of line λ + dλ coincides with the minimum of line λ.

The resolving power *R* is generally defined by the expression:

$$R = \frac{\lambda}{d\lambda}$$

For a prism, the following applies:

$$R = b \cdot \left|\frac{dn}{d\lambda}\right|,$$

where *b* is the base of the prism (see Fig. 4).

Resolving power *R* is determined in the 'yellow' and the 'blue' regions of the spectrum (Table 1) from the slope of the dispersion curve (Fig. 3) with the prism fully illuminated (*b* = 30 mm).

Table 1: The dispersions and resolving powers of glass prisms determined from the dispersion curve (Fig. 3).

Spectral region: yellow	$\frac{dn}{d\lambda}$/cm^{-1}	$\frac{\lambda}{d\lambda}$
Flint glass	691	2073
Crown glass	377	1131

Spectral region: blue	$\frac{dn}{d\lambda}$/cm^{-1}	$\frac{\lambda}{d\lambda}$
Flint glass	2365	7095
Crown glass	1126	3378

Example:
A prism with the resolving power

$$R = \frac{\lambda}{d\lambda} = 1000$$

is still able to separate the two sodium-D lines.