Moment of inertia and torsional vibrations

Related topics
Rigid body, moment of inertia, axis of rotation, torsional vibration, spring constant, angular restoring moment, moment of inertia of a sphere, moment of inertia of a disc, moment of inertia of a cylinder, moment of inertia of a long bar, moment of inertia of 2 point masses.

Principle and task
Various bodies perform torsional vibrations about axes through their centres of gravity. The vibration period is measured and the moment of inertia determined from this.

Equipment

Rotation axle	02415.01	1
Sphere	02415.02	1
Disk	02415.03	1
Hollow cylinder	02415.04	1
Solid cylinder	02415.05	1
Rod with movable masses	02415.06	1
Spring balance 2.5 N	03060.02	1
Light barrier with Counter	11207.08	1
Power supply 5 V DC/0.3 A	11076.93	1
Tripod base -PASS-	02002.55	1
Barrel base -PASS-	02006.55	1

Problems
The following will be determined:

1. The angular restoring moment of the spiral spring.
2. The moment of inertia
   a) of a disc, two cylinder, a sphere and a bar,
   b) of two point masses, as a function of the perpendicular distance to the axis of rotation. The centre of gravity lies in the axis of rotation.

Set-up and procedure
The experimental set-up is arranged as shown in Fig. 1. To determine the angular restoring moment, the bar is clamped in the torsion spindle and the two masses fixed symmetrically at a defined distance from the axis of rotation. With the spring balance, the bar is rotated by 180° about the axis in each case and the force is measured. During this measurement, the spring balance is held at right angles to the lever arm.

To measure the vibration period of the various bodies, a mask (width ≤ 3 mm) is stuck on. The light barrier is pushed over the mask while the body is at rest. Switch the light barrier to ⊓⊔-mode. Now the body is deflected through about 180°.

Fig. 1: Experimental set-up for determining moments of inertia of various bodies.

In each case, the time of a half-cycle is measured, several measurements being averaged. For reasons of safety and stability, it is recommended that the spring should not be twistedt beyond ± 720°.

Theory and evaluation

The relationship between the angular momentum $\vec{L}$ of a rigid body in a stationary coordinate system with its origin at the centre of gravity, and the moment $\vec{T}$ acting on it, is

$$\vec{T} = \frac{d}{dt}\vec{L}. \qquad (1)$$

The angular momentum is expressed by the angular velocity $\vec{\omega}$ and the inertia tensor $\hat{I}$:

$$\vec{L} = \hat{I} \odot \vec{\omega},$$

that is, the reduction of the tensor with the vector.

In the present case, $\vec{\omega}$ has the direction of a principal inertia axis (z-axis), so that $\vec{L}$ has only one component:

$$L_Z = I_Z \cdot \omega,$$

where I_Z is the z-component of the principal inertia tensor of the body. For this case, equation (1) reads:

$$T_Z = I_Z \frac{d\omega}{dt} = I_Z \frac{d^2\phi}{dt^2}.$$

where ϕ is the angle of rotation.

The moment of a spiral spring, in the Hooke's law range, is:

$$T_Z = -D \cdot \phi \qquad (2)$$

where D is the angular restoring constant.

From the regression line to the measured values of Fig. 2 with the linear statement

$$Y = A + B \cdot X$$

the slope

$$B = 0.0265 \text{ Nm/rad.} \qquad \text{(see (2))}$$

is obtained.

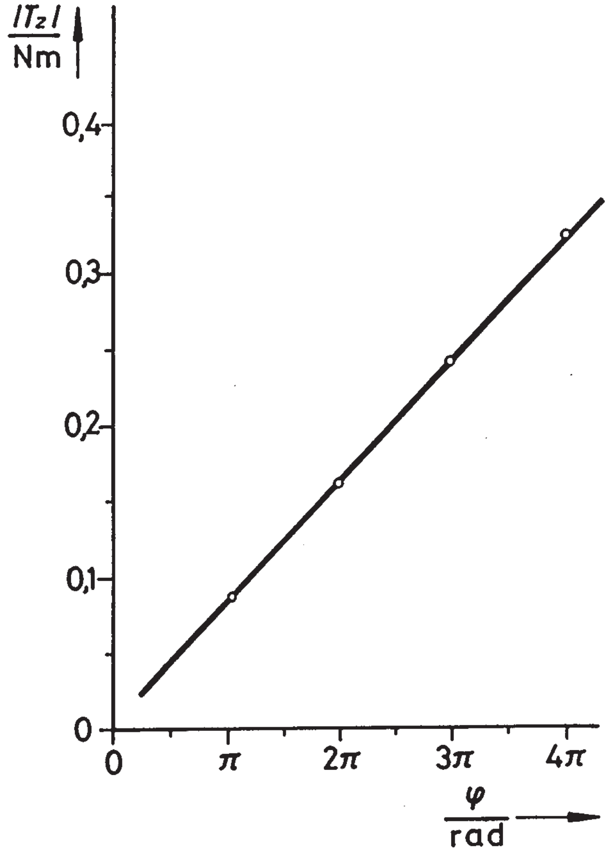
The angular restoring factor is

$$D = 0.0265 \text{ Nm/rad.}$$

The equation of motion reads:

$$\frac{d^2\phi}{dt^2} + \frac{D}{I_Z}\phi = 0.$$

Fig. 2: Moment of a spiral spring as a function of angle of rotation.



The period and frequency of this vibration are respectively

$$T = 2\pi \sqrt{I_Z/D}$$

$$f = \frac{1}{2}\pi \sqrt{D/I_Z}. \qquad (3)$$

If ρ (x, y, z) is the density distribution of the body, the moment of inertia I_Z is obtained as

$$I_Z = \int\int\int (x^2 y^2)\, \rho\, (x, y, z)\, dx\, dy\, dz.$$

The origin of coordinates is located at the centre of gravity.

For a sphere of radius

$$r = 0.070 \text{ m}$$

and of mass

$$m = 0.761 \text{ kg},$$

$$I_Z = \frac{2}{5} m r^2 = 1.49 \cdot 10^{-3} \text{ kgm}^2.$$

The measured value is

$$I_Z = 1.48 \cdot 10^{-3} \text{ kgm}^2.$$

For a circular disc of radius

$$r = 0.108 \text{ m}$$

and of mass

$$m = 0.284 \text{ kg},$$

$$I_Z = \frac{m}{2} r^2 = 1.66 \cdot 10^{-3} \text{ kgm}^2.$$

The measured value is

$$I_Z = 1.68 \cdot 10^{-3} \text{ kgm}^2.$$

For a solid cylinder of radius

$$r = 0.0495 \text{ m}$$

and of mass

$$m = 0.367 \text{ kg},$$

$$I_Z = \frac{1}{2} m r^2 = 0.45 \cdot 10^{-3} \text{ kgm}^2.$$

The measured value is

$$I_Z = 0.44 \cdot 10^{-3} \text{ kgm}^2.$$

For a hollow cylinder with the two radii

$$r_i = 0.046 \text{ m}$$

$$r_a = 0.050 \text{ m}$$

and of mass

$$m = 0.372 \text{ kg},$$

$$I_Z = \frac{1}{2} m (r_i^2 + r_a^2) = 0.86 \cdot 10^{-3} \text{ kgm}^2.$$

The measured value is

$$I_Z = 0.81 \cdot 10^{-3} \text{ kgm}^2.$$

For a thin rod of length

$$l = 0.6 \text{ m}$$

and of mass

$$m = 0.133 \text{ kg}$$

$$I_Z = \frac{m}{12} l^2 = 3.99 \cdot 10^{-3} \text{ kgm}^2.$$

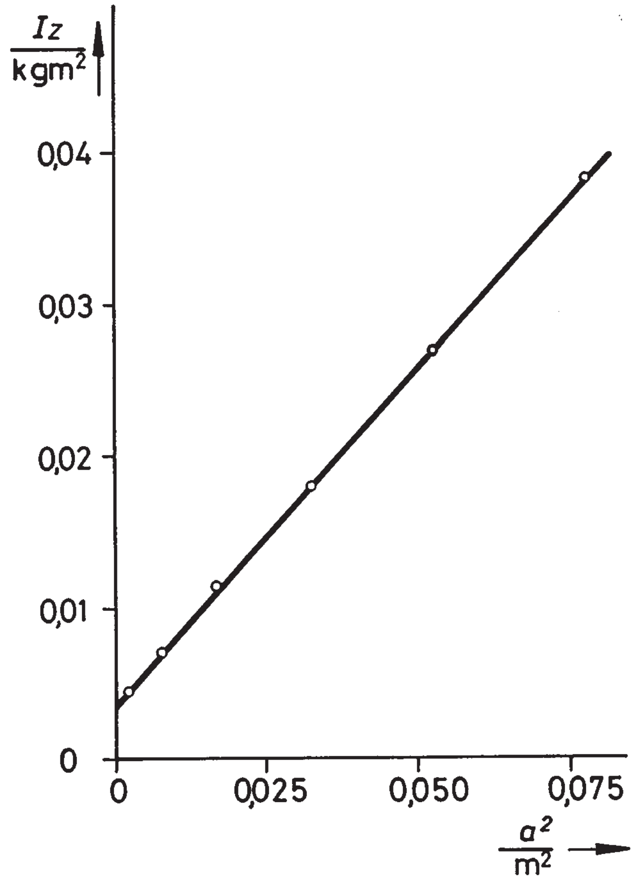
The measured value is

$$I_Z = 3.98 \cdot 10^{-3} \text{ kgm}^2.$$

For a point mass of mass m, at a distance a from the axis of rotation, one obtains:

$$I_Z = ma^2 \qquad (4)$$

Fig. 3: Moment of inertia of two equal masses, of 0.214 kg each, as a function of the distance between them.



From the regression line to the measured values of Fig. 3, with the statement

$$Y = A + BX^2 \qquad \text{(see (4))}$$

the slope

$$B = 0.441\text{kg}$$

and the axis intercept

$$A = 0.0043 \text{ kg/m}^2$$

are obtained.