Mathematical pendulum

LEP 1.3.21

Related topics
Duration of oscillation, period, amplitude, harmonic oscillation.

Principle and task
A mass, considered as of point form, suspended on a thread and subjected to the force of gravity, is deflected from its position of rest. The period of the oscillation thus produced is measured as a function of the thread length and the angle of deflection.

Equipment

Light barrier with Counter	11207.08	1
Power supply 5 V DC/0,3 A	11076.93	1
Steel ball with eyelet, d 24.4 mm	02465.01	1
Steel ball with eyelet, d 32 mm	02466.01	1
Meter scale, demo, l = 1000 mm	03001.00	1
Cursors, 1 pair	02201.00	1
Fish line, l 100 m	02090.00	1
Right angle clamp -PASS-	02040.55	2
Clamping pads on stem	02050.00	1
Support rod -PASS-, square, l 1250 mm	02029.55	1
Tripod base -PASS-	02002.55	1

Problems
1. For small deflections, the oscillation period is determined as a function of the cord length.
2. The acceleration due to gravity is determined.
3. The oscillation period is determined as a function of the deflection.

Set-up and procedure
The experimental set up is arranged as shown in Fig. 1. The steel ball is tied to the fishing line and the latter is fixed in the clamping pads on stem. With a new line,the ball should be allowed to hang for a few minutes, since the fishing line stretches slightly. The pendulum length should be measured before and after the experiment and averaged in each case. The radius of the ball should be taken into account in the measurement. For problem 1, the light barrier can be used to measure a half-cycle.

To measure the oscillation period as a function of the deflection, the pendulum is deflected to both sides and the half-cycle times are added. In each case switch the light barrier to -mode.

Fig. 1: Experimental set up for determining the oscillation period of a mathematical pendulum.

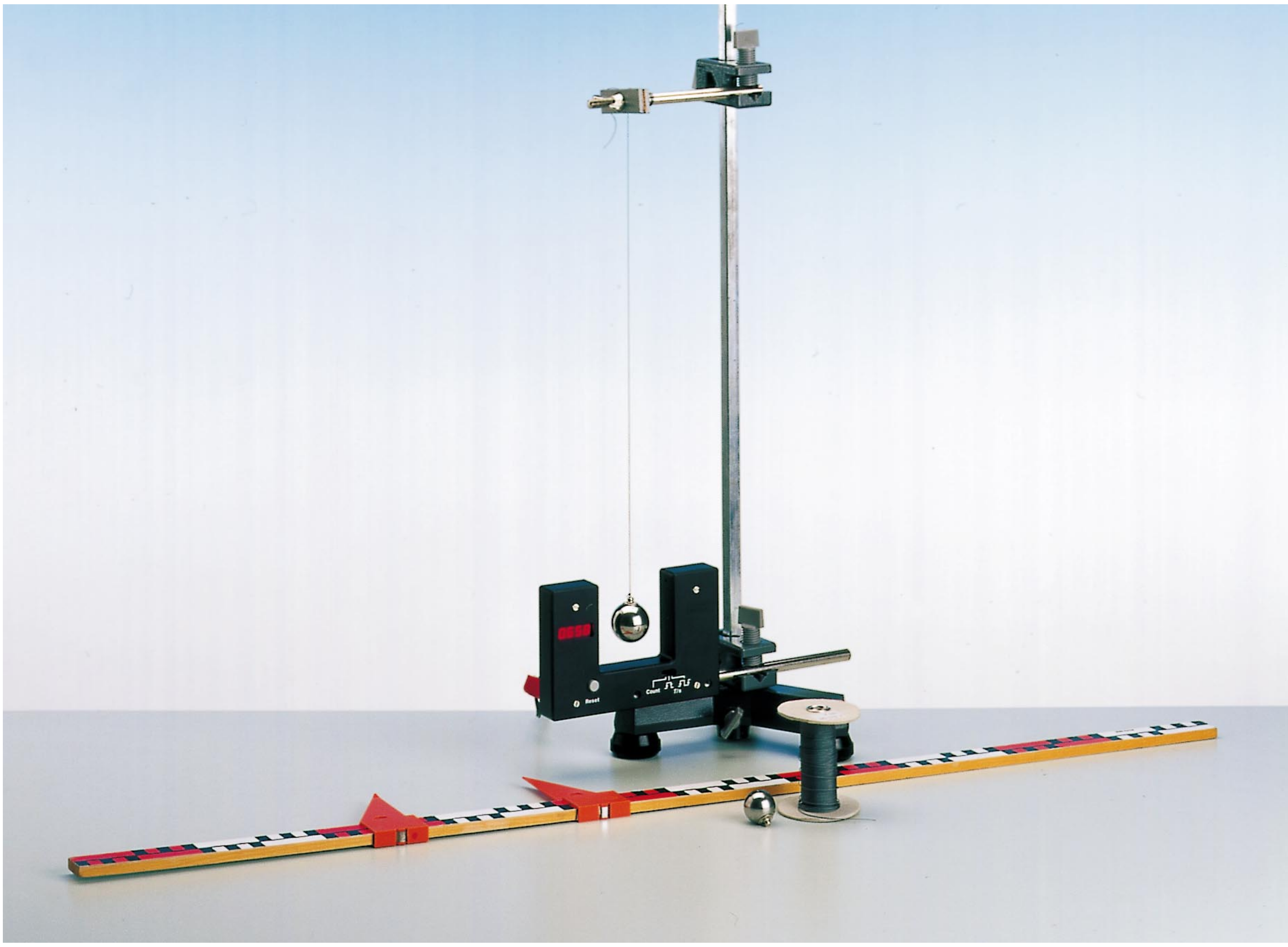
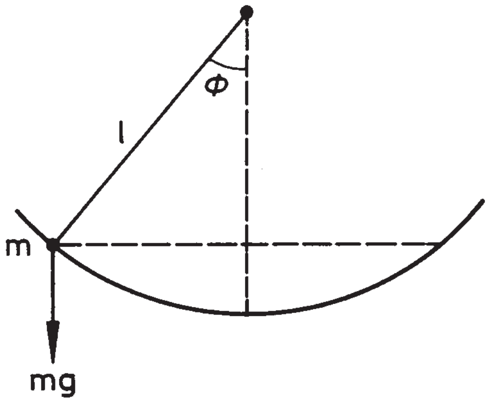


Fig. 2: Motion of the pendulum.



Theory and evaluation

From the energy equation there follows, with the notation of Fig. 2:

$$l^2 \left\{\frac{d\phi}{dt}\right\}^2 + 2gl\,(1\text{-}\cos\phi) = E_0 = \text{const.} \qquad (1)$$

Since the angular velocity vanishes at the reversal point, when

$$\phi = \alpha$$

then one obtains for E

$$E_0 = 2\,gl\,(1\text{-}\cos\alpha)$$

Therefore, from (1)

$$T/4 = \sqrt{\frac{l}{g}} \int_0^{\alpha} \frac{d\phi}{\sqrt{(cos\phi - cos\alpha)}}$$

With $k = \sin \alpha/2$, the period is obtained as

$$T = 4\,\sqrt{\frac{l}{g}} \cdot \int_0^{\pi/2} \frac{d\phi}{\sqrt{1-k^2\,sin^2\,\phi}} = 4\,\sqrt{\frac{l}{g}}\,K\,(k)$$

where K is the complete 1st-order elliptical integral.

Development of the series for $K(k)$ gives

$$T = 2\pi\,\sqrt{\frac{l}{g}} \left\{1 + \frac{1}{4}\sin^2\frac{\alpha}{2} + \ldots\right\} \qquad (2)$$

For small values of α ($\alpha \leq 2°$:

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (3)$$

From the regression line to the measured values of Fig. 3 with the exponential statement

$$Y = A \cdot X^B$$

the exponent is obtained

$$B = 0.502 \pm 0.001 \qquad \text{(see (3))}$$

and

$$A = 2.007\ s/\sqrt{m}$$

From this, with (3), the value for the acceleration due to gravity is obtained as

$$g = 9.80\ \text{m/s}^2$$

For larger angles α, T depends on α (2).

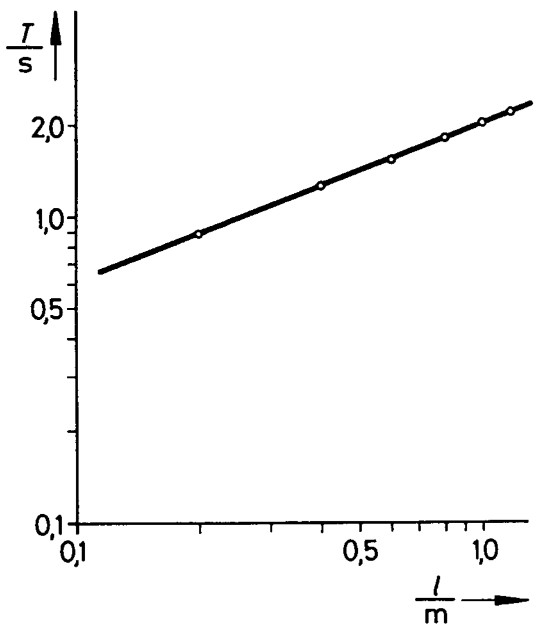


Fig. 3: Period of the pendulum as a function of length.

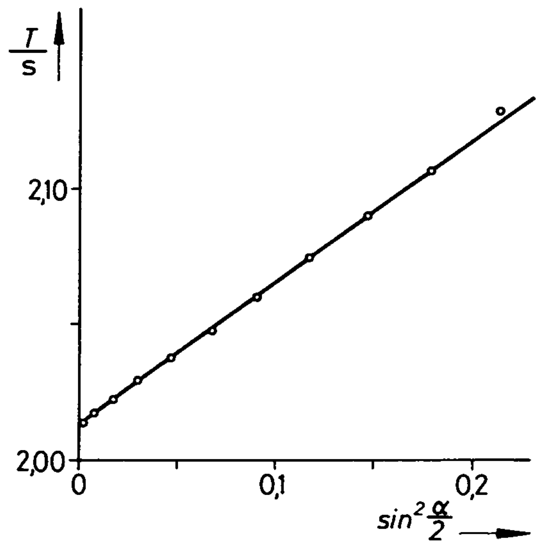


Fig. 4: Period of the pendulum as a function of the angle of deflection.