Variable g pendulum – Pendulum oscillations with PC interface

LEP 1.3.24

Related topics

Gravitational acceleration, gravitational force, oscillating plane, oscillating period, use of an interface.

Principle and task

Earth's gravitational acceleration *g* is determined for different lengths of the pendulum by means of the oscillating period. If the oscillating plane of the pendulum is not parallel to the gravitational field of the earth, only one component of the gravitational force acts on the pendulum movement.

Equipment

Protractor scale with pointer	08218.00	1
Pendulum f. movement sensor	12004.11	1
Steel ball with eyelet, d 32 mm	02466.01	2
Movement sensor with cable	12004.10	1
COBRA-interface 2	12100.93	1
PC COBRA data cable RS 232, 2 m	12100.01	1
Softw. COBRA Rotation (Win)	14295.61	1
Basic Softw. f. PHYWE Windows prog.	14099.61	1
Adapter, BNC-socket/4 mm plug pair	07542.27	1
Adapter, BNC socket - 4 mm plug	07542.20	1
Silk thread, 200 m	02412.00	1
Fish line, l 20 m	02089.00	1
Weight holder 1 g	02407.00	1
Circular level	02122.00	1
Measuring tape, l = 2 m	09936.00	1
Tripod base -PASS-	02002.55	1
Support rod -PASS-, square, l = 1000 mm	02028.55	1
Bench clamp -PASS-	02010.00	1
Stand tube	02060.00	1
Plate holder	02062.00	1
Right angle clamp -PASS-	02040.55	2

The PHYWE WINDOWS® Basic Software (14099.61) must have been installed once on the used computer for the software to work.

Problems

1. Determination of the oscillation period of a thread pendulum as a function of the pendulum length.
2. Determination of g.
3. Determination of the gravitational acceleration as a function of the inclination of the pendulum force.

Fig. 1: Experimental set up to determine Earth's gravitational acceleration from the oscillating period.

Set-up

— The oscillating periods of the thread pendulum are measured according to Fig. 1.

— The electric connection of the movement recorder is carried out according to Fig. 2 for the COBRA interface. The thread runs horizontally and is lead past the larger of the two thread grooves of the movement recorder.

— Fig.3 shows the set up to carry out measurements with the variable g pendulum.

Procedure

— COBRA: if the COBRA interface is connected to the computer port COM 1 "Osc_COM 1" is clicked on twice. If the serial port COM 2 is used "Osc_COM 2" is started.

— The maximum sampling rate <Delta t/ms> is adjusted according to computer speed. If a 486 (66 MHz) computer is used, 100 ms have proven to be a suitable choice, with faster computers, time may decrease, with slower computers, it may increase. Sampling time is set by shifting the grey slide with the mouse cursor. Modification of <Delta t/ms> must be repeated every time the programme is started. A too short sampling time is identified through the following effects:

  — No measurement values are recorded.

  — The measurement values are not equidistant on the time axis.

  — The distance in time of the measurement points is much larger than the set <Delta t/ms> value.

Recording of measurement values

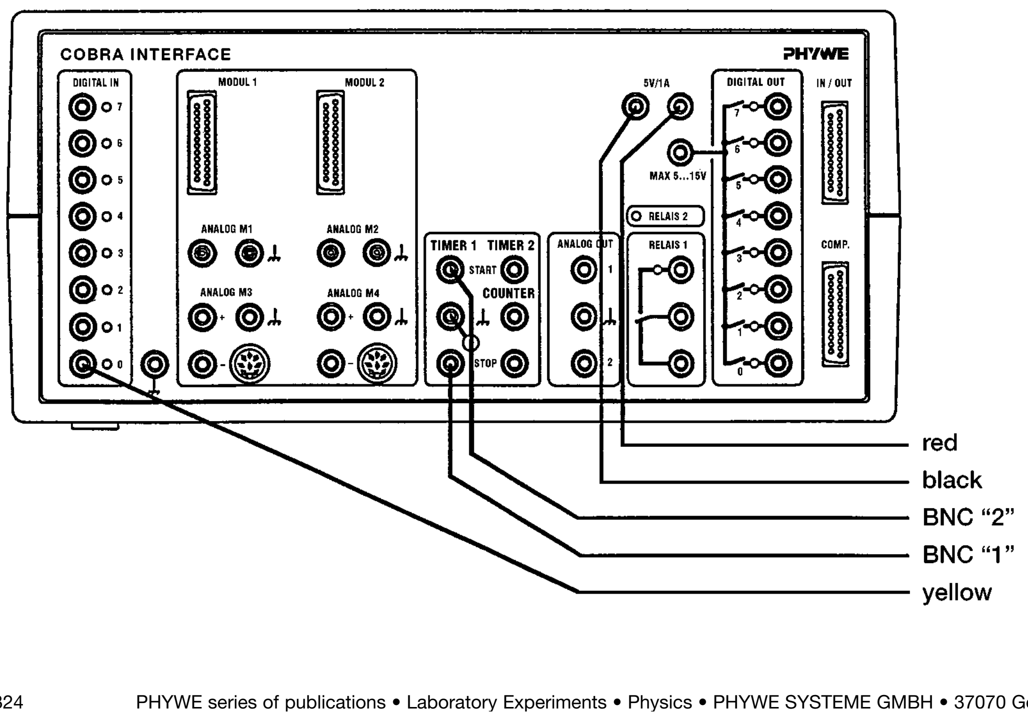
— The diameter of the thread groove of the movement recorder is entered into the input window d_0 (12 mm are set as a default value). In the first part of the experiment (thread penduum), d_1 is the double length of the pendulum in mm, that is, the diameter of the circle described by the centre of gravity of the pendulum. In this case, the measured deviations of the pendulum sphere are indicated directly in rad. If measurements are carried out with the g pendulum (experimental set up according to Fig. 4), 12 is entered for d_1 ($d_1 = d_0$), because the pendulum is now coupled 1:1 with the movement measuring unit. Each of these entries must be confirmed with the "Return" key on the keyboard.

— The 1 g weight plate which tenses the coupling thread between pendulum sphere and movement recorder, is deviated downwards manually and released. The <Start> button must then be pressed. A new measurement can be initiated any time with the <Reset> button, the number of measurement points "n" is reset to zero.

— In total, about n = 250 measurement values are recorded and then the <Stop> button is pressed.

— Now the newly appearing <Edit> button is pressed. After a few seconds, three new order buttons appear on the screen. Click on the <Edit> button. The vertical lines can be captured and shifted with the mouse button. They are positioned in such a way that all measurement values which are to be evaluated lie between the two lines. The <Edit> button is pressed again, followed by the <Evaluate> button. The computer now needs some time to evaluate all data, the length of which depends on the number of meas-

Fig. 2: Connection of the movement recorder to the COBRA interface.



urement points. The process is indicated by a counter running backwards.

— After the evaluation, a graph similar to Fig. 4 appears on the screen. The duration of the period Tand the circular frequency ω (w) of the oscillation are displayed. Press button <g>. To calculate Earth's gravitational acceleration g, the actual numerical value is entered into the input field for pendulum length *l.* In this experiment, the numerical values for "*theta*" are meaningless. Earth's gravitational acceleration is calculated for different pendulum lengths but constant pendulum mass. Then the mass of the pendulum is increased by suspending the second sphere from the eye of the first, keeping the length of the pendulum constant, and g is determined.

— The experimental set up is modified according to Fig. 3. The oscillating plane runs vertically to start with. A box bubble level set on the casing of the movement recorder facilitates exact adjustment. *g* is determined for different angles of deviation.

— *g* is determined every time with identical initial deviating angles (e.g. γ = 150); however, the plane of oscillation is not vertical, but makes an angle ϑ with the vertical direction. The following angles are recommended for measurements: ϑ = 0°,10°, 20°, 40°, 60°, 80°.

Result

— A sphere of mass *m* is suspended from a thread of length *l.* If the pendulum is deviated by an angle γ, it carries out an oscillation. For small deviations (sin(γ) ≈ γ), the following relation is valid for the period:

$$T = 2\pi \sqrt{\frac{l}{g}} \, .$$

This allows to obtain for the gravitational acceleration g:

$$g = \frac{4\pi^2 \cdot l}{T^2}$$

Table 1 contains examples of measurements for different pendulum lengths, the mass of the sphere being constant.

— An increase of the pendulum mass yields an identical value for Earth's gravitational acceleration, if the length of the pendulum remains constant. In one example of measurement, the length of the pendulum is *l* = 0.84 m:

Number of spheres	g / m/s^2
1	9.75
2	9.76

— The known relation to calculate the period of a thread pendulum,

$$T = 2\pi \sqrt{\frac{l}{g}}$$

is strictly valid for the boundary case of infinitesimal oscillating amplitudes. For larger amplitudes, this equation must be replaced through an expression of the kind:

$$T = 2\pi \sqrt{\frac{l}{g}} \left(1 + \frac{1}{4} \sin^2 \frac{\gamma}{2} + \frac{9}{64} \sin^4 \frac{\gamma}{4} + \ldots\right)$$

The period of oscillation thus increases with the angle γ. If the following relation:

$$g = \frac{4\pi^2 \cdot l}{T^2}$$

is used to calculate *g*, one obtains decreasing values for Earth's gravitational acceleration. The measurement values of table 2 display precisely this behaviour. However, table 2 also shows that angles of less than 20° can be considered to be small enough to fulfil

$$g = \frac{4\pi^2 \cdot l}{T^2} \, .$$

— In the case of a pendulum with vertical plane of oscillation (Fig. 5 the tangential component of the weight force *mg* sin γ is the restoring force of the oscillation. If, however, the plane of oscillation makes an angle ϑ with the vertical (Fig. 6), only the component *mg* sin γ cos ϑ of the force acts. For a period *T* one thus obtains:

$$T = 2\pi \sqrt{\frac{l}{g \cdot \cos \vartheta}} \, .$$

The measurement data given in Table 3 confirm this relation within the scope of measurement precision.

As the variable g-pendulum also allows to vary the acceleration acting on the pendulum, the experiment can be carried out in such a way as to obtain an idea of the oscillation velocity of the pendulum on the moon or on mars. On the moon, the gravitational pull is about 16.6% of that of Earth, which corresponds to an angle ϑ = 80.50°. The gravitational pull on Mars can be simulated by means of an angle ϑ = 69°, as the gravitational acceleration only is 38% of the corresponding value on Earth.

When evaluating the oscillating periods, the pendulum rod was considered to have no mass. The length of the pendulum *l* was the distance between the centre of the suspended mass from the centre of the rotation axis. If the mass of the pendulum rod is taken into account, the result is that the mass centre of gravity shifts nearer to the rotation axis; thus *l* becomes shorter. If an increase of measurement precision is wished, this effect must be taken into account.

Indications

— Masses suspended directly from the rotating axis should not be larger than 100 g and only be fastened to the movement recorder for short periods of time, in order to avoid damages to the bearing.

— If the oscillating period is very short, signal leaps or distortions may occur. These can be decreased by changing the sampling rate or by making the diagram invisible with <Clr>. Intervals recorded without faults may be cut out later from the measurement signal with <Edit>.

— Crescent shaped distortions of the oscillations are caused by the sliding of the thread over the thread groove of the movement recorder. This can be avoided winding the thread once around the thread groove.

— As the recording of movement is not free of direct contact, a small damping of the measured oscillations occurs.

— After the autoscale button situated on the left side above the diagram has been clicked on, all measurement data are represented so that they fill the whole screen. However, an unfortunate choice of the subdivisions of the axes may occur. In this case, the corresponding scaling values for the x and y axes may be entered manually by pressing the second or respectively the fourth button over the diagram (cf. Fig. 4). After every change of the y axis magnitudes, the autoscale button should be pressed, in order to adapt the scaling factors to the actually represented values.

— Primarily, the movement recorder is used to measure the angular velocity *omega* (t). From this, the programme calculates the angle φ (t) and the angular acceleration α(t). As an unknown initial angle must be taken into account when calculating φ(t), the programme requires a complete oscillating period to determine the zero position of the oscillation. This is displayed by the programme by the fact that the position of the first period on the y axis does not coincide with that of the other corrected periods. When calculating α(t), differences of small magnitudes must be calculated and divided by short time intervals. This leads to increased noise for the α(t) curve as compared to the ω(t) curve. Too small angular velocities are no longer detected by the movement recorder and are thus represented as zero line.

— The <Evaluate> button provides many evaluation possibilities and requires a certain time, before all results are calculated. For 2000 measurement values, this may take a few minutes.

— The measurement values can be saved by pressing the green <Save> button. They are written as ASCII symbols in columns into a data file, which must have the suffix "AFD" (**A**SCII **F**ile **D**ata). The following measurement values are found in the single columns: *t* / (s) running time, φ(*t*) / (rad) angle, ω(*t*) / (rad/s) angular velocity, α(*t*) / (rad/s^2) angular acceleration, maxphi (*t*) / (rad) positive peaks of the angle graph, *T* / (s) oscillation period.

The measurement data can easily be submitted to further processing or printed out under DOS or WINDOWS®.

— The <maxphi (t)> button selects the positive oscillation peaks and plots them beginning on the left side. The logarithmic oscillation decrement is obtained pressing the

Fig. 4: Typical figure generation to measure Earth's gravitational acceleration.

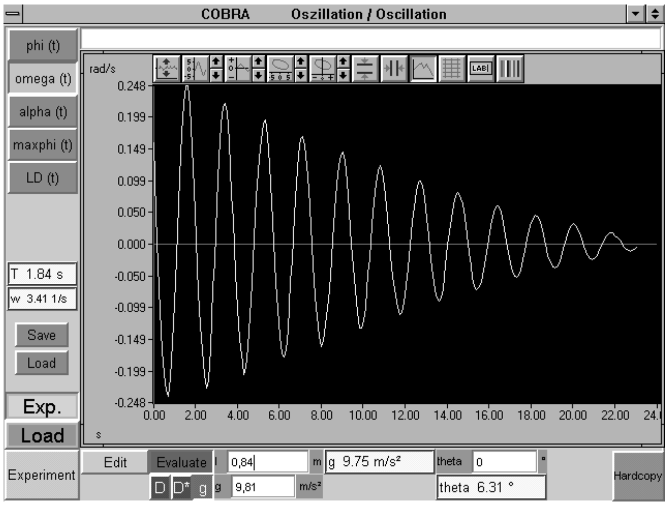


Fig. 3: Experimental set up for the variable g pendulum.

<LD> button. It must be noted that the first and last three values of “maxphi (t)” are not taken into account for the evaluation through <LD>.

— The green <Load> button allows to load measurement data into the running programme, so that the data are evaluated again and compared to actual data. The double button <Exp.> <Load> is used for this. <Exp.> shows the graph of the actual **exp**eriment, <Load> shows the loaded measurement.

— If a new measurement is to be carried out, the <Edit> and <Evaluate> buttons must be switched off to start with, after which the <Experiment> button is pressed. After a few seconds, the set up of the diagram already described for the recording of measurement values appears. The <Reset> button deletes the old measurement data. Once the measurement has been carried out, <Edit> is pressed. In the screen which appears then, the <Edit> button must be pressed to make the measurement values available for evaluation. If the <Evaluate> button is pressed first, no measurement values appear in the diagram. In this case, the <Evaluate> button is switched off again and <Edit> is pressed.

— It must be taken into account that the <Edit> and the <Evaluate> buttons cancel each other. This means that the button pressed first is active, the button pressed after this turns red, but is not activated. For this reason, it is recommended to switch off the button pressed first before pressing the other one.

— The <Clr> button in the measurement value recording screen deletes the diagram. This saves computing time and the maximum sampling rate can be slightly increased. Pressing this button twice will let the diagram appear again on the screen, the contents are deleted.

— An arbitrary text to comment or explain the experiment can be entered in the comment line on the upper edge of the screen, before a hardcopy printout is made. The hardcopy is printed out on the connected printer when <Hardcopy> is pressed. Before printing out a hardcopy, it is recommended to change the colours of the diagram, in order to save the ink ribbon or the cartridge of the printer. Diagram colours can be modified pressing the colour selection button situated on the right side above the diagram. The following colour combinations are recommended:

Scope: instead of black - white
Data: instead of yellow - black.

— Quitting the programme: if the programme is in the measurement value recording mode, it is sufficient to click twice at the upper left corner of the screen with the left mouse button. If not, the <Experiment> key must be pressed first, so the programme will return to the measurement value recording mode.

Table 1

l / m	g / m/s^2
0.84	9.75
0.69	9.61
0.43	9.86

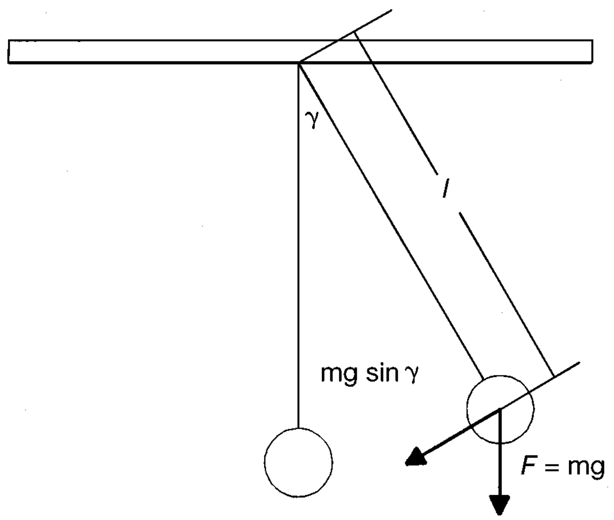


Fig. 5: Consideration of a standard pendulum.

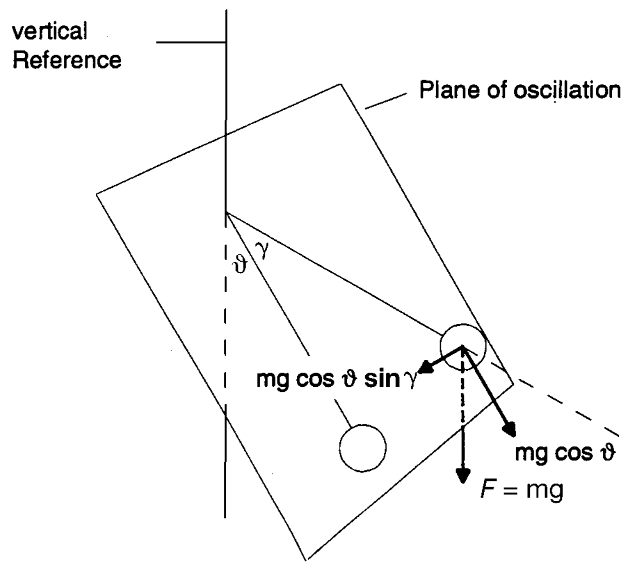


Fig. 6: Analysis of forces for a pendulum with inclined plane of oscillation (g pendulum).

Table 2 (Length of pendulum *l* = 0.138 m)

maximum deviation / °	*g* / m/s^2
5	9.87
10	9.82
20	9.82
45	8.67
90	7.07
approx. 180	3.95

Table 3 (Length of pendulum *l* = 0.138 m)

$\vartheta_{adiusted}$ / °	*g* / m/s^2	$\vartheta_{calculated}$ / °
0	9.87	0.00
10	9.69	9.11
20	9.34	17.89
40	7.74	37.93
60	5.22	57.85
80	2.07	77.81