Wavelengths and frequencies with a Quincke tube

Related topics

Transverse and longitudinal waves, wavelength, amplitude, frequency, phase shift, interference, velocity of sound in air, loudness, Weber-Fechner law.

Principle and task

If a sound wave of a particular frequency is divided into two coherent components (like, for example, light waves in an interferometer experiment), and if the path of one of the component waves is altered, it is possible to calculate the wavelength of the sound wave and its frequency from the interference phenomena recorded with a microphone.

Equipment

Interference tube, Quincke type	03482.00	1
Sound head	03524.00	1
Measuring microphone	03542.00	1
Power frequency generator 1 MHz	13650.93	1
Digital multimeter	07134.00	1
Vernier caliper	03010.00	1
Connecting cord, 1500 mm, blue	07364.04	2
Adapter, BNC-socket/4 mm plug pair	07542.27	1
Support base -PASS-	02005.55	1
Support rod -PASS-, square, l 630 mm	02027.55	2
Right angle clamp -PASS-	02040.55	4

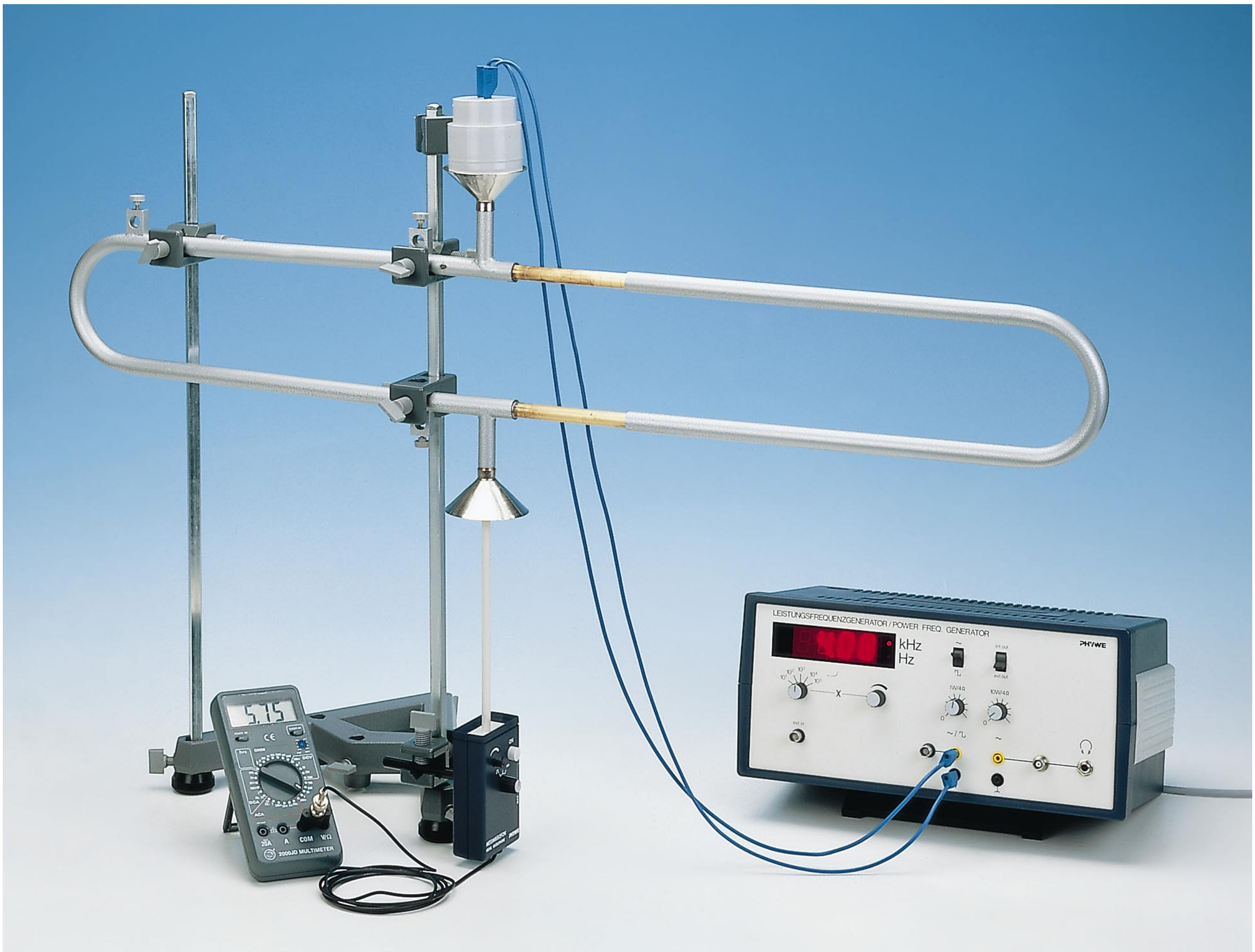
Problems

1. Record of the extension of a Quincke tube for given frequencies in the range 2000 Hz to 6000 Hz.
2. Calculation of the frequencies from the wavelengths determined, comparison with the given frequencies.

Set-up and procedure

1. The experiment is set up as in Fig. 1.
2. As many intensity minima as possible should be found for the frequencies 2000 Hz, 2200 Hz, 2400 Hz, and so on, up to 6000 Hz, which are set on the power frequency generator. The realtive extensions of the tube are measured with the vernier.

Fig. 1: Experimental set-up: Wavelengths and frequencies with a Quincke tube.



Theory and evaluation

If two harmonic linear waves have the same frequency and the same or opposite direction of propagation, the superimposition of the waves is obtained from:

$$A_r(x,\,t) = A_1(x,\,t) + A_2(x,\,t)$$

with

$$A_1(x,\,t) = A_1\, e^{i\,(k_1 x - \omega t)}$$

and

$$A_2(x,\,t) = A_2\, e^{i\,(k_2 x - \omega t - \Delta\varphi)},$$

where

$$|k_1| = |k_2| = k = \frac{2\pi}{\lambda}$$

are the values of the wave vectors in the x-direction or the opposite direction. The angular frequency is

$$\omega = \frac{2\pi}{T} = 2\,\pi f.$$

The resultant wave

$$A_r(x,\,t) = \left(A_1\, e^{i\,k_1 x} + A_2\, e^{i\,(k_2 x - \Delta\varphi)}\right) \cdot e^{i\omega t}$$

is likewise harmonic and has an angluar frequency ω.

If k_1 and k_2 are rectified, a progressive wave is always obtained:

$$A_r(x,\,t) = A_r\, e^{\,i\,(kx - \omega t)}$$

for $k_1 = k_2 = k$, with the amplitude

$$A_r = A_1 + A_2\, e^{-i\Delta\varphi}.$$

The amplitude of the resultant wave is a function of the phase difference $\Delta\varphi$.

If k_1 are opposed (Quincke tube), the resultant wave is a superimposition of a standing wave and a progressive wave:

$$A_r(x,\,t) = \left(A_1\, e^{i\,kx} + A_2\, e^{-\,i\,(kx - \Delta\varphi)}\right) \cdot e^{-\,i\omega t}$$

$$= \left(A_1\, e^{i\,(kx + \frac{\Delta\varphi}{2})} + A_2\, e^{-\,i\,(kx + \frac{\Delta\varphi}{2})}\right).$$

$$e^{-\,i\,(\omega t + \frac{\Delta\varphi}{2})}$$

$$= \left[(A_1 - A_2)\, e^{\,i\,(kx + \frac{\Delta\varphi}{2})} + A_2.\right.$$

$$\left.\left(e^{\,i(kx + \frac{\Delta\varphi}{2})} + e^{\,-i\,(kx + \frac{\Delta\varphi}{2})}\right)\right].$$

$$e^{-\,i\,(\omega t\, \frac{\Delta\varphi}{2})}$$

$$= A_{r1}\, e^{\,i\,(kx - \omega t)} + A_{r2} \cos\,(kx + \tfrac{\Delta\varphi}{2}).$$

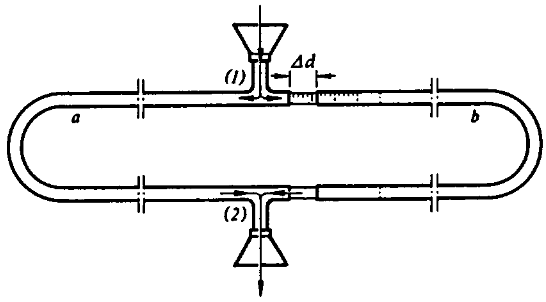
$$e^{-\,i\,(\omega t + \frac{\Delta\varphi}{2})}$$

for $k_1 = -k_2 = k$.

The amplitude of the progressive wave is

$$A_{r1} = A_1 - A_2$$

Fig. 2: Geometriy of the Quincke tube.



and that of the standing wave

$$A_{r2} = 2\,A_2.$$

It is assumed, without limitation, that $A_2 \leq A_1$.

In a Quincke tube, a sound wave is subdivided by the branching of the tube into two coherent components, which, after appropriate deflection, travel towards each other and are superimposed on each other (Fig. 2).
If the straigt sections of the tube a and b are the same size, then, because of the pressure drop along their path, the intensities of the sound waves impinging on the measuring point (2) will be equal ($A_1 = A_2$). In this case is $A_{r1} = 0$, so that only a standing wave exists in the region of the measuring point.

As the difference d in the lengths of the straight sections is increased, the intensity difference at the measuring point also becomes greater ($A_2 < A_1$). In this case the amplitude A_{r2} of the standing wave decreases, and that of the progressive wave increases.

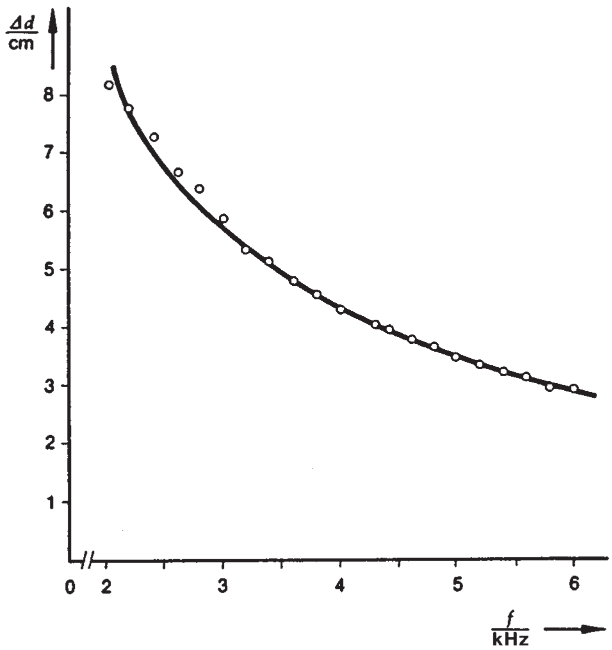
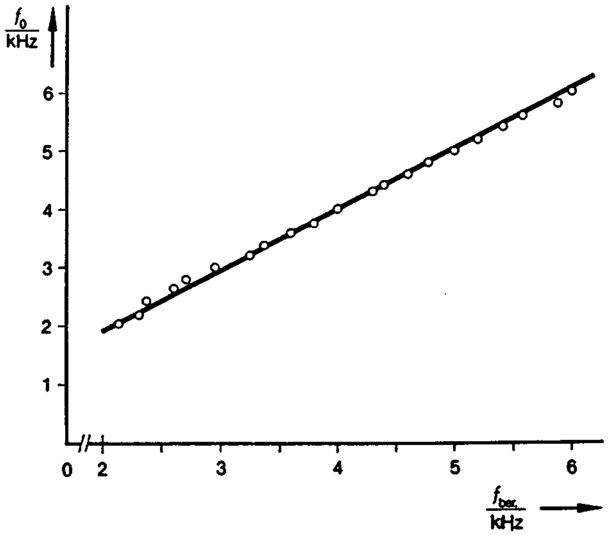


Fig. 3: Resonance wavelength dependent on the displacement Δd.

Fig. 4: Comparison of calculated frequency f_{cal} and measured frequency f_o.



If the measuring point is made $x = 0$, the sound pressure is

$$p = A_{r1} \cos \omega t + A_{r2} \cos \frac{\Delta\varphi}{2} (\cos \omega t + \frac{\Delta\varphi}{2}).$$

For small values of A_{r1}, the amplitude of the sound pressure is determined mainly by $A_{r2} \cos \frac{\Delta\varphi}{2}$. The sound pressure is always at its minimum, when

$$\frac{\Delta\varphi}{2} = \frac{2n+1}{2} \cdot \pi; \qquad n = 0, \pm 1, \pm 2, \ldots$$

The pressure minima are consequently expressed by:

$$d_n = \frac{2n+1}{4} \lambda; \qquad n = 0, \pm 1, \pm 2, \ldots$$

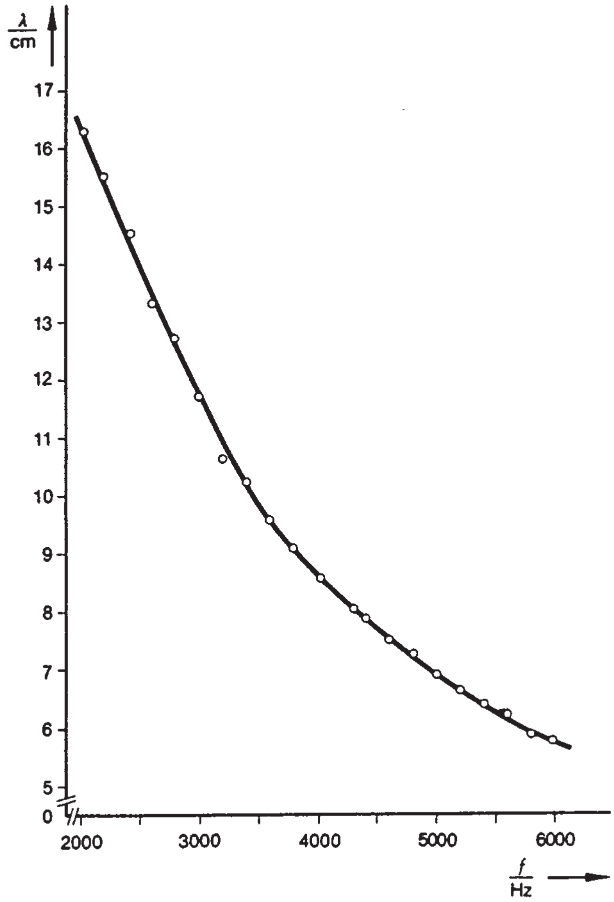
The gap between two minima corresponds exactly to half a wavelength:

$$\Delta d = d_{n+1} - d_n = \frac{\lambda}{2}.$$

When b is extended by a distance Δd, the path is increased by $2\Delta d$. Since $2\Delta d$ corresponds exactly to the wavelength, we obtain

$$\lambda = 2\Delta d \qquad \text{(Fig. 3).}$$

Fig. 5: Connection of wavelength λ and frequency f.



The frequency is obtained from the equation

$$c = f \cdot \lambda \rightarrow f = \frac{c}{\lambda} \text{ (Fig. 4, 5).}$$

The propagation velocity c of the sound waves in air is

$$c = 331.3 \text{ ms}^{-1}$$

at 0 °C, being dependent on temperature and air pressure. c is determined at room temperature from

$$c_{\text{room temp.}} = 331.3 \sqrt{1 + 0.004\,t} \text{ at } 1013 \text{ hPa},$$

with t measured in °C.