

Related topics

Rigid body, moment of inertia, centre of gravity, axis of rotation, torsional vibration, spring constant, angular restoring force.

Principle and Problems

- The moment of inertia of a circular disk related to the central axis perpendicular to the plane of the disk depends on the mass and the radius of the latter.
- The moment of inertia of a hollow cylinder depends on the sum of the squares of its exterior and interior radii.
The moment of inertia of a hollow cylinder is the result of the difference of the moments of inertia of two massive cylinders of the same density, the dimensions of which correspond to the exterior and to the interior dimensions of the hollow cylinder.
- The moment of inertia of a sphere related to an axis passing through the centre depends on the mass and the radius of the sphere. It is smaller than the moment of inertia of a cylinder with the same radius and mass as the sphere.
- The moment of inertia of a rod with masses which can be shifted depend on the position of the masses.
- The moment of inertia of a body related to a rotation axis which does not pass through its centre of gravity can be expressed as the sum of two characteristics moments of inertia:

1. the moment of inertia of the mass concentrated at the centre of gravity,
2. the moment of inertia of the body related to a rotation axis passing through its centre of gravity (characteristic moment of inertia).

Equipment

COBRA-interface 2	12100.93	1
PC COBRA data cable RS 232, 2 m	12100.01	1
Movement sensor with cable	12004.10	1
Adapter, BNC-socket/4 mm plug pair	07542.27	1
Adapter, BNC socket - 4 mm plug	07542.20	1
Softw. COBRA Rotation (Win)	14295.61	1
Basic Softw. f. PHYWE Windows prog.	14099.61	1
Rotation axle	02415.01	1
Sphere	02415.02	1
Disk	02415.03	1
Hollow cylinder	02415.04	1
Solid cylinder	02415.05	1
Rod with movable masses	02415.06	1
Disk, w. diametrical holes	02415.07	1
Portable balance Mod. LS 2000	46002.93	1
Flat cell battery, 9 V	07496.10	1
Silk thread, 200 m	02412.00	1
Weight holder 1 g	02407.00	1
Bench clamp -PASS-	02010.00	1
Tripod base -PASS-	02002.55	1

Fig. 1: Experimental set up: Moments of inertia of different bodies / Steiner's theorem with PC interface.



Stand tube	02060.00	1
Measuring tape, $l = 2$ m	09936.00	1

— The distance in time of the measurement points is much larger than the set $\langle \Delta t / \text{ms} \rangle$ value.

The PHYWE WINDOWS® Basic Software (14099.61) must have been installed once on the used computer for the software to work.

Set-up

According to Fig. 1. The electric connection of the movement recorder is carried out according to Fig. 2 for COBRA. Make sure the thread connecting the rotating axis with the movement recorder is horizontal. The thread is laid over the larger of the two thread grooves of the movement recorder. The thread is wound 7 times around the rotation axis of the rotation-oscillation unit, below the fixation screw.

Procedure

- COBRA: if the COBRA interface is connected to the computer port COM1 "Osc_COM1" is clicked on twice. If the serial port COM2 is used "Osc_COM2" is started.
- The maximum sampling rate $\langle \Delta t / \text{ms} \rangle$ is adjusted according to computer speed. If a 486 (66 MHz) computer is used, 100 ms have proven to be a suitable choice; with faster computers, time may decrease, with slower computers, it may increase. Sampling time is set by shifting the grey slide with the mouse cursor. Modification of $\langle \Delta t / \text{ms} \rangle$ must be repeated every time the programme is started. A too short sampling time is identified through the following effects:
 - No measurement values are recorded.
 - The measurement values are not equidistant on the time axis.

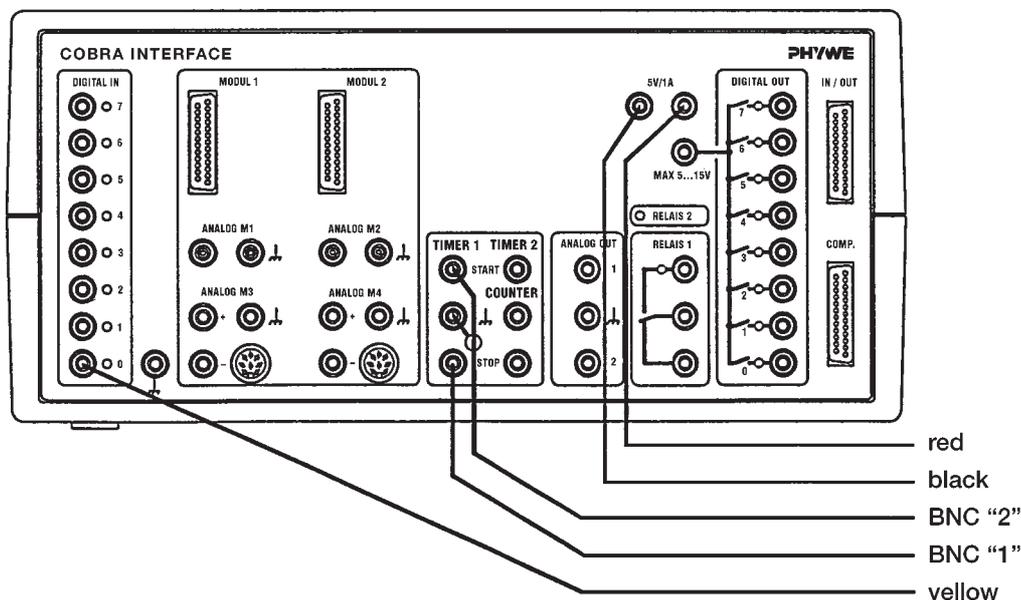
Recording of measurement values

- The diameter of the thread groove of the movement recorder is entered into the input window d_0 (12 mm are set as a default value). d_1 is the diameter of the rotary axis which carries the test bodies and around which the silk thread is wound (20 mm). Each of these entries must be confirmed with the "Enter" key on the keyboard.
- The test body is deviated manually about 180° and released. Press the $\langle \text{Start} \rangle$ button. A new measurement can be initiated any time with the $\langle \text{Reset} \rangle$ button, the number of measurement points "n" is reset to zero.

The following bodies are successively set onto the rotary axis: circular disk without holes, hollow cylinder, massive cylinder, sphere, rod, circular disk with holes. The masses are fixed symmetrically on the rod and shifted successively to different distances of the rotation axis. Beginning at the centre, the circular disk with holes is turned through all holes which are situated on a radius.

- In total, about $n = 250$ measurement values are recorded and then the $\langle \text{Stop} \rangle$ button is pressed.
- Now the newly appearing $\langle \text{Edit} \rangle$ button is pressed. After a few seconds, three new order buttons appear on the screen. Click on the $\langle \text{Edit} \rangle$ button. The vertical lines can be captured and shifted with the mouse button. They are positioned in such a way that all measurement values which are to be evaluated lie between the two lines. The $\langle \text{Edit} \rangle$ button is pressed again, followed by the $\langle \text{Evaluate} \rangle$ button. The computer now needs some time to evaluate all data, the length of which depends on the number of meas-

Fig. 2: Connection of the movement recorder to the COBRA interface.



urement points. The process is indicated by a counter running backwards.

- After the evaluation, a graph similar to Fig. 3 appears on the screen. The duration of the period T and the circular frequency ω (w) of the oscillation are displayed. Press button $\langle D^* \rangle$. If the angular directive force D^* is known (here $D = 2.4$ Ncm), it is entered into the input window for D^* (after entering the number, press "Enter" on the keyboard). If the value "0.0" appears in the input field for the moment of inertia J , the actual numerical value for J is calculated and displayed.

Indication:

If the angular directive force D^* of the used spiral spring is unknown, a body of known moment of inertia J is set on the rotatory axis and the movement is measured as described above.

After that, the $\langle \text{Evaluate} \rangle$ and $\langle D^* \rangle$ buttons are pressed and the known numerical value is entered into the input window for J (do not forget to press "Enter" on the keyboard). If the value "0.0" is displayed in the input window D^* , then D^* is calculated. This value of D^* is entered into the input window for D^* and then 0.0 is entered in the input window for J . The measurement system for the measurement of unknown moments of inertia is now calibrated.

Result

- For small oscillation amplitudes, the angle of rotation and the corresponding returning momentum are proportional; one obtains harmonic circular oscillations with the frequency

$$\omega = \sqrt{\frac{D^*}{J}} \cdot$$

The angular directive force D^* is given through the restoring spring (the spiral spring in our case). The duration of oscillation

$$T = 2\pi \sqrt{\frac{J}{D^*}} \cdot$$

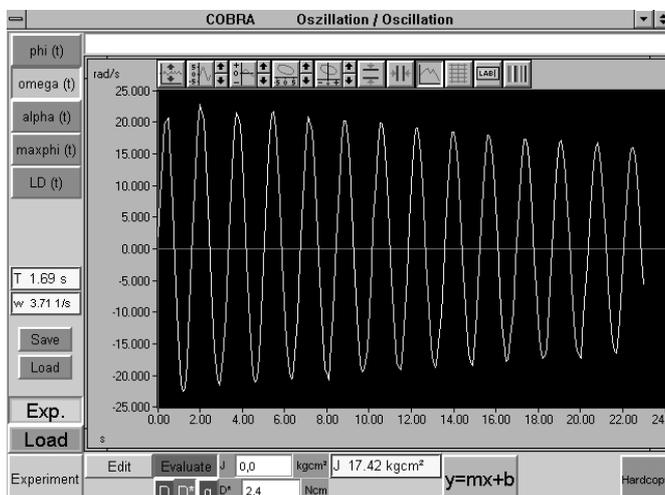


Fig. 3: Typical diagram for the measurement of moments of inertia due to rotational oscillations.

of the circular oscillation is determined through the measurement. The unknown momentum of inertia can be calculated from the values for the angular directive force D^* and the period of oscillation T by means of the following equation;

$$J = \frac{T^2}{4\pi^2} D^*.$$

Theoretical considerations yield the following relations for the moments of inertia J of the used test bodies:

- Circular disk

The moment of inertia J of a circular disk depends on its mass m and its radius r according to

$$J = \frac{1}{2} m \cdot r^2.$$

- Massive and hollow cylinder

The moment of inertia J_v of a massive cylinder depends on its mass m_v and its radius r . The following relation is valid:

$$J_v = \frac{1}{2} m_v \cdot r^2.$$

For a hollow cylinder of comparable mass and exterior radius, the moment of inertia must be larger. It depends both on the mass m_H of the hollow cylinder and on the radii r_i and r_a :

$$J_H = \frac{1}{2} m_H (r_a^2 + r_i^2) \text{ (hollow cylinder)}$$

For a density ρ and a height h , the mass of the hollow cylinder can be expressed through

$$m_H = \rho \cdot h \cdot \pi (r_a^2 + r_i^2).$$

For the moment of inertia J_H one obtains the following equation:

$$J_H = \frac{1}{2} \rho \cdot h \cdot \pi (r_a^2 - r_i^2) (r_a^2 + r_i^2)$$

which yields:

$$J_H = \frac{1}{2} \rho \cdot h \cdot \pi r_a^4 - \frac{1}{2} \rho \cdot h \cdot \pi r_i^4.$$

This equation expresses that the moment of inertia of the hollow cylinder can be expressed as moment of inertia of two massive cylinders of the same density. The mass of the larger massive cylinder of radius r_a is:

$$m_a = \rho \cdot h \cdot \pi \cdot r_a^2$$

and the mass of the smaller massive cylinder of radius r_i is:

$$m_i = \rho \cdot h \cdot \pi \cdot r_i^2.$$

One thus obtains

$$J_H = \frac{1}{2} m_a \cdot r_a^2 - \frac{1}{2} m_i \cdot r_i^2.$$

— Sphere

The moment of inertia J_K of a homogeneous spherical body related to an axis passing through its central point is:

$$J_K = \frac{2}{5} m \cdot r^2.$$

Thus, the moment of inertia of a cylinder with the same radius r and the same mass m is larger than that of a comparable sphere.

Next to the dimensions and masses of the test bodies, table 1 also contains the measured and theoretically calculated moments of inertia.

— Rod with movable masses

The moment of inertia of a rod with masses which can be shifted depends on the distance between the masses and the rotating axis. The measured moments of inertia of the rod without masses and with the masses set at different points are listed in table 2. The table also gives the calculated moments of inertia of the rod without masses:

$$J = \frac{1}{12} M L^2$$

$$= \frac{1}{12} \cdot 0.1715 \cdot 60^2 \text{ kg cm}^2 = 51.45 \text{ kg cm}^2,$$

the moments of inertia of the masses without the rod:

$$2 \cdot m \cdot l^2$$

and finally, the sum of the moments of inertia of the rod and of the masses:

$$\frac{1}{12} M L^2 + 2 \cdot m \cdot l^2 = J_0 + 2 \cdot m \cdot l^2.$$

The moment of inertia measured every time corresponds to the sum of the single moments of inertia.

The following parameters are valid for the measurement examples described here:

total length of the rod L	60 cm
mass M of the rod	0.1715 kg.
Mass of a single weight m	0.2115 kg.

— Steiner's theorem

According to Steiner's theorem, the total moment of inertia J of a body rotated about an arbitrary axis is composed of two parts according to

$$J = J_0 + m \cdot r.$$

J_0 is the moment of inertia related to an axis running parallel to the rotation axis and passing through the centre of gravity (axis of gravity). The expression mr^2 is the moment of inertia of the mass of the body concentrated at the centre of gravity, related to the rotation axis. Table 3 compares the measured moment of inertia J_{measured} with the sum of the single moments of inertia $J_0 + mr^2$ and shows the validity of Steiner's theorem.

Indications

- Signal distortions may occur if the angular velocity is too small. In this case, it is recommended to connect the thread with the smaller thread groove. The figure 6 is now entered into the input window "d₀" If the thread slides in the thread groove, it must be wound once around the thread groove.
- If the oscillating period is very short, signal leaps or distortions may occur. These can be decreased by changing the sampling rate or by making the diagram invisible with <Clr>. Intervals recorded without faults may be cut out later from the measurement signal with <Edit>.
- As the recording of movement is not free of direct contact, a small damping of the measured oscillations occurs. The result is that the determined moments of inertia are a bit too large. In most cases, the effect lies however within the statistical error lines.
- After the autoscale button situated on the left side above the diagram has been clicked on, all measurement data are represented so that they fill the whole screen. However, an unfortunate choice of the subdivisions of the axes may occur. In this case, the corresponding scaling values for the x and y axes may be entered manually by pressing the second or respectively the fourth button over the diagram (cf. fig. 3). After every change of the y axis magnitudes, the autoscale button should be pressed, in order to adapt the scaling factors to the actually represented values.
- The <Evaluate> button provides many evaluation possibilities and requires a certain time, before all results are calculated. For 2000 measurement values, this may take a few minutes.
- The measurement values can be saved by pressing the green <Save> button. They are written as ASCII symbols in columns into a data file, which must have the suffix "AFD" (ASCII File Data). The following measurement values are found in the single columns: t / (s) running time, $\phi(t)$ / (rad) angle, $\omega(t)$ / (rad/s) angular velocity, $\alpha(t)$ / (rad/s²) angular acceleration, $\max\phi(t)$ / (rad) positive peaks of the angle graph, T / (s) oscillation period.
The measurement data can easily be submitted to further processing or printed out under DOS or WINDOWS®.
- The <maxphi (t)> button selects the positive oscillation peaks and plots them beginning on the left side. The logarithmic oscillation decrement is obtained pressing the <LD> button. It must be noted that the first and last three values of "maxphi (t)" are not taken into account for the evaluation through <LD>.
- The green <Load> button allows to load measurement data into the running programme, so that the data are evaluated again and compared to actual data. The double button <Exp.> <Load> is used for this. <Exp.> shows the graph of the actual experiment, <Load> shows the loaded measurement.
- If a new measurement is to be carried out, the <Edit> and <Evaluate> buttons must be switched off to start with, after which the <Experiment> button is pressed. After a few seconds, the set up of the diagram already described for the recording of measurement values appears. The <Reset> button deletes the old measurement data. Once the measurement has been carried out, <Edit> is pressed. In the screen which appears then, the <Edit> button must be pressed to make the measurement values available for

evaluation. If the <Evaluate> button is pressed first, no measurement values appear in the diagram. In this case, the <Evaluate> button is switched off again and <Edit> is pressed.

- It must be taken into account that the <Edit> and the <Evaluate> buttons cancel each other. This means that the button pressed first is active, the button pressed after this turns red, but is not activated. For this reason, it is recommended to switch off the button pressed first before pressing the other one.
- The <Clr> button in the measurement value recording screen deletes the diagram. This saves computing time and the maximum sampling rate can be slightly increased. Pressing this button twice will let the diagram appear again on the screen, the contents are deleted.
- An arbitrary text to comment or explain the experiment can be entered in the comment line on the upper edge of the screen, before a hardcopy printout is made. The hardcopy is printed out on the connected printer when <Hardcopy> is pressed. Before printing out a hardcopy, it is recommended to change the colours of the diagram, in order to save the ink ribbon or the cartridge of the printer. Diagram colours can be modified pressing the colour selection button situated on the right side above the diagram. The following colour combinations are recommended:
Scope: instead of black - white
Data: instead of yellow - black.
- Quitting the programme: if the programme is in the measurement value recording mode, it is sufficient to click twice at the upper left corner of the screen with the left mouse button. If not, the <Experiment> key must be pressed first, so the programme will return to the measurement value recording mode.

Table 1

Body	m / kg	r / cm	J / kgcm^2	J_T / kgcm^2
Circular disk	0.292	11	17.42	17.66
Massive cylinder	0.370	4.9	4.78	4.44
Hollow cylinder	0.390	$r_a/r_i : 5/4.6$	8.15	7.49
Sphere	0.798	7	16.14	15.64

Table 2

l / cm	$J_{\text{measured}} / \text{kgcm}^2$	$I^2 m / \text{kgcm}^2$	$I^2 m / \text{kgcm}^2 + J_0$
–	42.14	–	51.45
5	53.81	10.58	62.03
10	86.28	42.3	93.75
15	144.52	95.18	146.63
20	223.77	169.2	220.65
25	322.75	264.38	315.83

Tabelle 3 (Mass of disk $m = 0.454 \text{ kg}$)

r / cm	$J_{\text{measured}} / \text{kgcm}^2$	$m r^2 / \text{kgcm}^2$	$J_0 + m r^2 / \text{kgcm}^2$
0	50.44	–	–
3	55.25	4.09	54.53
6	68.91	16.34	66.78
9	106.47	36.77	87.21
12	119.00	65.38	115.82