Surface of rotating liquids

LEP
1.4.02

Related topics
Angular velocity, centrifugal force, rotary motion, paraboloid of rotation, equilibrium.

Principle and task
A vessel containing liquid is rotated about an axis. The liquid surface forms a paraboloid of rotation, the parameters of which will be determined as a function of the angular velocity.

Equipment

Rotating liquid cell	02536.01	1
Bearing unit	02845.00	1
Driving belt	03981.00	1
Motor, with gearing, 12 VDC	11610.00	1
Power supply 0-12V DC/6 V, 12 V AC	13505.93	1
Light barrier with Counter	11207.08	1
Power supply 5 V DC/0.3 A	11076.93	1
Bench clamp, -PASS-	02010.00	2
Barrel base -PASS-	02006.55	1
Methylene blue sol., alkal. 250 ml	31568.25	1

Problems
On the rotating liquid surface, the following are determined:

1. the shape,
2. the location of the lowest point as a function of the angular velocity,
3. the curvature.

Set-up and procedure
The experimental set up is arranged as shown in Fig. 1. Water, to which a little methylene blue has been added, is put into the cell. The height of the liquid surface is selected so that it corresponds with the horizontal line on the “Plexiglas” plate, on which are printed 3 parabolas.

To determine the lowest point and the curvature of the liquid surface, the foil with crossed coordinate axes is pushed, together with the Plexiglas plate, into the guide of the cell.

To ensure constant speed, it is important that the drive belt between the motor and bearing unit is taut and that the cell is screwed tightly to the bearing unit.

Fig. 1: Experimental set up for determining the parameters of a rotating liquid surface.

Since the cell is closed at the top, except for a small filling aperture, higher speeds can also be selected.

For the measurement of angular velocity Ω a screen made of stiff card board of 1 cm width is pasted to the bottom or one edge of the cell. On rotation, the screen interrupts the light path of the fork type light barrier, which is operatrd in mode ⌐⌐. The counter starts and stops only if the cell completes a full rotation and the scren is once again led into the light path. The angular velocity Ω is calculated from the rotational time T

$$\Omega = 2 \pi T$$

Theory and evaluation

The surface of a liquid sets itself so that the sum of the external forces acting on the particles in the surface, is. perpendicular to the surface.

Two external forces act on a particle of mass m at point $\vec{r}$ the gravitational force $\vec{f_1}$

$$\vec{f_1} = m\vec{g}\,; \quad \vec{g} = \text{acceleration due}$$

to gravity

and the centrifugal force $\vec{f_2}$

$$\vec{f_2} = m\vec{\omega} \times (\vec{r} \times \vec{\omega})$$

where $\vec{\omega}$ denotes the angular velocity.

(Figure 2, in the rotating reference system.)

From Fig. 2, one obtains

$$\tan \alpha = \frac{dy}{dx} = \frac{\omega^2 x}{g}$$

and from this

$$y = \frac{1}{2}\,\frac{\omega^2 x^2}{g} + c \quad \text{(parabola).}$$

If the x-axis of Fig. 2 is located in the surface of the liquid at $\omega = 0$ and the y-axis is in the axis of rotation, then because of the conservation of mass and the assumed incompressibility of the liquid, one obtains:

$$\int_o^a y\,dx = 0,$$

if $2a$ is the width of the cell, and from this the location of the lowest point of the rotating liquid,

$$c = -\frac{1}{6}\,\frac{\omega^2 a^2}{g}\,. \tag{1}$$

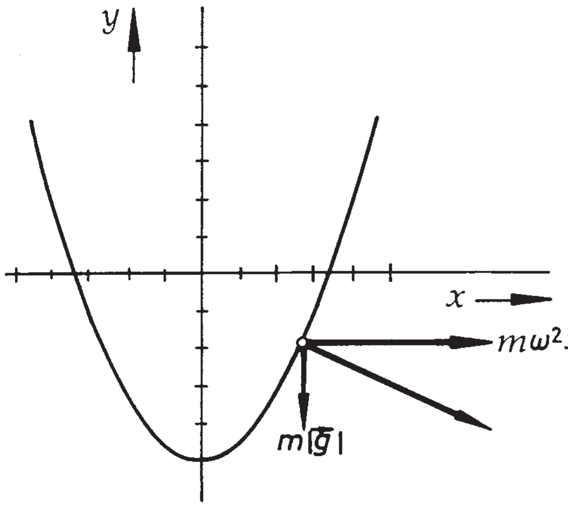
From the regression line to the measured values of Fig. 3, with the exponential statement

$$Y = A \cdot X^B$$

the exponent is obtained

$$B = 1.99 \qquad \text{(see (1))}$$

Fig. 2: Equilibrium of forces on a particle in the surface of a rotating liquid.



From the slope of the curve in Fig. 4, one obtains the

proportionality factor

$$\frac{a^2}{6g} = 8.08 \cdot 10^{-5}\ \text{ms.}$$

From the general equation

$$y = ax^2 - 8.08 \cdot 10^{-5}\,\omega^2 \tag{2}$$

and from the fact that all parabolas for any value of ω pass through the point

$$(x = 0.0398\ \text{m},\ y = 0),$$

it follows that

$$\alpha \sim \omega^2.$$

From this, and from equation (2), one obtains, for $y = 0$

$$\alpha = \frac{8.08 \cdot 10^{-5}}{3.98^2 \cdot 10^{-4}} \cdot \omega^2 = 0.0510\,\omega^2 \tag{2}$$

and from this the acceleration due to gravity is

$$g = 9.805\,\frac{\text{m}}{\text{s}^2}\,.$$

Fig. 3: Location of the lowest point $|c|$ of the liquid as a function of the angular velocity.

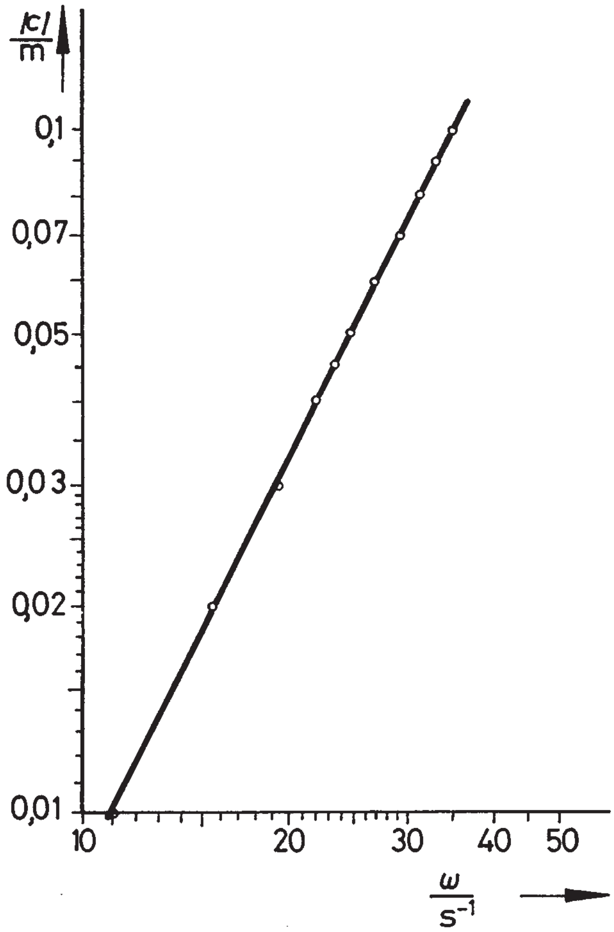


Fig. 4: Location of the lowest point $|c|$ of the liquid as a function of the square of the angular velocity.