

Related topics

Potential and kinetic energy, rotational energy, moment of inertia, inelastic collision, principle of conservation of momentum and angular momentum, measurement of projectile velocities .

Principle and task

A classic method of determining the velocity of a projectile is to shoot the projectile into a resting mass which is large compared to the projectile's mass and hung as a pendulum. In the process, the projectile remains in the pendulum mass and oscillates with it. This is an inelastic collision in which the momentum remains unchanged. If the pendulum's mechanical data are known, one can infer the velocity of the pendulum's mass (including the projectile's mass) at the lowest point of the pendulum's oscillation from the amplitude of the pendulum's oscillation. The momentum of the two masses in this phase of the oscillation must thus be equal to the impulse of the projectile before it struck the pendulum. If one knows the masses of the pendulum and the projectile, one can calculate the projectile's velocity.

In order to be able to use this measuring principle without danger, the following set-up is used here: A steel ball is shot at the mass of a pendulum with the aid of a spring catapult. The pendulum mass has a hollow space in which the steel ball is held.

If, additionally, two light barriers and a time measuring device are available, an independent, direct measurement of the initial velocity of the ball can be made.

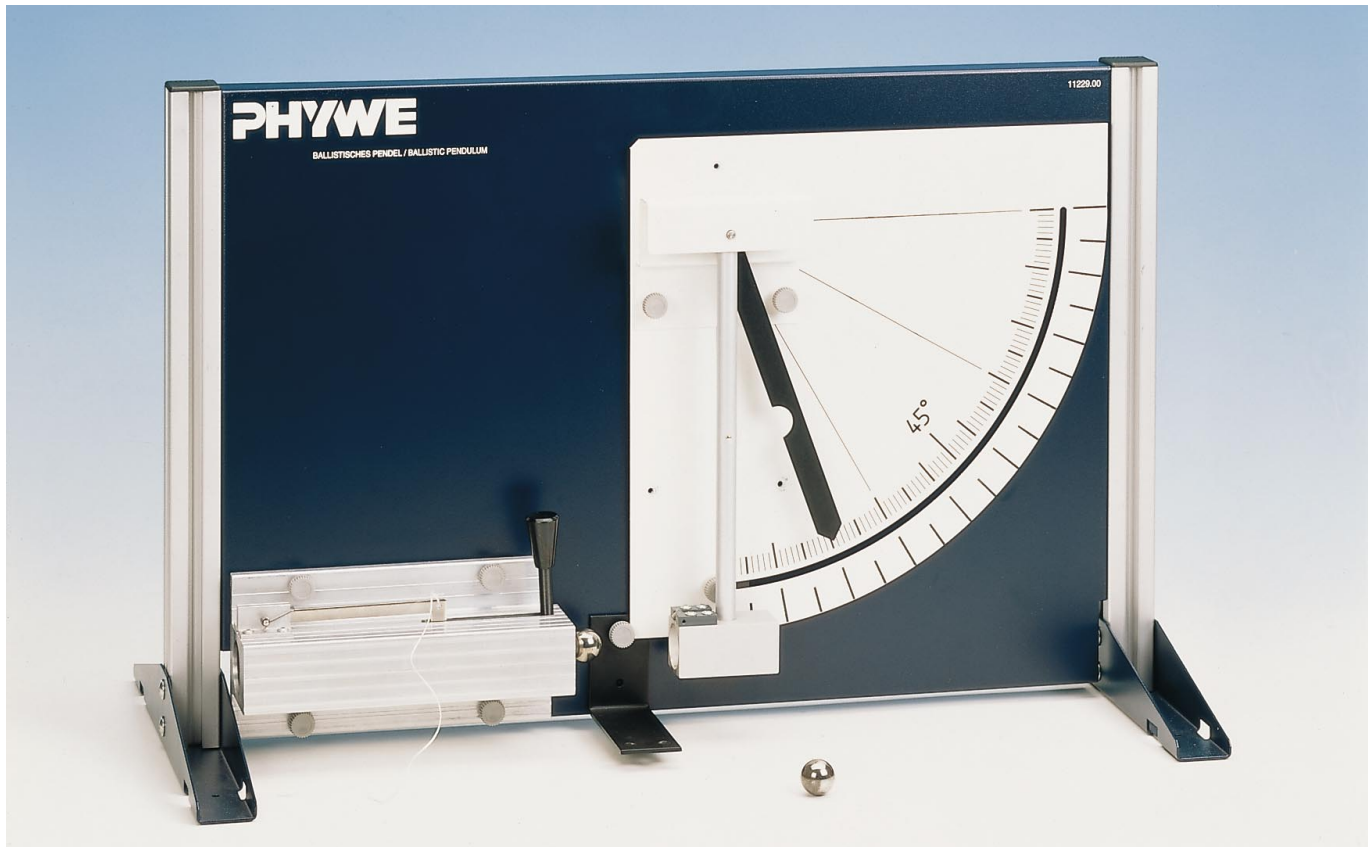
Equipment

Ballistic pendulum	11229.00	1
Steel ball, d 19 mm	02502.01	2
Timer 4-4	13605.99	1
Speed measuring attachment	11229.30	1
Connecting cord, 750 mm, red	07362.01	1
Connecting cord, 750 mm, yellow	07362.02	2
Connecting cord, 750 mm, blue	07362.04	1

Problems

1. Measurement of the oscillation amplitudes of the ballistic pendulum after capturing the steel ball for the three possible tension energies of the throwing device.
2. Calculation of the initial velocities of the ball from the measured oscillation amplitudes and the mechanical data of the pendulum is performed using the approximation formula (3).
3. Plotting of the velocity of the steel ball v as a function of the maximum deflection φ ($0 \dots 90^\circ$) of the pendulum according to formula (3), taking into consideration the special mechanical data of the experiment.
4. Determination of the correction factor f_{cor} for the utilised pendulum for the conversion of the velocities determined by using the approximation formula into the values obtained from the exact theory. Correction of the velocity values from Problem 2.

Fig. 1: Experimental set-up: Ballistic Pendulum.



5. If the supplementary devices for the direct measurement of the initial velocity are available, measure the initial velocities corresponding to the three tension steps of the throwing device by performing 10 measurements each with subsequent mean value calculation. Plot the measured points in the diagram from Problem 3. Give reasons for contingent systematic deviations from the theoretical curve.

Set-up and Procedure

Assemble the device according to Fig. 1. It must be standing on a stable table during the measurements. Information on the operation of the ballistic pendulum is to be obtained from the appropriate operating instructions.

Before stretching the spring of the throwing device, affix the steel ball to the holding magnet of the bolt. Then pull the bolt back until the desired lock-in position has been reached. Now, without touching the pendulum's pulling pin, ensure that the pendulum is at rest and that the trailing pointer indicates nearly zero. After these preparations have been completed, trigger the shot by pulling the release lever. The amplitude of the pendulum's oscillation can be read from the trailing pointer.

Note: Due to the friction involved in the functioning of the trailing pointer, the measured amplitude may be can be slightly low. It is therefore advisable to shoot at the resting pendulum using the same spring tension for a second and a third time without resetting the trailing pointer. When the trailing pointer is not moved any further, one can assume that the angle indicated has not been falsified by friction.

Theory and evaluation

For the evaluation, we proceed from the approximation that the pendulum's mass M and the Mass m of the captured ball move as one mass point at the location of their common centre of gravity. If we then set the potential energy of the pendu-

lum in its resting position to zero, the following is valid for the potential energy at the highest point of the oscillation:

$$E_{\text{pot}} = (m + M) \cdot g \cdot \Delta h ;$$

where g is the acceleration of gravity and Δh the height by which the centre of gravity was raised. In Fig. 2 one sees that with r as the distance between the pivot point and the centre of gravity, one can also write this formula as:

$$E_{\text{pot}} = (m + M) \cdot g \cdot (1 - \cos \varphi) \quad (1)$$

This potential energy must be equal to the kinetic energy E_{kin} immediately after the collision:

$$E_{\text{kin}} = \frac{1}{2} \cdot (m + M) \cdot v_p^2 \quad (2)$$

where v_p is the velocity of the pendulum's mass (including the ball) immediately after the collision. If one substitutes the momentum $p = (m + M) \cdot v_p$, in this equation, one obtains the following:

$$E_{\text{kin}} = \frac{p^2}{2 \cdot (m + M)}$$

$$\text{or } p = \sqrt{2 \cdot (m + M) \cdot E_{\text{kin}}}$$

Before the collision, the pendulum was at rest. Due to the principle of conservation of momentum, the momentum $p = m \cdot v$ must be equal in magnitude to the momentum of the ball before the collision. One obtains the following for the velocity v of the ball before the start, i. e. for the parameter to be determined:

$$v = \frac{m + M}{m} \cdot \sqrt{2 \cdot g \cdot r \cdot (1 - \cos \varphi)} \quad (3)$$

Fig. 3 shows the function $v(\varphi)$. Between 0 and 90° , the curve deviates only slightly from a straight line, i. e. the deflection of the pendulum is, as a good approximation, proportional to the velocity of the ball.

For the evaluation, determine the mass m of the ball. The position of the centre of gravity with captured steel ball is marked on the pendulum's body. the mass M of the pendulum's body can only be determined together with its mounting. The exact value is slightly less and can be obtained from the operating instructions

Information for an exact evaluation of the experiment

The theoretical treatment of the processes occurring on the ballistic pendulum, given above, is based on a certain idealisation. Formula (3) is consequently only approximately valid, but is very useful for practical purposes. At this time, we only wish to briefly show how one can achieve an exact evaluation. To begin with, Equation (2) for the kinetic energy is to be replaced by the equation for the rotational energy of a physical pendulum:

$$E_{\text{kin}} = \frac{1}{2} \cdot I \cdot \omega^2 \quad (2a)$$

where I is the moment of inertia of the pendulum with captured ball and ω the angular velocity. If one substitutes the angular momentum $L = I \cdot \omega$, the following is obtained.

$$E_{\text{kin}} = \frac{L^2}{2I} \cdot \text{or } L = \sqrt{2I \cdot E_{\text{kin}}}$$

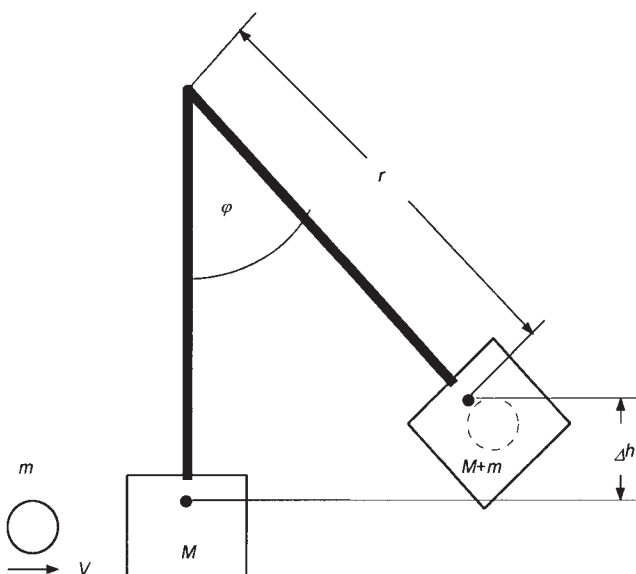
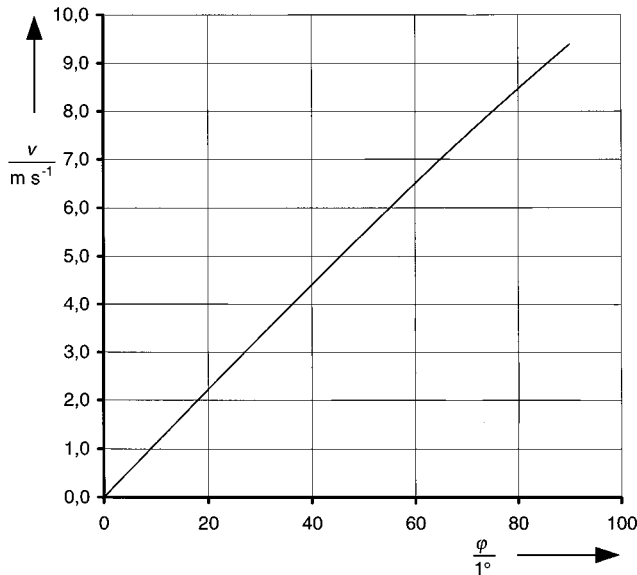


Fig. 2: Diagram on the theory of the ballistic pendulum.

Fig. 3: Theoretical calibration curve for a velocity measurement with a ballistic pendulum. This is only an exemplary curve as the manufacturer reserves the right to make technical alterations with regard to the properties of the device.



This angular momentum must be equal to the angular momentum L_b of the ball before the collision with reference to the pivot point of the pendulum. If r_b is the distance of the ball from the pivot point at the instant of capture, then

$$L_b = m \cdot r_b^2 \cdot \omega_b = m \cdot r_b \cdot v.$$

By setting the two angular momentums equal to each other, one obtains

$$v = \frac{1}{m \cdot r_b} \cdot \sqrt{2 \cdot I \cdot (m + M) \cdot g \cdot r \cdot (1 - \cos \varphi)}$$

The determination of the moment of inertia I of a physical pendulum is achieved by measuring its oscillation period T . For our pendulum the following results:

$$I = \frac{(m + M) \cdot g \cdot r \cdot T^2}{4\pi^2}$$

$$\text{and thus } \frac{m + M}{m} \cdot \frac{r}{r_b} \cdot \frac{g \cdot T}{2\pi} \cdot \sqrt{2 \cdot (1 - \cos \varphi)} \quad (3a)$$

If we now term the velocity calculated with the approximation formula (3) v_{app} and the exact value calculated with the exact formula (3a) v_{ex} , The two differ in accordance with $v_{ex} = f_{cor} \cdot v_{app}$ by the correction factor:

$$f_{cor} = \frac{T}{2\pi \cdot r_b} \cdot \sqrt{r \cdot g}.$$

This formula becomes more illustrative if one substitutes the following expression for T , in which r_m is the length of a mathematical pendulum having the oscillation period T :

$$T = 2\pi \cdot \sqrt{\frac{r_m}{g}}.$$

The following is now obtained:

$$f_{cor} = \frac{\sqrt{r \cdot r_m}}{r_b} \quad (4)$$

For an imaginary experiment with a projectile in the form of a mass point, which collides inelastically with a mathematical pendulum at rest, the three lengths used in Formula (4) are equal, i.e. $f_{cor} = 1$. By measuring r and r_b for the physical pendulum used as well as by calculating r_m from the measured oscillation period, one can easily determine the correction factor for the velocities obtained with Equation (3) from Equation (4).



Fig. 4: Experimental set-up with supplement for direct measurement of the initial velocity of the ball.