Forced oscillations – Pohl's pendulum

Related topics

Angular frequency, characteristic frequency, resonance frequency, torsion pendulum, torsional vibration, torque, restoring torque, damped/undamped free oscillation, forced oscillation, ratio of attenuation/decrement, damping constant, logarithmic decrement, aperiodic case, creeping.

Principle and task

If an oscillating system is allowed to swing freely it is observed that the decrease of successive maximum amplitudes is highly dependent on the damping. If the oscillating system is stimulated to swing by an external periodic torque, we observe that in the steady state the amplitude is a function of the frequency and the amplitude of the external periodic torque and of the damping. The characteristic frequencies of the free oscillation as well as the resonance curves of the forced oscillation for different damping values are to be determined.

Equipment

Torsion pendulum after pohl	11214.00	1
Power supply, universal	13500.93	1
Bridge rectifier, 30 V AC/1 A DC	06031.10	1
Stopwatch, digital, 1/100 sec.	03071.01	1
Digital multimeter	07134.00	1
Connecting cord, 250 mm, yellow	07360.02	2
Connecting cord, 750 mm, red	07362.01	2
Connecting cord, 750 mm, blue	07362.04	3

Problems

A. Free oscillation

1. To determine the oscillating period and the characteristic frequency of the undamped case.
2. To determine the oscillating periods and the corresponding characteristic frequencies for different damping values. Successive, unidirectional maximum amplitudes are to be plotted as a function of time. The corresponding ratios of attenuation, the damping constants and the logarithmic decrements are to be calculated.
3. To realize the aperiodic case and the creeping.

B. Forced oscillation

1. The resonance curves are to be determined and to be represented graphically using the damping values of A.
2. The resonance frequencies are to be determined and are to be compared with the resonance frequency values found beforehand.
3. The phase shifting between the torsion pendulum and the stimulating external torque is to be observed for a small damping value assuming that in one case the stimulating frequency is far below the resonance frequency and in the other case it is far above it.

Fig. 1a: Experimental set-up for free and forced torsional vibration.

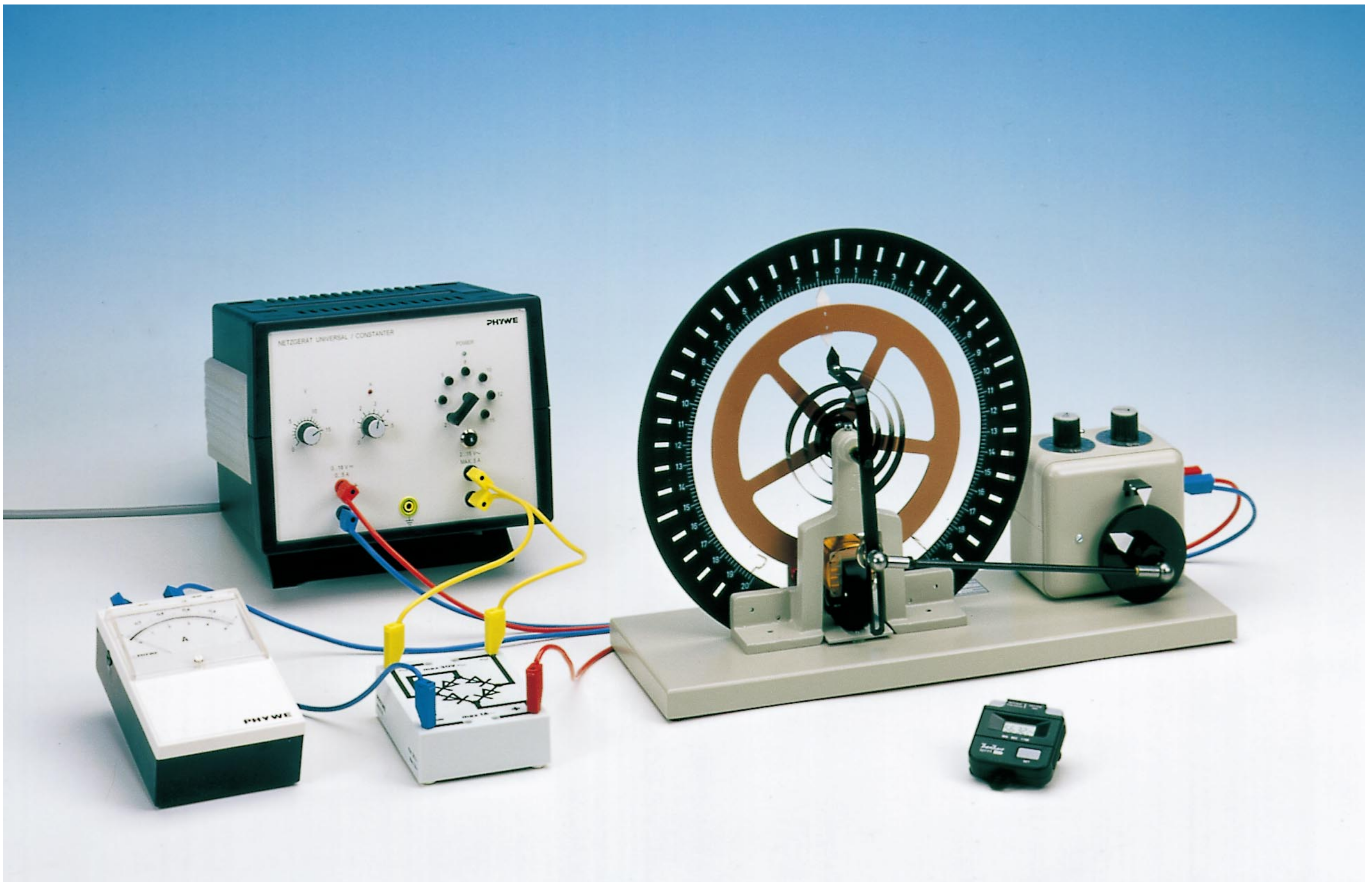
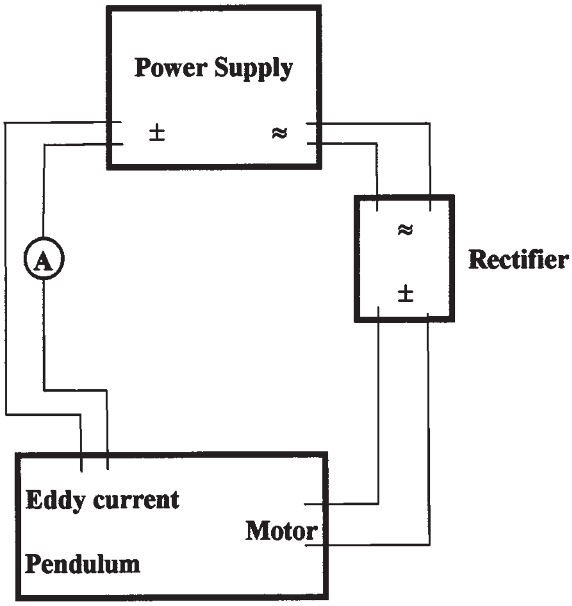


Fig. 1b: Electrical connection of the experiment.



Set-up and procedure

The experiment is set up as shown in Fig. 1a und 1b. The DC output $U_{–}$ of the power supply unit is connected to the two upper sockets of the DC motor. The eddy current brake also needs DC voltage. For this reason a rectifier is inserted between the AC output $U_{–}$ of the power supply unit and the entrance to the eddy current brake. The DC current supplied to the eddy current brake, I_B, is indicated by the ammeter.

To determine the characteristic frequency w_0 of the torsion pendulum without damping ($I_B = 0$), the time for several oscillations is measured repeatedly and the mean value of the period $\overline{T}_0$ calculated. In the same way the characteristic frequencies for the damped oscillations are found using the following current intensities for the eddy current brake:

$I_B \sim 0.25$ A, ($U_{\sim} =$ 4 V)
$I_B \sim 0.40$ A, ($U_{\sim} =$ 6 V)
$I_B \sim 0.55$ A, ($U_{\sim} =$ 8 V)
$I_B \sim 0.9$ A, ($U_{\sim} =$ 12 V)

To determine the damping values for the above mentioned cases the decrease in amplitude is measured by deflecting the pendulum completely to one side while observing the magnitude of successive amplitudes on the other side. Initially it has to be ensured that the pendulum pointer at rest coincides with the zero-position of the scale. This can be achieved by turning the eccentric disc of the motor.

To realize the aperiodic case ($I_B \sim 1.5$ A) and the creeping case ($I_B \sim 2.0$ A) the eddy current brake is briefly connected directly to the DC output Uof the power supply unit.

To stimulate the torsion pendulum, the connecting rod of the motor is fixed to the upper third of the stimulating source. The DC voltage $U_{–}$ of the power supply unit must be set to maximum. The stimulating frequency ω_a of the motor can be found by using a stopwatch and counting the number of turns. The measurement begins with small frequencies. ω_a is increased by means of the motor-potentiometer setting "coarse". In the vincinity of the maximum ω_a is changed in small steps using the potentiometer setting "fine". In each case, readings should only be taken after a stable pendulum amplitude has been established. In the absence of damping or for only very small damping values, ω_a must be chosen in such a way that the pendulum does not exceed its scale range.

Theory and evaluation

A. Undamped and damped free oscillation In case of free and damped torsional vibration torques M_1 (spiral spring) and M_2 (eddy current brake) act on the pendulum. We have

$$M_1 = -D^0 \phi \text{ and } M_2 = -C\phi$$

ϕ = angle of rotation
$\dot{\phi}$ = angular velocity
D^0 = torque per unit angle
C = factor of proportionality depending on the current which supplies the eddy current brake

The resultant torque

$$M = -D^0\phi - C\phi$$

leads us to the following equation of motion:

$$I\ddot{\phi} + C\dot{\phi} + D^0\phi = 0 \quad (1)$$

I = pendulum's moment of inertia
$\ddot{\phi}$ = angular acceleration

Dividing Eq. (1) by I and using the abbreviations

$$\delta = \frac{C}{2I} \text{ and } \omega_0^2 = \frac{D^0}{I}$$

results in

$$\ddot{\phi} + 2\delta\phi + \omega_0^2\phi = 0 \quad (2)$$

δ is called the "damping constant" and

$$\omega_0 = \sqrt{\frac{D^0}{I}}$$

the characteristic frequency of the undamped system.

The solution of the differential equation (2) is

$$\phi(t) = \phi_0 e^{-\delta t} \cos \omega t \quad (3)$$

with

$$\omega = \sqrt{\omega_0^2 - \delta^2} \quad (4)$$

Eq. (3) shows that the amplitude $\phi(t)$ of the damped oscillation has decreased to the e-th part of the initial amplitude ϕ_0 after the time $t = 1/6$ has elapsed. Moreover, from Eq. (3) it follows that the ratio of two successive amplitudes is constant.

$$\frac{\phi_n}{\phi_{n+1}} = K = e^{\delta T} \quad (5)$$

K is called the "damping ratio" and the quantity

$$\Lambda = \ln K = \delta T = \ln \frac{\phi_n}{\phi_{n+1}} \quad (6)$$

is called the "logarithmic decrement". Eq. (4) has a real solution only if

$$\omega_0^2 \geqslant \delta^2.$$

For $\omega_0^2 = \delta^2$, the pendulum returns in a minimum of time to its initial position without oscillating (aperiodic case). For $\omega_0^2 < \delta^2$, the pendulum returns asymptotically to its initial position (creeping).

B. Forced oscillation

If the pendulum is acted on by a periodic torque $M_a = M_0 \cos \omega_a t$ Eq. (2) changes into

$$\ddot{\phi} + 2\delta\dot{\phi} + \omega_0^2\phi = F_0 \cos \omega_a t \quad (7)$$

where $F_0 = \frac{M_0}{I}$

In the steady state, the solution of this differential equation is

$$\phi(t) = \phi_a \cos(\omega_a t - \alpha) \quad (8)$$

where

$$\phi_a = \frac{\phi_0}{\sqrt{\left\{1 - \left[\frac{\omega_a}{\omega_0}\right]^2\right\}^2 + \left[2\frac{\delta}{\omega_0}\frac{\omega_a}{\omega_0}\right]^2}} \quad (9)$$

and $\phi_0 = \frac{F_0}{\omega_0^2}$

Furthermore:

$$\tan \alpha = \frac{2\,\delta\omega_a}{\omega_0^2 - \omega_a^2}$$

respectively

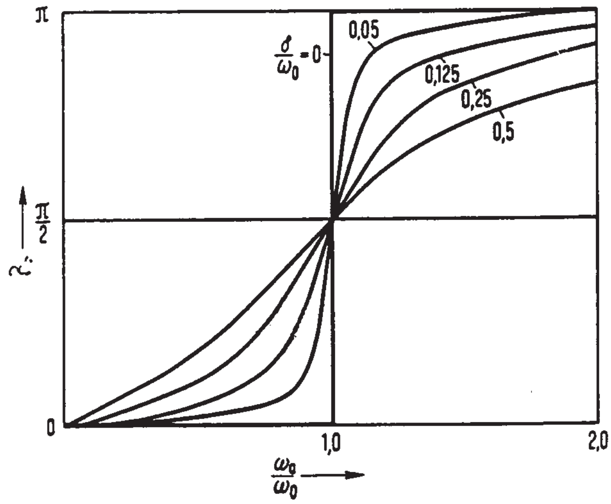
$$\alpha = \arctan \frac{2\,\delta\omega_a}{\omega_0^2 - \omega_a^2} \quad (10)$$

An analysis of Eq. (9) gives evidence of the following:

1. The greater F_0, the greater ϕ_a
2. For a fixed value F_0 we have:
   $\phi \rightarrow \phi_{max}$ for $\omega_a \simeq \omega_0$
3. The greater δ, the smaller ϕ_a
4. For $\delta = 0$ we find:
   $\phi_a \rightarrow \infty$ if $\omega_a = \omega_0$

Fig. 2 shows the phase difference of the forced oscillation as a function of the stimulating frequency according to Eq. (10). For very small frequencies ω_a the phase difference is approximately zero, i.e. the pendulum and the stimulating torque are "inphase". If ω_a is much greater than ω_0,pendulum and stimulating torque are nearly in opposite phase to each other.

Fig. 2: Phase shifting of forced oscillation for different dampings.



The smaller the damping, the faster the transition from swinging "inphase" to the "in opposite phase" state can be achieved.

The mean value of the period $\overline{T}_0$ and the corresponding characteristic frequency $\bar{\omega}_0$ of the free and undamped swinging torsional pendulum are found to be

$$\overline{T}_0 = (1.817 \pm 0.017)\ \text{sec}; \quad \frac{\Delta\overline{T}_0}{\Delta\overline{T}_0} = \pm 1\,\%$$

and $\bar{\omega}_0 = (3.46 \pm 0.03)\ \text{sec}^{-1}$

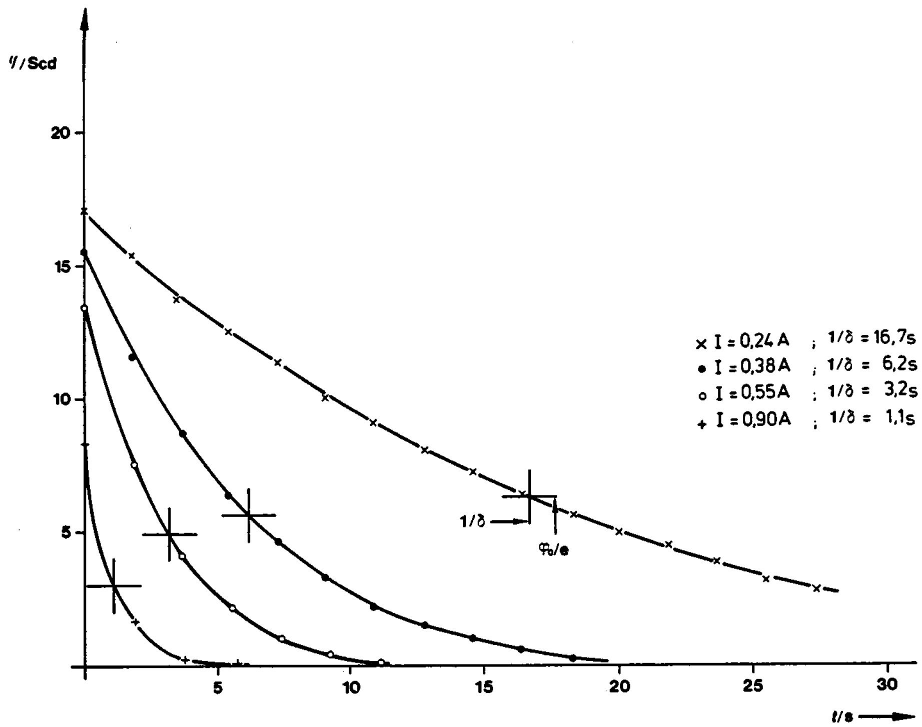
Fig. 3 illustrates the decrease in the unidirectional amplitude values as a function of time for free oscillation and different dampings.

Fig. 4 shows the resonance curves for different dampings. Evaluating the curves leads to medium resonance frequency $\omega = 3.41\ \text{s}^{-1}$.

Table 1 indicates the characteristic damping values.

I/A	$\frac{I}{\delta}$/s	δ/s^{-1}	$\omega = \sqrt{\omega_0^2 - \delta^2}\,/s$	$K = \frac{\phi_n}{\phi_{n+1}}$	Λ
0.25	16.7	0.06	3.46	1.1	0.12
0.40	6.2	0.16	3.45	1.4	0.31
0.55	3.2	0.31	3.44	1.9	0.64
0.90	1.1	0.91	3.34	5.6	1.72

Fig. 3: Maximum values of unidirectional amplitudes as a function of time for different dampings.



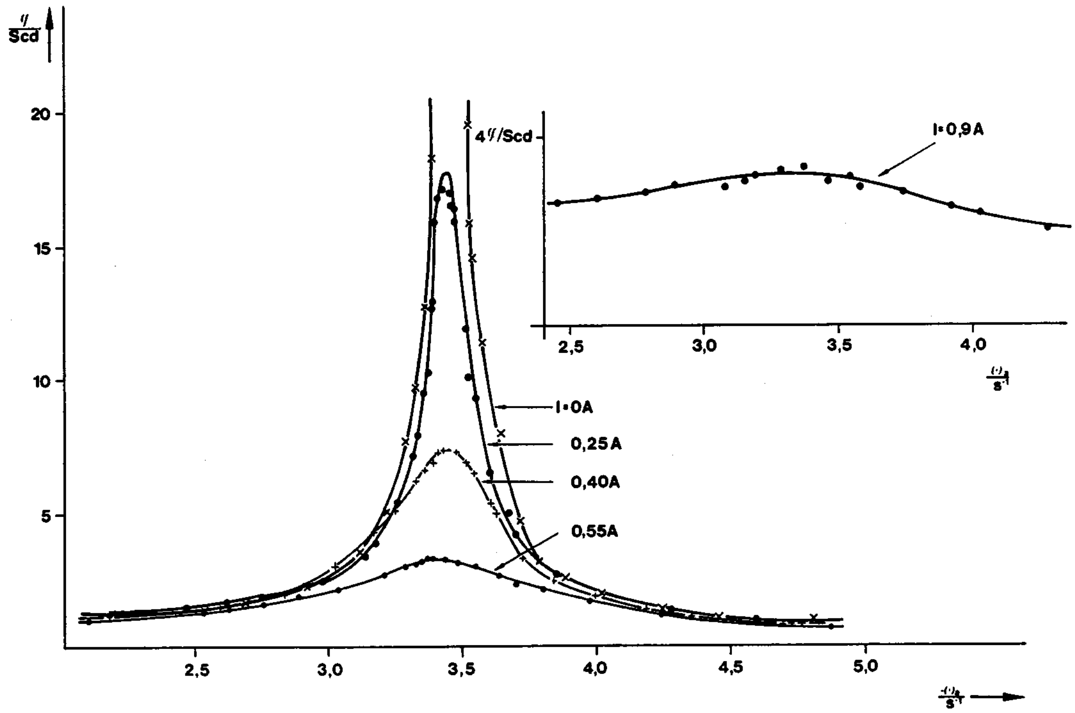


Fig. 4: Resonance curves for different dampings.