Moment of inertia and angular acceleration

Related topics

Angular velocity, rotary motion, moment, moment of inertia of a disc, moment of inertia of a bar, moment of inertia of a mass point.

Principle and task

A moment acts on a body which can be rotated about a bearing without friction. The moment of inertia is determined from the angular acceleration.

Equipment

Turntable with angle scale	02417.02	1
Aperture plate for turntable	02417.05	1
Air bearing	02417.01	1
Inertia rod	02417.03	1
Holding device w. cable release	02417.04	1
Precision pulley	11201.02	1
Blower	13770.93	1
Pressure tube, l 1.5 m	11205.01	1
Light barrier with Counter	11207.08	1
Power supply 5 V DC/0, 3 A	11076.93	1
Supporting blocks, set of 4	02070.00	1
Slotted weight, 1 g, natur. colour	03916.00	20
Slotted weight, 10 g, black	02205.01	10
Slotted weight, 50 g, black	02206.01	2
Weight holder 1 g	02407.00	1
Silk thread, 200 m	02412.00	1
Tripod base -PASS-	02002.55	2
Support rod -PASS-, square, l = 1000 mm	02028.55	1
Support rod -PASS-, square, l 400 mm	02026.55	1
Right angle clamp -PASS-	02040.55	3
Bench clamp, -PASS-	02010.00	2

Problems

From the angular acceleration, the moment of inertia are determined as a function of the mass and of the distance from the axis of rotation.

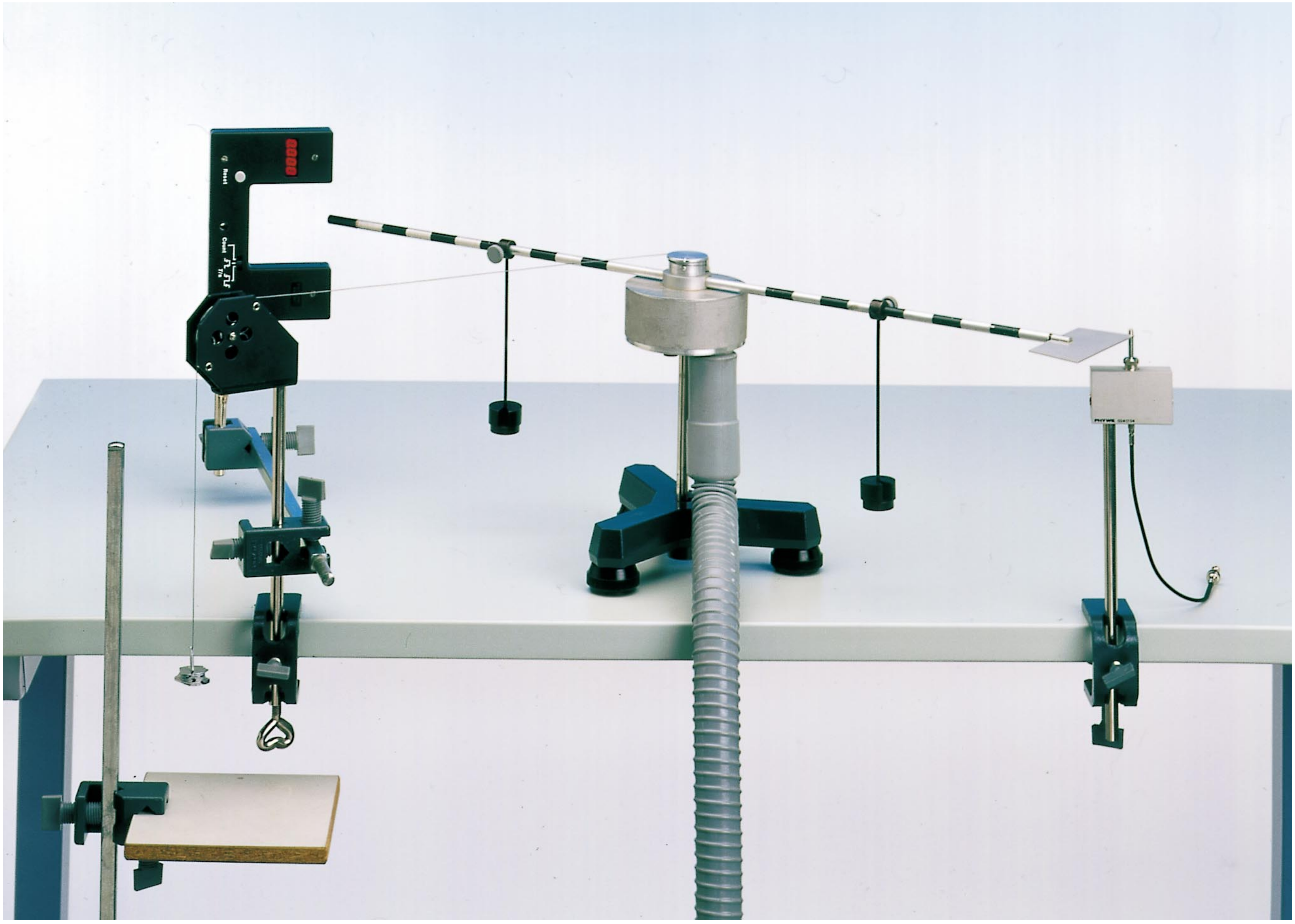
1. of a disc,
2. of a bar,
3. of a mass point.

Set-up and procedure

The experimental set-up is arranged as shown in Fig. 1. The rotary bearing, with the blower switched on, is alignet horizontally with the adjusting feet on the tripod. The release trip must be so adjusted that it is in contact with the inserted sector mark in the set condition.

The precision pully is clamped so that the thread floats horizontally above the rotating plane and is aligned with the pulley.

Fig. 1: Experimental set-up for investigating the moment of inertia of bodies.



While using the bar, the fork type light barrier is positioned in such a way that, the end of the bar, lying opposite to the angular screen standing just in front of the light path, is held by the holder.
While using the turntable, before the beginning of the experiment, the pin of the holder through the bore hole near the edge fixes the turntable. The fork type light barrier is positioned such that, the screen connected to the turntable, is in front of the light ray.
Whenever the holder is released, the light barrier must be interrupted at that moment.

The measurement is done in the following manner:

1. Measurement of the angular velocity ω:

– Set the light barrier selection key at "⌐⌙" and press the "Reset" button
– Release the holder to start the movement flow
  The light barrier measures at first, the initial darkening time which is of no great importance.
– During the flow movement, press the "Reset" button after the screen has attained end velocity but before the screen passes the light barrier. The time measured now, Δt is used for the measurement of angular velocity ($\Delta\varphi$ is the angle of the used rotary disc shutter)
  $\omega = \Delta\varphi / \Delta t$

2. Measurement of angular acceleration α:

– The experiment is repeated under the same conditions, required for the measurement of angular velocity. However, the light barrier key must be set at "⌐⌙⌐" and the "Reset" button is pressed.
– The time 't' indicated is the time for the acceleration. According to $\alpha = \omega / t$
  the acceleration is obtained.

Note

It is to be noted, that the supporting block stops the weight holder used for acceleration at that moment, when the screen enters the path of light of the fork type light barrier. More acceleration should not be effected during the measurement of Δt.

Theory and evaluation

The relationship between the angular momentum $\vec{L}$ of a rigid body in the stationary coordinate system with its origin at the centre of gravity, and the moment $\vec{T}$ acting on it, is

$$\vec{T} = \frac{d}{dt} \vec{L}. \qquad (1)$$

The angular momentum is expressed by the angular velocity $\vec{\omega}$ and the inertia tensor $\hat{I}$ from

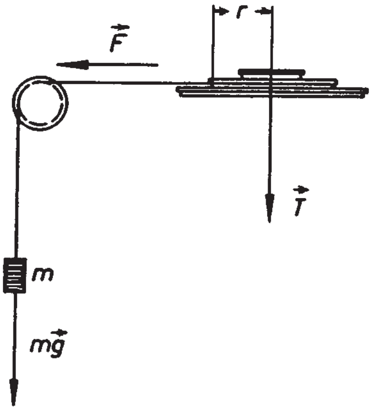
$$\vec{L} = \hat{I} \cdot \vec{\omega},$$

that is, the reduction of the tensor with the vector.

In the present case, $\vec{\omega}$ has the direction of a principal inertia axis (Z-axis), so that $\vec{L}$ has only one component:

$$L_Z = I_Z \cdot \omega$$

Fig. 2: Moment of a weight force on the rotary plate.



where I_Z is the Z-component of the principal inertia tensor of the body. For this case, equation (1) reads

$$T_Z = I_Z \frac{d\omega}{dt}.$$

The moment of the force $\vec{F}$ (see Fig. 2)

$$\vec{T} = \vec{r} \times \vec{F}$$

gives for $\vec{r} \perp \vec{F}$:

$$T_Z = r \cdot m \cdot g,$$

so that the equation of motion reads

$$mgr = I_Z \frac{d\omega}{dt} \equiv I_Z \cdot \alpha.$$

From this, one obtains

$$I_Z = \frac{mgr}{\alpha}.$$

The moment of inertia I_Z of a body of density $\rho(x, y, z)$ is

$$I_Z = \iiint \rho(x, y, z)(x^2 + y^2)\, dx\, dy\, dz$$

a) For a flat disc of radius r and mass m, one obtains

$$I_Z = \frac{1}{2} m r^2.$$

From the data of the disc

$2r = 0.350$ m
$m = 0.829$ kg

one obtains

$$I_Z = 12.69 \cdot 10^{-3} \text{ kgm}^2.$$

The mean value of the measured moment of inertia is

$$I_Z = 12.71 \cdot 10^{-3} \text{ kgm}^2.$$

b) For a long rod of mass m and length l, one obtains

$$I_Z = \frac{1}{12} m l^2.$$

From the data for the rod

$m = 0.158$ kg
$l = 0.730$ m

one obtains

$$I_Z = 7.017 \cdot 10^{-3}\ \text{kgm}^2.$$

The mean value of the measured moment of inertia is

$$I_Z = 6.988 \cdot 10^{-3}\ \text{kgm}^2.$$

c) For a mass point of mass m at a distance r from the axis of rotation, one obtains

$$I_Z = m r^2.$$

For the measurements, a distance

$r = 0.15$ m

was selected.

From the regression line to the measured values of Fig. 3, with the exponential statement

$$Y = A \cdot X^B + I_O$$

(I_o is the moment of inertia of the rod), the exponent

$B = 1.00 \pm 0.02$ (see (2))

is obtained.

The measurement was carried out with $m = 0.2$ kg.

From the regression line to the measured values of Fig. 4,

$B = 1.93 \pm 0.03$ (see (2))

is obtained.

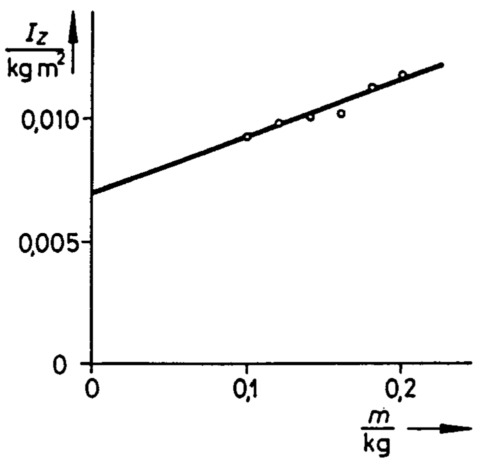
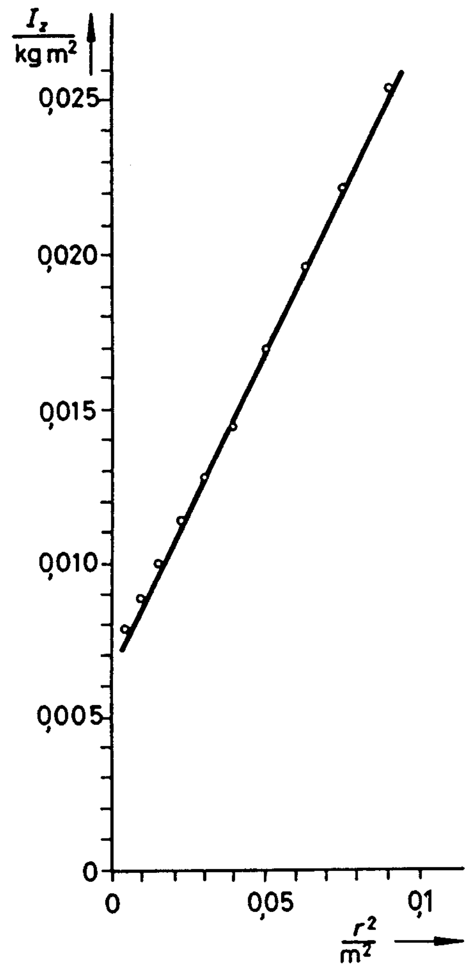


Fig. 3: Moment of inertia of a mass point as a function of the mass.

Fig .4: Moment of inertia of a mass point as a function of the square of its distance from the axis of rotation.



Note

The pivot pin is not taken into account for the theoretical calculation of the moment of inertia, since with a mass of 48 g, it has a moment of inertia of only $4.3 \cdot 10^{-6}$ kgm^2.

The "support face" and bar retaining ring are balanced by the sector mask and plug, so that a uniform mass distribution can be assumed for the bar over its whole length.