Moment of inertia and angular acceleration with PC interface

LEP 1.3.14

Related topics

Angular velocity, rotary motion, moment of inertia of a disc, moment of inertia of a bar, moment of inertia of a mass point.

Principle and task

If a free body which can rotate is let to rotated without interference of exterior forces around its fixed axis, the angle of rotation will increase proportionally with time, whereas its angular velocity is independent of time.

If a constant torque acts on a body which can rotate without friction around a fixed axis, the angle of rotation, which changes during rotation, increases proportionally to the square of time and the angular velocity increases proportionally to time.

If a given torque thrust acts on a body which can rotate, the latter is accelerated to an angular velocity which depends on its inertia (moment of inertia). A torque thrust will cause an equally large angular momentum.

Equipment

Turntable with angle scale	02417.02	2
Aperture plate for turntable	02417.05	1
Air bearing	02417.01	1
Holding device w. cable release	02417.04	1
Pressure tube, l 1.5 m	11205.01	1
Blower	13770.93	1
Slotted weight, 1 g, natur. colour	03916.00	9
Slotted weight, 10 g, black	02205.01	3
Slotted weight, 50 g, silver bronze	02206.02	2
Silk thread, 200 m	02412.00	1
Weight holder 1 g	02407.00	1
Bench clamp -PASS-	02010.00	2
Tripod base -PASS-	02002.55	1
Stand tube	02060.00	1
Measuring tape, l = 2 m	09936.00	1
Circular level	02122.00	1
Adapter, BNC-socket/4 mm plug pair	07542.27	1
Adapter, BNC socket - 4 mm plug	07542.20	1
COBRA-interface 2	12100.93	1
PC COBRA data cable RS232, 2 m	12100.01	1
Movement sensor with cable	12004.10	1
Softw. COBRA Rotation (Win)	14295.61	1
Basic Softw. f. PHYWE Windows prog.	14099.61	1

The PHYWE WINDOWS® Basic Software (14099.61) must have been installed once on the used computer for the software to work.

Fig. 1: Experimental set up: Moment of inertia and angular acceleration with PC interface.

Problems

1. Measurement of the laws of angle and angular velocity according to time for a uniform rotation movement.
2. Measurement of the laws of angle and angular velocity according to time for a uniformly accelerated rotational movement.
3. Rotation angle φ is proportional to the time t required for the rotation:
4. Measurement of the laws of angle and angular velocity according to time for a uniformly accelerated rotational movement.

Set-up

— As shown in Fig .1. Electric connection of the movement recorder is carried out according to Fig. 2 for the COBRA interface. Make sure the thread connecting the rotating axis with the movement recorder is horizontal. The thread is laid over the larger of the two thread grooves of the movement recorder.

— The feet of the tripod must be adjusted so that the turning plate is horizontal.

— Set the air inlet so that the rotor just lifts off due to air pressure and slides without vibrations on the air cushion.

Procedure

— COBRA: if the COBRA interface is connected to the computer port COM 1 “Rot_COM 1” is clicked on twice. If the serial port COM 2 is used, “Rot_COM 2” is started.

— The maximum sampling rate <Delta t/ms> is adjusted according to computer speed. If a 486 (66 MHz) computer is used, 100 ms have proven to be a suitable choice; with faster computers, time may decrease, with slower computers, it may increase. Sampling time is set by shifting the grey slide with the mouse cursor. Modification of <Delta t/ms> must be repeated every time the programme is started. A too short sampling time is identified through the following effects:

— No measurement values are recorded.

— The measurement values are not equidistant on the time axis.

— The distance in time of the measurement points is much larger than the set <Delta t/ms> value.

As the rotational movement is slow, a sampling time of about 200 ms is recommended, in order to limit the number of measurement data.

— The <Start> button is pressed to record the measurement value. The aperture is brought to the initial position and held by means of the holding device. The diameter of the thread groove of the movement recorder is entered into the input window d_0 (12 mm are set as a default value). d_1 is the value of the axis of the rotating plate around which the silk thread is wound (30 mm are set as a default value). Each of these entries must be confirmed with the “Return” key on the keyboard.

— *Measurement of the laws of angle and angular velocity according to time for a uniform rotation movement.*

— The end of the silk thread is weighted only with one or two paper clips. This small mass is sufficient to tense the thread without causing a noticeable acceleration to the rotating plate.

— Set <Average> to about 1...20.

— Switch on the fan, release the wire trigger and give the rotating disk an angular velocity by giving it a shove with your hand. The plate must not start vibrating. If the plate runs smoothly, press the <Reset> button. The displayed number of measurement points “n:” is “0” and starts increasing. After a few seconds, and before the paper clip has reached the floor, press the <Stop> button.

Fig. 2: Connection of the movement recorder to the COBRA interface.

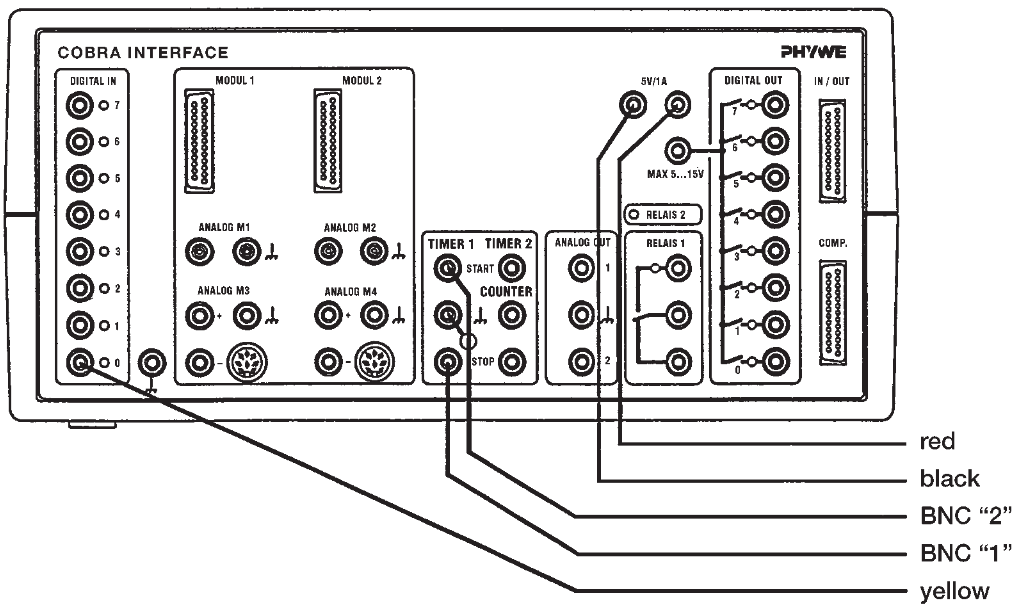


Fig. 3: Angular time dependence law of a non accelerated rotational movement.

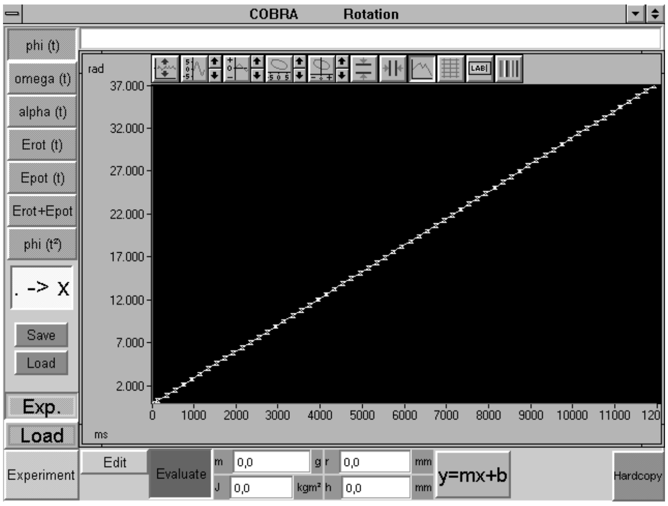
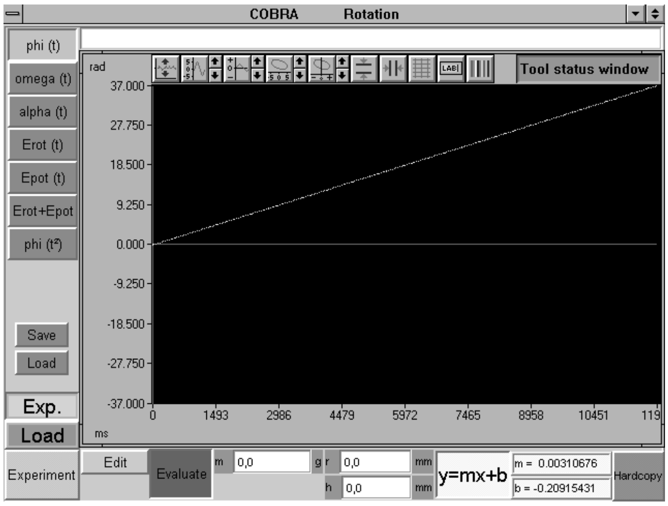


Fig 4: Regression line in the ω*t* diagram.



Indications:

— A negative angular moment can be converted to a positive one by means of the <Invert> button.

— If the rotating plate does not run smoothly, check whether inverting the direction of rotation causes an improvement. If not, the number of averaging operations can be increased under <Average>.

Measurement of the laws of angle and angular velocity according to time for a uniformly accelerated rotational movement

— The end of the silk thread is weighted with the weight plate and by adding supplementary slot weights. The mass may not swing during the recording of the measurement value.

— Set <Average> to 1.

— Switch on the fan and press the <Reset> button. The number of displayed measurement points “n:” is “0”. If non valid measurement points are recorded due to inadvertent minimal movements, press the <Reset> button again.

— Release the wire trigger. The rotating plate starts turning, accelerates and recording of measurement values starts automatically as soon as a minimal angular velocity is passed. After about 2 revolutions, the pin of the wire trigger is pressed out again, so that the angular aperture is stopped abruptly during the next revolution. The number of measurement points “n:” and the angular velocity <omega> are updated during the course of the rotational movement. At the end of the movement recording, angular velocity decreases again below the minimal value and recording of measurement values stops.

— Once the recording of measurement values is concluded, press the red <Stop> button.

Results

— The <Edit> button is pressed to start with. This only will appear if more than three measurement values have been recorded. After a few seconds, a new screen surface appears. The <Edit> button is pressed again in this screen. The measurement data are represented in an ωt diagram.

Measurement of the laws of angle and angular velocity according to time for a uniform rotation movement:

— Switch off the <Edit> button through pressing it again and switch on the <Evaluate> button.

— After the following buttons are pressed, a graph as in Fig. 3 appears:<phi (t)>, <Autoscale (1st button above the diagram)>, <Join points (8th button)>, <manual x-, y- axis scaling (4th, 2nd, button)>, <.->x>, (poss.) <Hardcopy>.

— Law of the angle according to time.

Rotation angle φ is proportional to the time t required for the rotation:

$$\varphi = \omega \cdot t$$

From the slope *m* of the ω *t*-Graph (switch off <.->x> button and switch on <y~mx+b>) one obtains the angular velocity ω. In Fig. 4, time is plotted in ms on the x axis, so that the unit of slope *m* = 0.00311 is rad/ms. One thus finds: ω = 3.11 rad/s. One also obtains this value ω by pressing the <omega (t)> button (cf. Fig. 5). In the ωt-representation, the law of the angle according to time is described by a horizontal straight line. The section *b* of the y axis represents angular velocity ω = 3.09 rad/s. If a rigid body rotates freely around a fixed axis, its angular velocity is constant.

Measurement of the laws of angle and angular velocity according to time for a uniformly accelerated rotational movement:

— After clicking on the autoscale button situated on the left side above the diagram, all measurement data are represented over the full screen (cf. Fig. 6). Next to the measurement points which actually are of interest (increasing branch of the graph), other measurement points have also been recorded, which must be assigned to the collision between the angular aperture and the diaphragm. These measurement points can be deleted step by step pressing the <Del> button. If too many measurement points were deleted by mistake, switch the <Edit> button off and on again. All measured values are represented again for new processing and correction. After editing is concluded, the <Edit> button is switched off.

Fig 5: Angular velocity time dependence law of a non accelerated rotational movement.

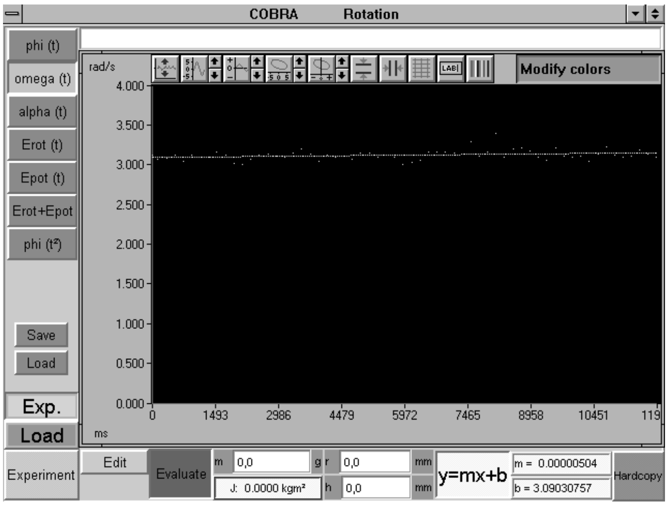


Fig. 6: Representation of all measurement values, accelerated rotational movement and abrupt breaking of the rotating.

— The <Evaluate> key makes available a multitude of evaluation possibilities:

— <omega (t)> displays the angular velocity versus time graph, a straight line which obeys the relation $\omega = \alpha \cdot t$ (Fig. 7). The factor of proportionality or represents angular acceleration $\omega = \alpha$. If the <y=mx+b> key is pressed, a regression line is laid through the measurement points; the slope m gives the angular acceleration α. In this case, α has the unit rad/(s*ms), due to the time unit of the x axis, ms. α = 0.00081 rad/(s*ms) = 0.81 rad/s^2. (If the autoscale button situated on the left side above the diagram is pressed, all measurement values are displayed; however, axis subdivisions may have been selected unfortunately. In this case, corresponding scaling values forthe y and x axes can be entered manually by pressing the second or the fourth button above the diagram).

Indication:

every time the y measurement magnitudes are switched over, the autoscale button should be pressed, in order to adapt the scalings to the actually represented values.

— The evolution of angular acceleration in time can be displayed by means of <alpha (t)>. Fig. 9 shows a linear regression through the angular acceleration data (<y=mx+b>). The y axis portion b = 0.74 rad/s^2 gives the initial value of angular acceleration or. As shown in Fig. 9 angular acceleration as a function of time is constant for a uniformly accelerated rotational movement.

— The law of angular time dependence is obtained pressing the <phi (t)> button. Fig. 10 shows a parabolic curve in which the measurement points have been enhanced. This is obtained by pressing the <. -> x> button. This buttons freezes the green selection switches for the y axis. Other measurement magnitudes can only be displayed once the <. -> x> has been switched off again. The <. -> x> button is not active in the regression line representation.

— The parabolic course of the law of angular time dependence can be verified by means of the <phi (t^2)> button. In this case, the square of time is plotted on the x axis and one obtains a straight line; this confirms the relation $\varphi(t) = 0.5\ \alpha\ t^2$. Clicking on the <y=mx+b> button, one can check that $m = 0.5\ \alpha$ is valid for the slope of the regression line.

— The rotating plate is accelerated by the vertically moving mass. The effective torque M is calculated according to

$$M = r \cdot m \cdot g$$

with

r radius of the axle bolt or of the used driving disk

m accelerating Mass

g = 9.81 m/s^2 earth acceleration

The relation between torque thrust $M\ t$, moment of inertia J and angular velocity ω is

$$M \cdot t = J \cdot \omega .$$

The moment of inertia J thus obeys the following relation:

$$J = \frac{M}{\omega/t} .$$

In an ω-t illustrative representation, the relation ω-t is precisely the slope of the regression line. Fig. 12 shows the regression parameters m (slope) and b (y axis section) on the one hand, and the moment of inertia J on the other. In order to calculate J, it is still necessary to enter the accelerating mass m and the radius r of the rotating axle around which the thread is wound ("Enter" must be pressed on the keyboard after every input!).

Fig. 7: Angular velocity versus time diagram.

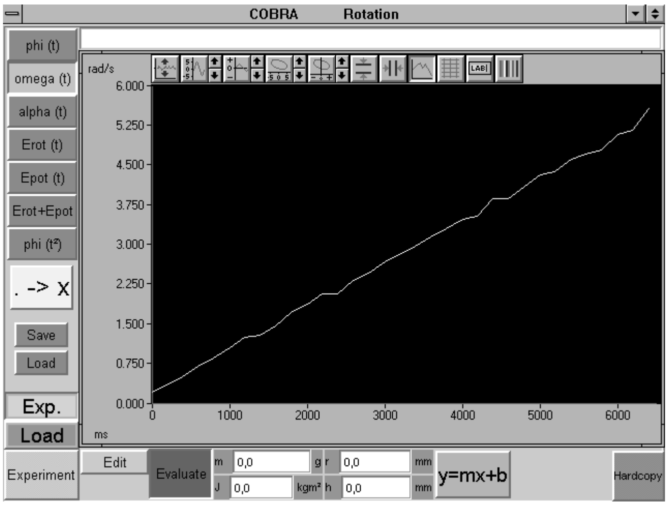
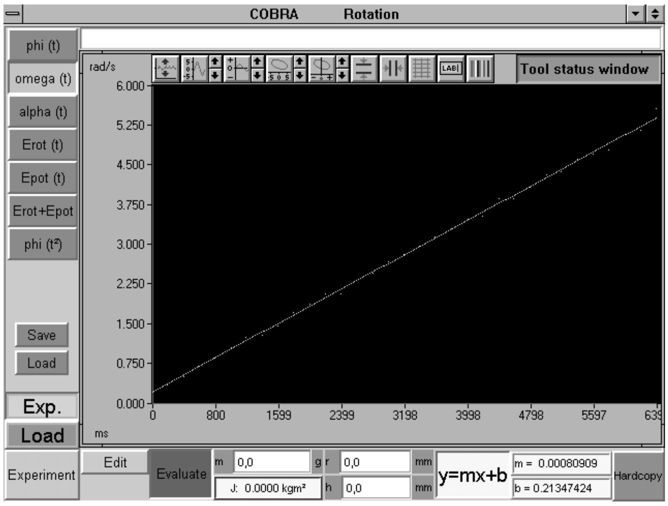


Fig. 8: Angular velocity versus time diagram with regression line.



The following is valid for this measurement example

m (slope) = 0.809 rad/s^2 $= \omega/t$

M = r m g = 0.015 m · 0.081 kg · 9.81 m/s^2 = 0.0119 kg m^2/s^2

$$J = \frac{M}{\omega/t} = \frac{0.0119 \text{ kgm}^2 \text{ /s}^2}{0.809 \text{ rad/s}^2} = 0.0147 \text{ kg m}^2.$$

There is a second way to obtain the moment of inertia J: the dynamic effect of a torque is the angular acceleration. Torque and angular acceleration are proportional to one another:

$$M = J \cdot \alpha .$$

so

$$J = \frac{M}{\alpha} = \frac{0.0119 \text{ kg m}^2 \text{ /s}^2}{\alpha} = 0.0143 \text{ kg m}^2.$$

If the <alpha (t)> and <y=mx+b> buttons are pressed (Fig. 9, the programme automatically calculates M (after entering m and r), the average value of the angular acceleration $\bar{\alpha}$ and from this, the moment of inertia J.

— In order to verify the energy balance, the accelerating mass m, the radius of the rotation axis rand the total falling height h of the mass m must be entered into the corresponding input fields (conclude every input by pressing ENTER on the keyboard!). <Erot (t)> shows the parabola shaped increase of rotation energy

$E_{rot}(t) = 0.5\ J\ \omega^2$ (Fig. 13).
<Epot (t)> gives the potential energy
$E_{pot}(t) = m\ g\ (h - s(t))$, with $s(t)$ = phi(t) r,
which decreases in time (Fig. 14. The law of conservation of energy says that the sum of the rotation energy and of potential energy must be constant in this closed system. The <Erot+Epot> button allows to verify this statement quite easily (Fig. 15).

Indication:
the accelerating mass m increases its speed during the course of the experience and thus becomes an increasing kinetic energy. This kinetic energy is, however, negligible in comparison to the other two types of occurring energy and is thus not taken into account in the <Erot+Epot> calculation.

Indications

— The measurement values can be saved by pressing the green <Save> button. They are written as ASCII symbols in columns into a data file, which must have the suffix “AFD” (**A**SCII **F**ile **D**ata). The following measurement values are found in the single columns: t / (ms), *phi* (t) / (rad), *omega* (t) / (rad/s), *alpha* (t) / (rad/s^2), *Erot*(t) / (g m^2/s^2), Epot(t) / (g m^2/s^2), Erot + Epot / (g m^2/s^2).

The measurement data can easily be submitted to further processing or printed out under DOS or WINDOWS®.

— The green <Load> button allows to load measurement data into the running programme, so that the data are evaluated again and compared to actual data. The double button <Exp.> <Load> is used for this. <Exp.> shows the graph of the actual **exp**eriment, <Load> shows the loaded measurement.

— If a new measurement is to be carried out, the <Edit> and <Evaluate> buttons must be switched off to start with, after which the <Experiment> button is pressed. After a few seconds, the set up of the diagram already described for the recording of measurement values appears. The <Reset> button deletes the old measurement data. Once the measurement has been carried out, <Edit> is pressed. In the screen which appears then, the <Edit> button must be pressed to make the measurement values available for evaluation. If the <Evaluate> button is pressed first, no measurement values appear in the diagram. In this case, the <Evaluate> button is switched off again and <Edit> is pressed.

Fig. 9: Angular acceleration versus time diagram.

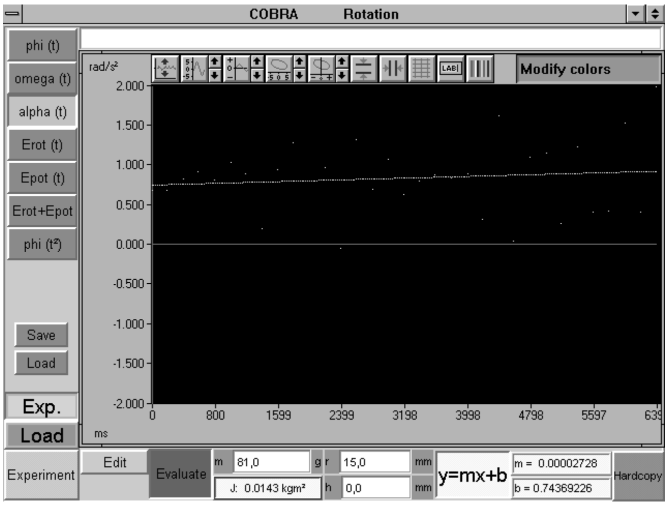


Fig. 10: Angle versus time diagram.

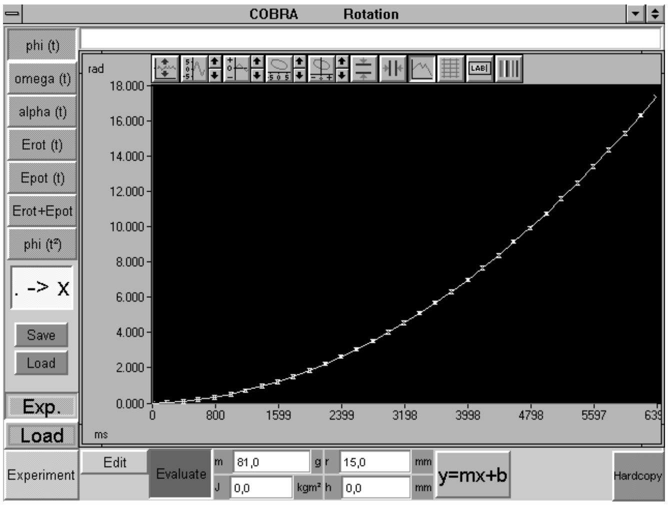
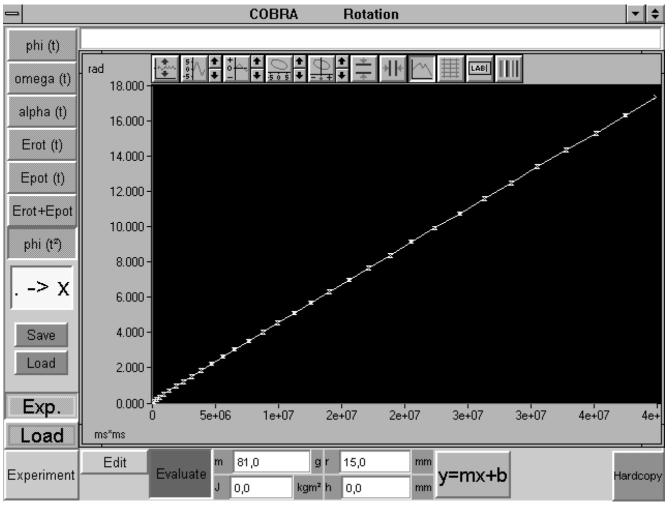


Fig. 11: Angle versus time² diagram.



— An arbitrary text to comment or explain the experiment can be entered in the comment line on the upper edge of the screen, before a hardcopy printout is made. The hardcopy is printed out on the connected printer when <Hardcopy> is pressed. Before printing out a hardcopy, it is recommended to change the colours of the diagram, in order to save the ink ribbon or the cartridge of the printer. Diagram colours can be modified pressing the colour selection button situated on the right side above the diagram. The following colour combinations are recommended:

Scope: instead of black - white
Data: instead of yellow - black.

— The experiments can also be carried out with the inertial rod (02417.03). The second rotating plate is used to increase the moment of inertia for the experiment described here.

— It must be taken into account, that the <Edit> and <Evaluate> keys cancel each other mutually. This means that the button pressed first is active, the button pressed after this turns red, but is not activated. For this reason, it is recommended to switch off the first pressed button before pressing the other one.

— Quitting the programme: if the programme is in the measurement value recording mode, it is sufficient to click twice at the upper left corner of the screen with the left mouse button. If not, the <Experiment> key must be pressed first, so the programme will return to the measurement value recording mode.

Fig. 12: Angular velocity versus time diagram with regression line and calculation of the moment of inertia.

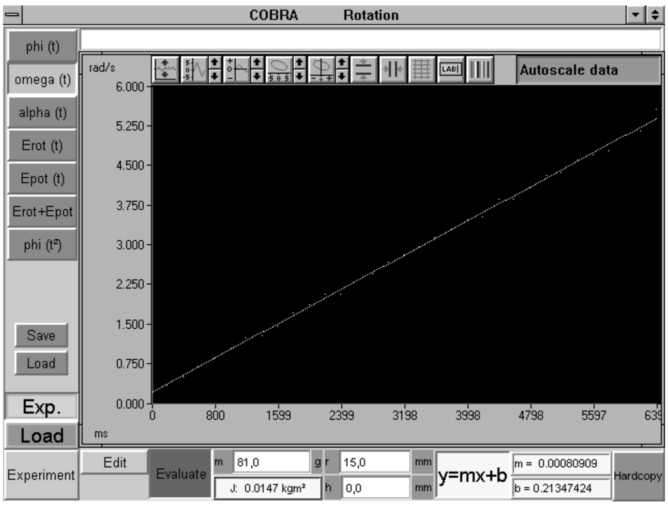


Fig. 13: Rotation energy of an accelerated rotational movement.

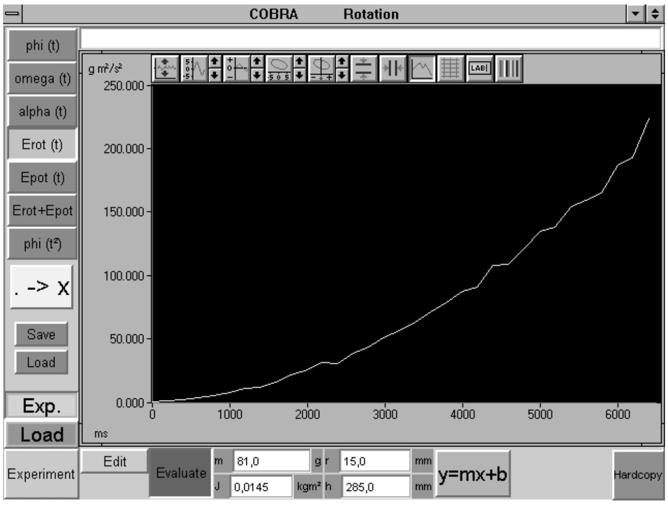


Fig. 14: Potential energy of the accelerating mass.

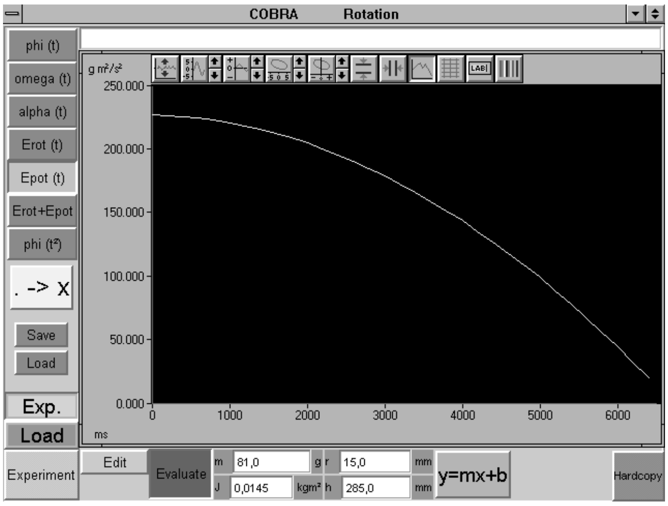


Fig. 15: Energy balance