Related topics

Circular motion, angular velocity, angular acceleration, moment of inertia, Newton's Laws, rotation.

Principle and task

The angle of rotation and angular velocity are measured as a function of time on a body which is pivoted so as to rotate without friction and which is acted on by a moment. The angular acceleration is determined as a function of the moment.

Equipment

Turntable with angle scale	02417.02	1
Aperture plate for turntable	02417.05	1
Holding device w. cable release	02417.04	1
Air bearing	02417.01	1
Precision pulley	11201.02	1
Pressure tube, l 1.5 m	11205.01	1
Blower	13770.93	1
Light barrier with Counter	11207.08	1
Power supply 5V DC/0.3 A	11076.93	1
PEK capacitor/case 1/0.1 mmF/500 V	39105.18	1
Adapter, BNC-plug/socket 4 mm	07542.26	1
Weight holder 1 g	02407.00	1
Slotted weight, 1 g, natur. colour	03916.00	20
Silk thread, 200 m	02412.00	1
Connecting cord, 1000 mm, blue	07363.04	1
Connecting cord, 1000 mm, red	07363.01	1
Circular level	02122.00	1
Tripod base -PASS-	02002.55	1
Barrel base -PASS-	02006.55	1
Bench clamp -PASS-	02010.00	2

Problems

With uniformly accelerated rotary motion, the following will be determined:

1. the angle of rotation as a function of time,

2. the angular velocity as a function of time.

3. the angular acceleration as a function of time,

4. the angular acceleration as a function of the lever arm.

Set-up and procedure

The experimental set-up is arranged as shown in Fig. 1. By means of a spirit level, and with the blower switched on, the rotary bearing is aligned horizontally with the two adjusting feet on the support base. The release switch should be so adjusted that in the set condition it touches the curve of the sector mask. The triggering range and the electrical switching

Fig. 1: Experimental set-up for investigating uniformly accelerated rotary motion.
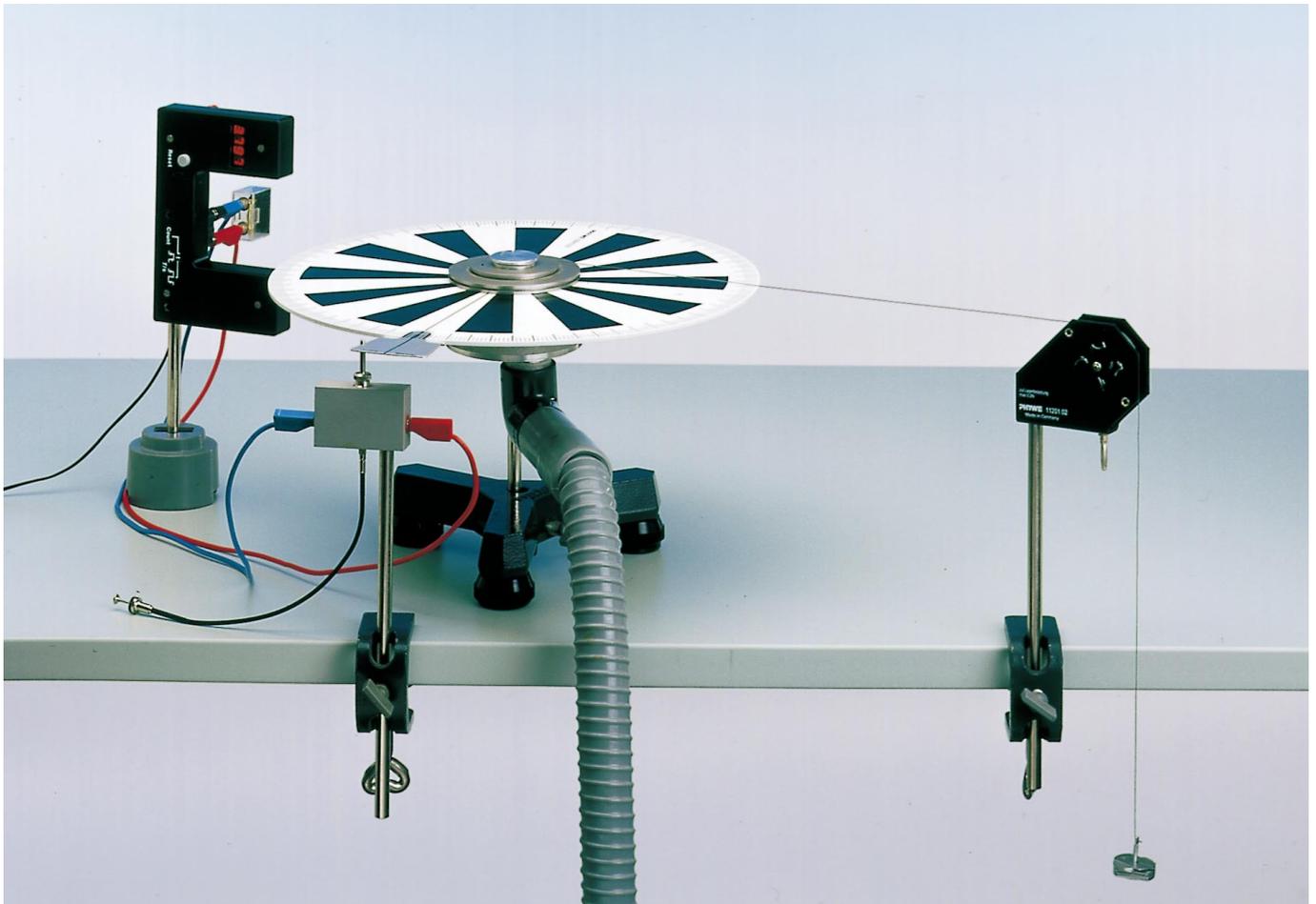
point can be adjusted with the knurled screw. The angular range to be measured is fixed by moving the light barrier. The release switch and light barrier should be connected as shown in Fig. 2. The precision pulley is clamped so that the thread is suspended horizontally over the plate and is flush with the pulley.

Measurement of the angle of rotation φ as a function of time *t*:
- Place the selection key on ⌐⌐⌐ of the fork type light barrier.
- Install the light barrier in a desired angle φ with the shutter release.
- Press the wire release and lock in place.
- Press the "Reset" key of the light barrier.
- Loosening the wire release stopper; sets in motion the plate of rotation with the screen and the counter starts.
- After the screen has passed the pin of the shutter release, the wire release is pressed again and locked.
- As soon as the screen enters the light path of the light barrier, the counter stops.

Note

If the counter stops on loosening, viz. pressing the wire release, a capacitor of high capacitance should be connected parallel to the wire release.
- Measurement of the angular velocity Ω as a function of time *t*.

The dark time of the sector mask in the light barrier is used (gate time Δ*t*) for measuring velocity.

Immediately, after the measurement of the time of rotation *t* for an angle of rotation φ, the corresponding angular velocity Ω is measured.
- Place the selection key to the light barrier at ⌐⌐.
- Press the "Reset" key of the light barrier.
- Loosening the locked wire release; sets into motion the plate of rotation with the screen, the counter indicates "0000".
- As soon as the screen enters the path of the light of the fork type light barrier, the counter starts; it stops when the screen moves out of the light path.

Measurement of Δ*t*

Calculation of the angular velocity
$\Omega = \Delta\phi / \Delta t = 0.262 /\Delta t$

If Δφ is the angle of the sector mask, then

$$\omega \left(t_1 + \frac{\Delta t}{2} \right) = \frac{\Delta\phi}{\Delta t},$$

where t_1 is the time from the start until the sector mask reaches the light barrier (angle time).

To determine the acceleration as a function of the force, the mass on the weight pan is increased by 1 g at a time. The dependence of the acceleration on the radius is determined at constant mass. The instantaneous velocities are measured at a fixed location. The precision pulley must be aligned to the corresponding radius (height, alignment).

Fig. 2: Connection of the light barrier (Lb).
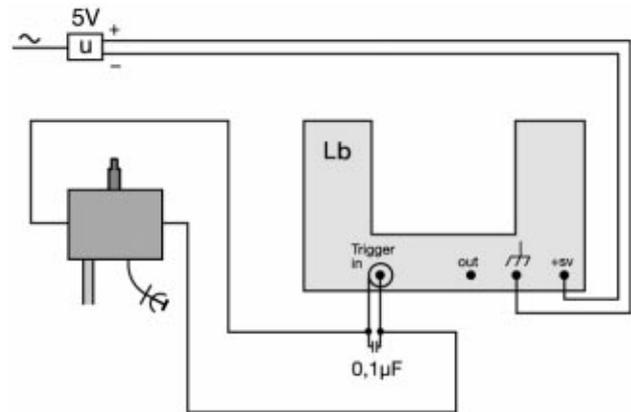
Theory and evaluation

The relationship between the angular momentum $\vec{L}$ of a rigid body in a stationary coordinate system with its origin at the centre of gravity, and the moment $\vec{T}$ acting on it, is

$$\vec{T} = \frac{d}{dt} \vec{L}. \qquad (1)$$

The angular momentum is expressed by the angular velocity $\vec{\omega}$ and the inertia tensor $\hat{I}$ from

$$\vec{L} = \hat{I} \cdot \vec{\omega},$$

that is, the reduction of the tensor with the vector.

In the present case, $\vec{\omega}$ has the direction of a principal inertia axis (z-axis), so that $\vec{L}$ has only one component:

$$L_Z = I_Z \cdot \omega,$$

where I_Z is the z-component of the principal inertia tensor of the plate.

For this case, equation (1) reads:

$$T_Z = I_Z \frac{d\omega}{dt}.$$
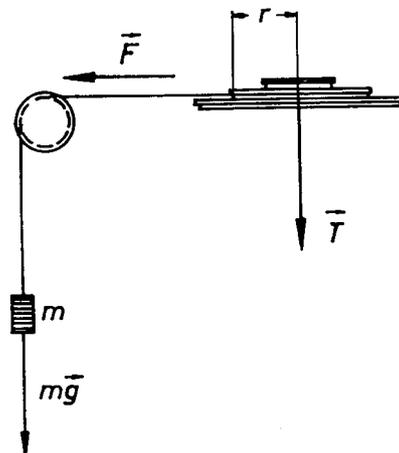
Fig. 3: Moment of a weight force on the rotary plate.

Fig. 4: Angle of rotation as a function of time with uniformly accelerated rotary motion for $m = 0.01$ kg, $r = 0.015$ m.
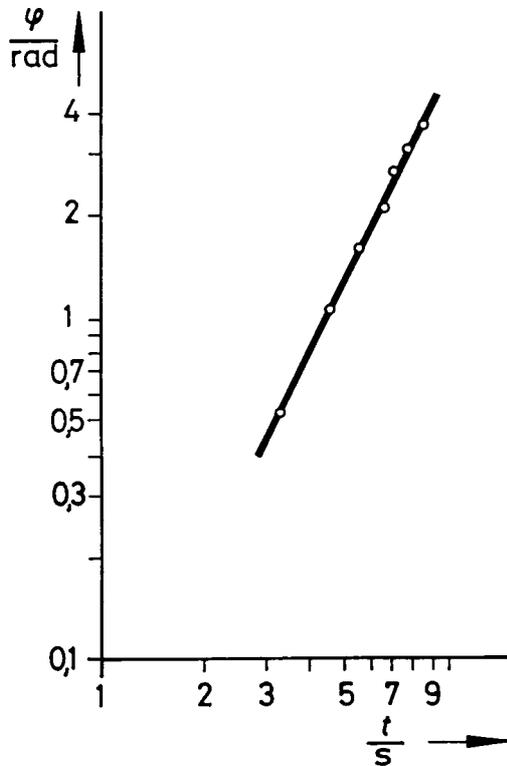
Fig. 5: Angular velocity as a function of time in uniformly accelerated rotary motion, with $m = 0.01$ kg, $r = 0.015$ m.
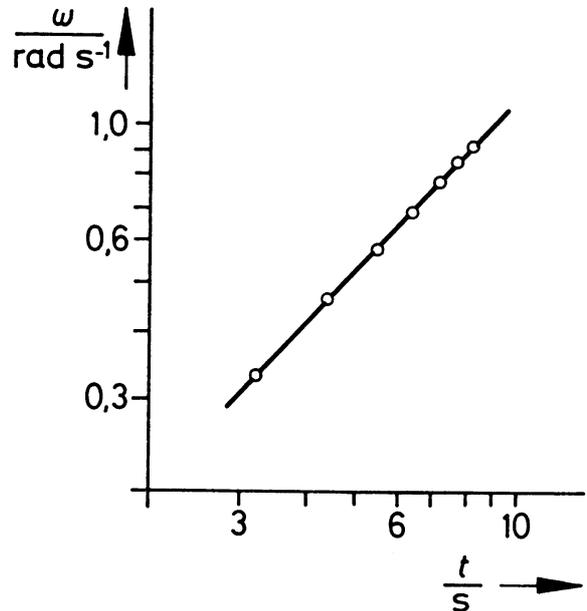
The moment of the force F (see Fig. 2)

$$\vec{T} = \vec{r} \cdot \vec{F}$$

gives, as shown in Fig. 3,

$$T_Z = r \cdot m \cdot g,$$

so that the equation of motion reads

$$mgr = I_Z \frac{d\omega(t)}{dt} \equiv I_Z \cdot \alpha. \tag{2}$$

With the initial condition

$$\omega(0) = 0$$

one obtains

$$\omega(t) = \frac{mgr}{I_Z} t \tag{3}$$

and for the angle of rotation φ with the initial condition

$$\varphi(0) = 0$$

$$\varphi(t) = \frac{1}{2} \frac{mgr}{I_Z} t^2. \tag{4}$$

From the regression line to the measured values of Fig. 4, with the exponential statement

$$Y = A \cdot X^B$$

the exponent is obtained

$$B = 2.005 \qquad \text{(see (4))}$$

From the regression line to the measured values of Fig. 5, with the linear statement

$$Y = A + B \cdot X$$

the slope is obtained

$$B = 0.114 \qquad \text{(see (3))}$$

From this, a moment of inertia

$$I_Z = 0.0129 \text{ kgm}^2$$

is obtained in accordance with (2).

Fig. 6: Angular acceleration $\alpha = \omega/t$ as a function of the active force with uniformly accelerated rotary motion for $r = 0.03$ m.
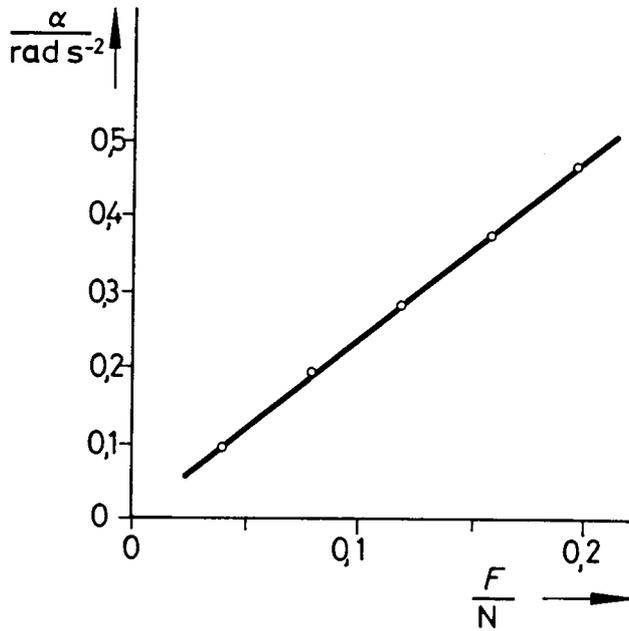
Fig. 7: Angular acceleration as a function of the lever arm with uniformly accelerated rotary motion for $m = 0.016$ kg.