

Related topics

Propagation of sound waves, superimposition of sound waves, Doppler shift of frequency, beat frequency.

Principle and task

If a source of sound is in motion relative to its medium of propagation, the frequency of the waves that it emits is displaced due to the Doppler effect. With the superimposition of a moving and a stationary source of sound which are emitting waves of the same frequency, beats occur whose frequency is equal to the Doppler shift of the frequency of the moving source of sound. The Doppler shift of the frequency is measured and compared with the theoretical value for different velocities of the sound emitting source.

Equipment

Measuring microphone	03542.00	1
Sound head	03524.00	2
Power frequency generator 1 MHz	13650.93	1
tY recorder, 1 channel	11414.95	1
Car, motor driven	11061.00	1
Attachment for car	11061.02	1
Round cell, 1.5 V	07922.01	2
Meter scale, demo, l = 1000 mm	03001.00	1
Stopwatch, digital, 1/100 sec.	03071.01	1
Barrel base -PASS-	02006.55	2
Stand tube	02060.00	1
Connecting cord, 750 mm, yellow	07362.02	2
Connecting cord, 1500 mm, blue	07364.04	2

Problems

1. The Doppler shift of the frequency of the moving source of sound is to be determined by measuring the beat frequency.
2. The measured values are to be compared with the calculated ones for velocities in the range between 0.06 and 0.16 m/sec.

Set-up and procedure

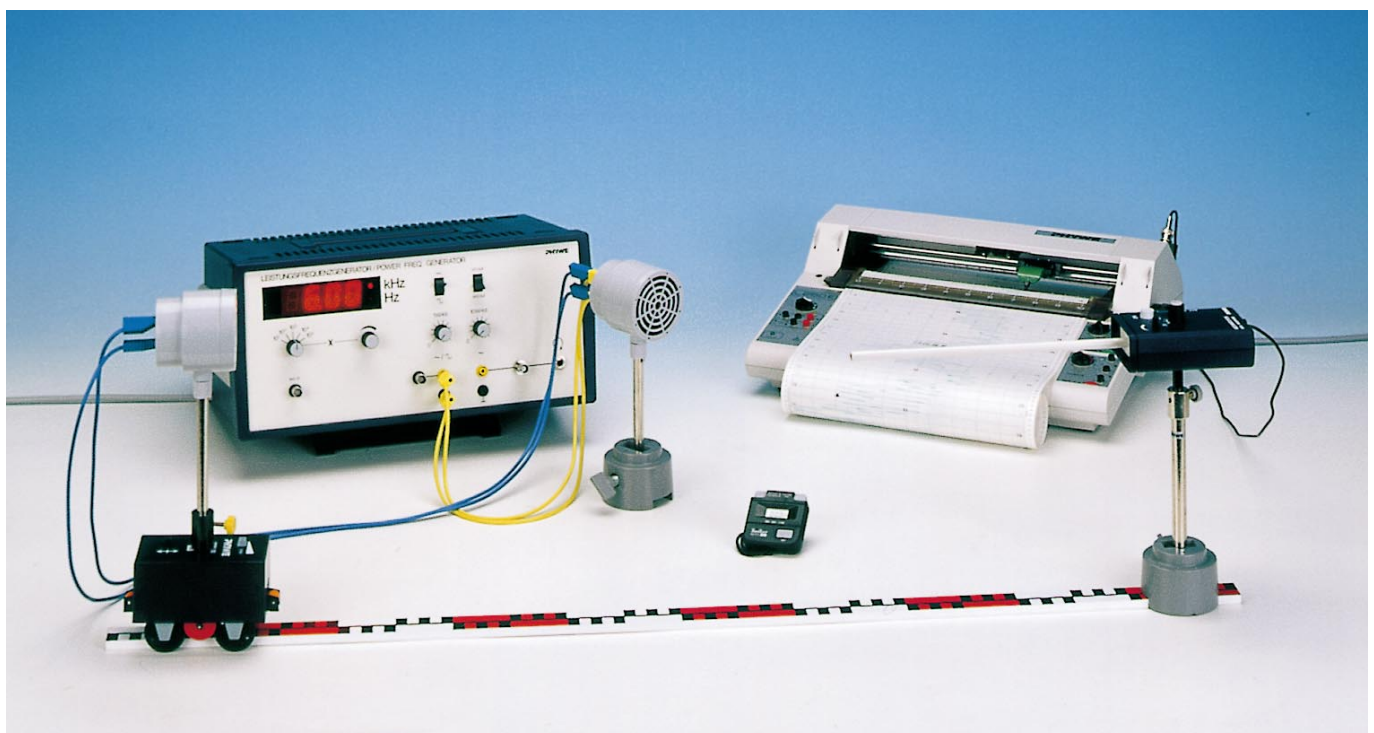
The equipment is set up as shown in Fig. 1. A sound head is fitted to a barrel base. The second sound head is mounted with the adaptor on a motor driven car which moves along the meter scale.

The microphone is fitted to a barrel base at the same height as the sound heads.

The sound heads are connected to output (4 V, 4 Ohm) of the frequency generator. The microphone is connected to the input of the tY pen recorder.

Other than the carriage, no moving object is to be in the vicinity of the sound probe (microphone). A sound frequency in the range of 5 to 10 kHz is used during the experiment. The velocity of the moving sound head should not exceed 0.16 m/s in order to facilitate the recording of the beat frequency. The velocity of the moving sound head is established in a preliminary test by measuring the time it takes for the carriage to travel a distance with a stop watch.

Fig. 1: Experimental set-up for measuring the Doppler shift of the frequency.



After the carriage carrying the sound head has returned to its starting point, the carriage is then started with the velocity unchanged. The pen recorder is also started simultaneously and it records the beat frequency.

The experiment is repeated with the sound head at various velocities, whereby the carriage is moved both towards and away from the receiver.

Theory and evaluation

If a sound source Q with frequency f_Q moves with velocity V_Q towards an observer B who is stationary relative to the medium of propagation, he hears the frequency

$$f_B = \frac{f_Q}{1 - \frac{V_Q}{C}} \quad (1)$$

C is the velocity of sound in the propagation medium. If the source moves away from the observer at a velocity of $-V_Q$, the frequency is then

$$f_B = \frac{f_Q}{1 + \frac{V_Q}{C}} \quad (2)$$

For $V_Q \ll C$ the equations (1) and (2) can be approximated by

$$f_B = \frac{f_Q}{1 \pm \frac{V_Q}{C}} \approx f_Q \left(1 \pm \frac{V_Q}{C}\right)$$

and therefore the frequency shift

$$\Delta f = f_B - f_Q \approx \pm \frac{V_Q}{C} f_Q \quad (3)$$

The dependence on time of the sound pressure generated by the radiated waves of a stationary sound source 1 and a moving sound source 2 which are assumed to be of the same amplitude, can be described by the following equations for a fixed point in space:

$$p_1 = p_0 \sin 2\pi f_Q t$$

and

$$p_2 = p_0 \sin 2\pi f_B t = p_0 \sin 2\pi (f_Q + \Delta f) t$$

If both waves have the same direction of propagation, the variation of pressure with time follows the equation below for superimposition at the observed point of the sound field according to the addition theorem for sine functions

$$p = p_1 + p_2 = 2p_0 \cos 2\pi \Delta f t \cdot \sin 2\pi \cdot \frac{2f_Q + \Delta f}{2} t$$

Whereas the sine-term describes an oscillation with a frequency only slightly different from f_Q , the term

$$2p_0 \cos 2\pi \Delta f t$$

which only changes slowly can be regarded as an amplitude of this oscillation. It varies with beat frequency Δf between the values 0 and $2p_0$. If the superimposed waves have the different pressure amplitudes p_{01} and p_{02} , the total amplitude changes with the beat frequency between $|p_{01} - p_{02}|$ and $p_{01} + p_{02}$.

The frequency shift caused by the Doppler effect

$$\Delta f = \frac{V_Q}{C} f_Q \quad (4)$$

of the wave radiated by the moving sound head can therefore be determined as the beat frequency.

Two sound heads connected in parallel to an audio-frequency generator serve as the sound sources in the experiment; one of the sound heads is mounted on a carriage which can be uniformly moved. The sound is measured with a microphone. The measurement voltage, which is proportional to the pressure, is amplified and, after rectification of the high frequency part, is passed to a tY recorder that registers the variation of the beats.

The test series was carried out with a sound frequency of

$$f_0 = 10 \text{ kHz}$$

Using a measurement where the sound head is moving away from the microphone as an example the following is found:

$$V_Q = 0.098 \text{ m/s}$$

With the velocity of sound

$$C = 345 \text{ m/s}$$

the expected frequency shift according to equation (4) would be

$$\Delta f = 2.84 \text{ Hz}$$

Fig. 2 shows the variation of the beat frequency recorded on the tY recorder. The deviations of amplitude during the measurement can be attributed to the fact that the sound pressure

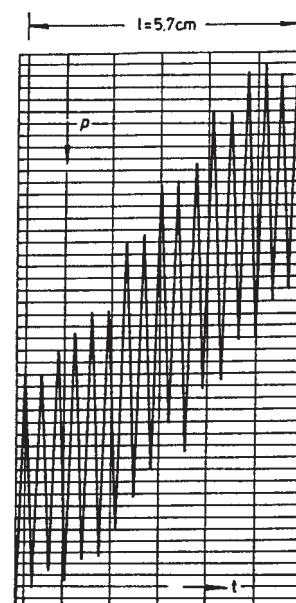
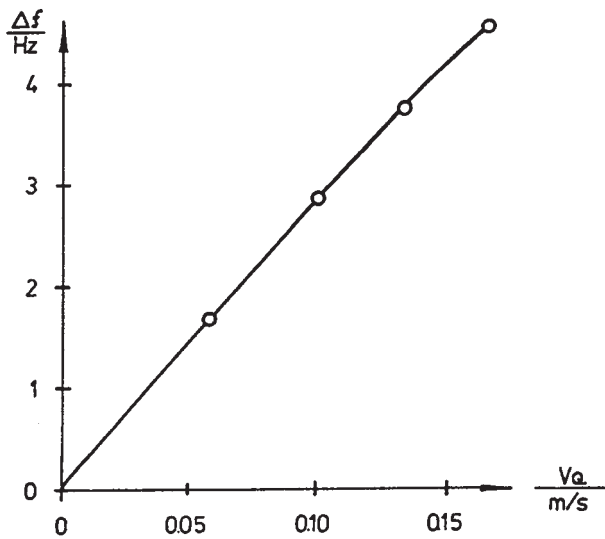


Fig. 2: Recording of the beat frequency.

Fig. 3: Beat frequency as a function of velocity V_Q .



amplitude of waves emitted by both sound heads alters at the location of the microphone when one sound head is moved. This is because the amplitude of the moving sound head decreases.

The diagram produced on the tY recorder shows an amplitude of $N = 17$ for a paper length of $l = 5.7$ cm. At a paper transport speed of 60 cm/min, this corresponds to 16 oscillations in a time of $t = 5.7$ s, and thus a beat frequency of

$$\Delta f = 2.8 \text{ Hz}$$

Tab. 1 lists the beat frequencies Δf which were measured by moving the sound head away from the microphone at various velocities. The Doppler shift was calculated using equation (4). The beat frequencies measured are entered in Fig. 3 above velocity V_Q . The full line shows the theoretical curve which would be expected according to equation (4).

$\frac{-V_Q}{\text{m/s}}$	0.065	0.098	0.13	0.16
$\frac{\Delta f \text{ meas.}}{\text{Hz}}$	1.82	2.80	3.73	4.53
$\frac{\Delta f \text{ cal.}}{\text{Hz}}$	1.88	2.84	3.77	4.64

Tab. 1: Measured and calculated beat frequencies

As the results of the experiment show, the measurement of the beat frequency permits an extremely accurate definition of the frequency shift brought about by the Doppler effect. The relationship described by equation (4) between the velocity of a sound source and the frequency shift is confirmed by the amount of shift.

However, the measuring procedure does not permit conclusions to be drawn from the frequency shift, as to whether the sound source is approaching or moving away from the receiver.

The approximation applied for $V_Q \ll C$ during the derivation of the frequency shift Δf is acceptable, because for the typical values in the example

$$f_B = 10 \text{ kHz} \quad V_Q = 0.1 \text{ m/s} \quad C \approx 345 \text{ m/s}$$

the two frequencies

$$\frac{f_Q}{1 \mp \frac{V_Q}{C}} \quad \text{and} \quad f_Q \left(1 \pm \frac{V_Q}{C}\right)$$

only differ by 0.001 Hz.

Remarks

The measurement process described in the experiment can also be used to determine the Doppler shift of electro-magnetic waves.

It permits the measurement of the velocity of moving bodies without physical contact. Furthermore, the body itself does not need to emit waves. The waves reflected from it can be superimposed with the incoming primary waves.

The reflected wave exhibits a Doppler shift at a velocity V of the body towards or away from the direction of propagation. The magnitude of this Doppler shift is

$$2 \frac{V}{C} f,$$

whereby f is the frequency of the primary wave and c its propagation velocity. From the appropriate beat frequency corresponding to this frequency, velocity V of the body can be established.