Related topics

Motion along a straight line subject to constant acceleration, laws governing falling bodies, acceleration due to gravity.

Principle and task

The fall times t are measured for different heights of fall s. s is represented as the function of t or t^2 such that the distance-time law of the free fall results as

$$s = \frac{1}{2} \cdot g \cdot t^2$$

Then the measured values are taken to determine the acceleration due to gravity g.

Equipment

Falling sphere apparatus	02502.88	1
consisting of		
Release unit	02502.00	1
Impact switch	02503.00	1
COBRA-interface 2	12100.93	1
PC COBRA data cable RS232, 2 m	12100.01	1
Softw.COBRA Free fall	14267.51	1
Tripod base -PASS-	02002.55	1
Support rod -PASS-, square, l = 1000 mm	02028.55	1
Right angle clamp -PASS-	02040.55	2
Meter scale, demo, l = 1000 mm	03001.00	1

Fig. 1: Experimental set-up (the computer which must be connected is not shown).

Connecting cord, 750 mm, yellow	07362.02	1
Connecting cord, 750 mm, blue	07362.04	1
Connecting cord, 1500 mm, yellow	07364.02	1
Connecting cord, 1500 mm, blue	07364.04	1

Problems

Determination of:

1. Distance time law for the free fall.
2. Velocity-time law for the free fall.
3. Precise measurement of the acceleration due to for the free fall.

1) Distance time law for the free fall

Set-up and procedure

— Arrange the ball free fall unit according to Fig.1 The releasing device must be positioned directly over of the impact switch so that the fixed ball is in a centered position above the impact plate.
— Connect the releasing device to the start and earth socket of timer 1.
— Connect the impact switch to the earth socket of timer 1.
— Carry out some releasing tests for different increasing heights of fall. If the ball does not hit the plate roughly in its centre for each height of fall, then the support rod might be slightly inclined. In this case the adjusting screws of the support base must be readjusted. It might be helpful to use a plumb bob (03114.00).

As far as this experimental set-up is concerned, the measuring accuracy is mainly determined by the accuracy when measuring the height of fall. We recommend the following procedure (see Fig.1) with the ball not yet being clamped into the releasing device:

— Place one end of the meter scale onto the plate of the impact switch.
— Pull the plate upwards until it touches the stopper and hold it in this position.
— Measure the distance between the plate and the red reference mark on the releasing device.
— Subtract the ball radius (e.g. 9.5 mm) from this distance in order to obtain the height of fall.

When the COBRA program was started according to the program description, the following steps are necessary to measure the fall time:

— Check whether the locking device at the cable release of the starting device is activated. When the releasing device is pressed down, it must remain in its new position. (However, if this is not the case, unscrew the knurled plate under the pressure pin of the releasing device by turning it in the counter-clockwise direction.) Then rerelease the locking device by pressing down the knurled ring.
— Place the ball in a centered position on the moving plate of the releasing device and fix it by pressing down the cable release.
— Pull up the plate of the impact switch until it touches the stopper.
— Start the "Free Fall" program.
— Select the program function <measure> <parameters>.
— Enter an appropriate name for the measurement (e.g. "distance-time law"). This name is then used for all graphical representations.

— Select "sequence - *s* variable" in the selective window <type of measurement>.
— In the beginning, set a small height of fall (appr. 0.1 m). Measure the height of fall and enter it into the input field "height of fall".
— Hit <ok> to close the window.
— Activate the program function <measure> <start> (in the most comfortable way simply by means of the key F10).
— Prepare the free fall unit so that it is ready for the measurement.
— Hit <ok> to trigger the measurement.
— Release the ball from the free fall unit, when the programs prompts "timer ready" in the status line on the screen.
— Increase the height of fall step by step and prepare additional measurements by means of <measure> <start>. Enter the new height of fall before triggering the measurement with <ok>. Attention: Do not use the menu function <measure><parameters> since in this case the program would start a new series of measurements and overwrite all the preceding measurements.

Faulty measurements can always be repeated with the aid of the <measure> <correction> function. The new measurements must be carried out with the same heights of fall as the initial measurements. In order to simplify this procedure, corrections should be done before changing the height of fall. After the measurement, a graphical representation "*s* (*t*)" is created by selecting <graphics> <function>.

Theory and evaluation

Figure 2 shows an example of the measured relationship between the height of fall and the fall time. In order to obtain the representation shown, the function <autorange> must be activated according to the presetting (extension over the whole screen) under <graphics> <options>. The parabolic slope of the curve cannot be clearly recognized in this representation since the origin of coordinates is not shown. In order to show that s is proportional to t^2 ,the representation "$s(t^2)$" must be selected. Then a regression line is plotted with the aid of <graphics> <curves> (Fig. 3). It can now be seen that all measuring points are rather precisely approximated by the straight line.
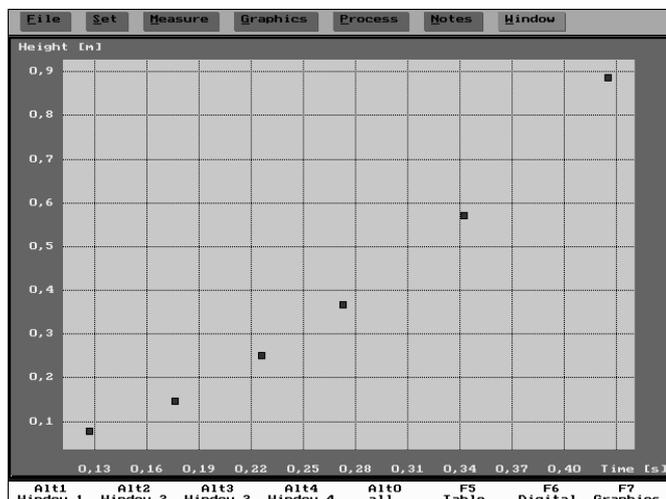
But now let's get back to figure 2 which can be displayed again on the screen by means of <graphics> <function> and "*s*(*t*)". The coordinate interval which is displayed is now to be extended in such a way that the origin of coordinates is shown, too. This can be done in two different ways:

Either
— Activate the functions <graphics> <zoom> and select "minimum - maximum (X;Y)".
— Enter 0 into the input fields of the coordinates Xmin and Ymin.
— Hit <ok> to close the window.
or
— Activate the functions <graphics> <options>.
— Select "no" in the selective window <autorange>.
— Hit <ok> to close the window.
— Activate the functions <graphics> <zoom> and select "interactive".
— Select the desired cross-section of the image by means of the mouse.

Figure 4 show the result with a theoretical curve and a given *g* (9.81 m/s^2) obtained with the aid of <graphics> <curves>.

Instead of diagrams the program can also display the value tables which form the basis of each graphical representation. Figure 5 shows the complete table including the values calculated by the program which are required for special representations. Window 4 of the screen shows the average acceleration due to gravity calculated from the 6 measurements. The error limits correspond to the standard deviations of the individual values.

Select <graphics> <hardcopy> to print out the table. If only the measured values without the evaluation columns are to be printed out (e.g. for a manual evaluation by the students), the representation in four quadrants must be selected (use <window> and the Alt-0 key or click on "all" in the lower screen line). The table is then reduced to two columns. One column comprises the height of fall, the second column can be chosen by means of the → and ← keys (Fig. 6).
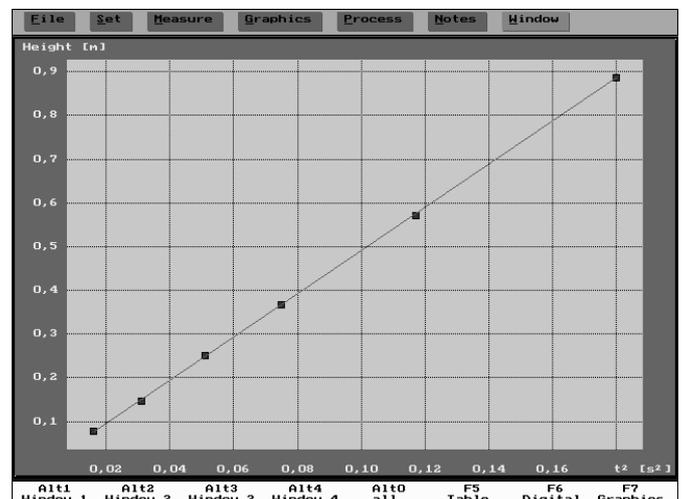

Fig. 2: Height of fall s as a function of the fall time *t*.

Fig. 3: Height of fall s as a function of t^2 with regression line.

Note

Sometimes, a simultaneous display of several graphical representations of the measurement series carried out might be desired for comparison purposes. Figure 7 shows how this is possible in a very easy way with the aid of the 4-window system of the COBRA software.

In order to obtain the representation shown in figure 6 act as follows:

— Starting with figure 4, switch to a 4-window representation ("all").

— Activate the program functions <process> <new table>.

— Enter a new name for the new window (e.g. „s(t)") and select "window 2".

— Hit <ok> to trigger the <new table> function.

— Hit <Retum>: The first measuring point which is marked by a cross in window 1 is copied into window 2.

— Use the cursor keys to shift the cross successively to each of the other points. Always hit <Return> to copy them. Then hit the <Esc> key to return to the main menu.

— Select the corresponding type of representation by means of <graphics> <function>.

— Optimize the range of representation with the aid of <graphics> <zoom> and plot a compensating line.

Proceed in the same way to obtain the representation g(s) in window 3, set Ymin = 6.0 by means of <graphics> <zoom> (scale intervals optimized by "autorange" make no sense since the Y-values are nearly constant!) and plot the theoretical curve with the aid of <graphics> <curve>. Then the accel-

eration due to gravity of each of the three windows can be entered into window 4 starting with <process> <acceleration>.

Another very comfortable way for processing the same measurement in several windows is to store the measurement on the hard disk of the computer (<file> <save>) and then loading it into the other windows.

Figure 7 (third quadrant) shows that the two first measurements are obviously less accurate than the other measurements. It is clear that with the relatively small heights of fall errors during the distance measurements have a higher impact. For this reason, these measurements should not be considered, if the acceleration due to gravity in window 4 is to be determined in a very precise way. For this purpose, the program offers the options <process> <erase>. The measuring point which is marked is erased from the value table. Attention: This process cannot be undone. For this reason, the measurement series should be stored on the hard disk. If, in the example shown the first two measured values are removed, the result is

$$g = 9{,}81 \pm 0{,}03 \text{ m/s}^2.$$

Fig. 5: Complete table of the measured values and the calculated data used internally.
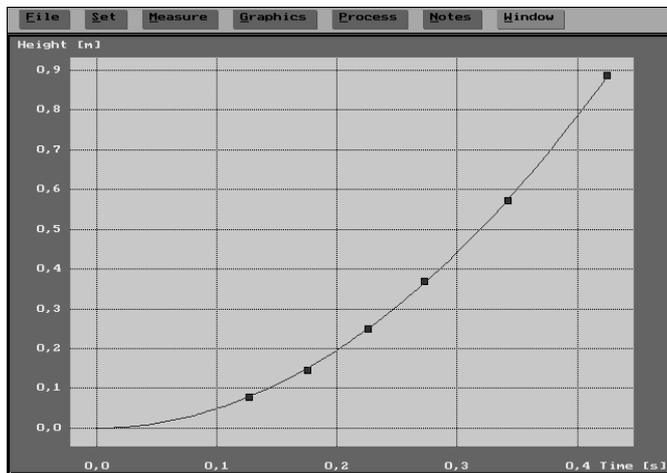
Fig. 4: Distance-time function for the free fall with the origin of coordinates being considered.
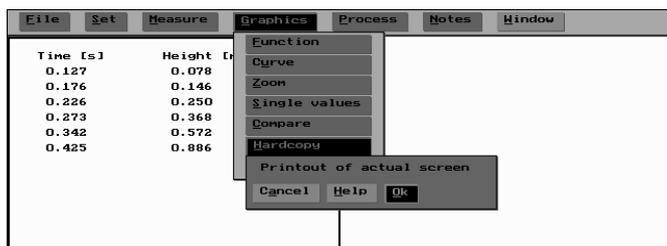
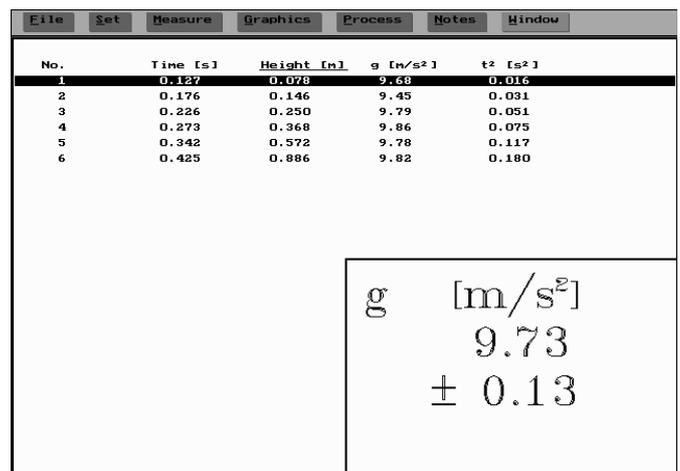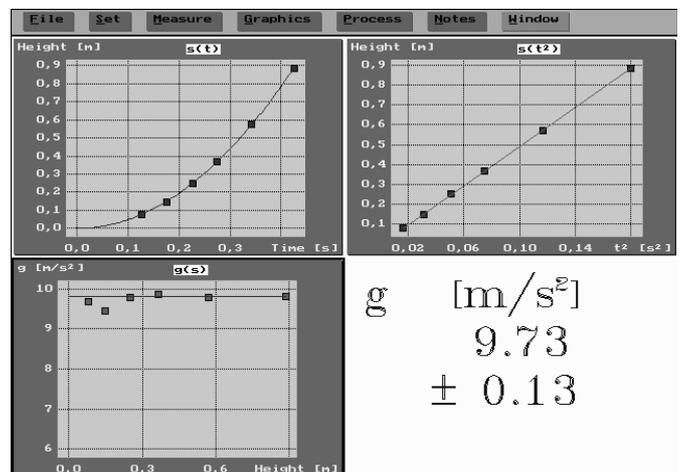Fig. 6: Table with two columns, prepared for being printed out for a manual evaluation.

Fig. 7: Four different types of representation of the same series of measurements.

2) The velocity-time law for the free fall

Principle

Starting with a minimum height of fall, the fall distance is increased step by step by the same interval. The corresponding fall times are measured. The software used is able to calculate a table with the velocities corresponding to certain time values from the measurement data table. In this way, the velocity v can be represented as the function of the time t. This representation underlines the linear relationship between the two quantities

$$v = g \cdot t$$

Then the measured values are taken to determine to acceleration due to gravity g.

Set-up and procedure

The experiment is set up as shown in Part A.

— Start the "Free Fall" program.

  Select the program function <measure> <parameters>.

— If desired, enter a name for the measurement. This name is then used for all graphical representations.

— Select "s variable (auto)" in the selective window <type of measurement>.

— Enter the smallest desired height of fall into the input field "height of fall" (starting value, e.g. 0.1 m).

— Enter the step by which the height of fall is to be increased from measurement to measurement (e.g. 0.1 m or 0.2 m) into the input field "step (auto)".

— Hit <ok> to close the window.

— Activate the program functions <measure> <start> (in the most comfortable way simply by means of the F10 key).

— Precisely adjust the height displayed in the window at the free fall unit. In this operating mode it is not possible to change the displayed value of the height of fall.
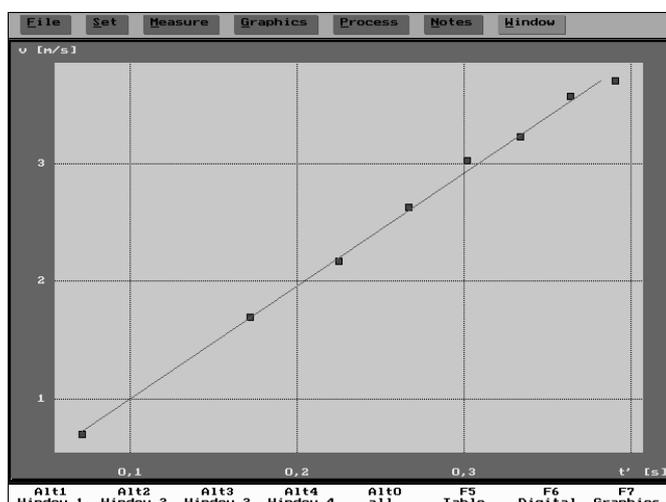
— Prepare the free fall unit so that it is ready for the measurement.

— Hit <ok> to trigger the measurement.

— Release the ball from the free fall unit, when the program prompts "timer ready" in the status line on the screen.

— Prepare additional measurements by means of <measure> <start>. Adjust the new height of fall before triggering the measurement with <ok>. Attention: Do not use the menu function <measure> <parameters> since in this case the program would start a new series of measurements and overwrite all the preceding measurements.

Faulty measurements can always be repeated with the aid of the <measure> <correction> function. The new measurements must be carried out with the same heights of fall as the initial measurements. In order to simplify this procedure, corrections should be done before changing the height of fall.

When the measurement series is completed, <graphics> <function> must be selected to obtain the graphical representation of "v(t)". If the graphic shows individual values which clearly deviate from the straight line expected, these faulty measurements can be repeated with the aid of <measure> <correction>.

Theory and evaluation

Figure 8 shows the measured relationship between velocity and time with a regression line (<graphics> <curves>). The individual values deviate to a greater extent from the regression line than it is the case in, for instance, the graphical representation of "$s(t^2)$". This is due to the fact that for calculating the velocities the program must use the relatively small differences of the fall times measured.

In order to obtain the representation shown, the function <auto-range> must be activated according to the presetting (extension over the whole screen) under <graphics> <options>. Figure 9 shows basically the same graphical representation, but in this case the origin of coordinates is shown, too.

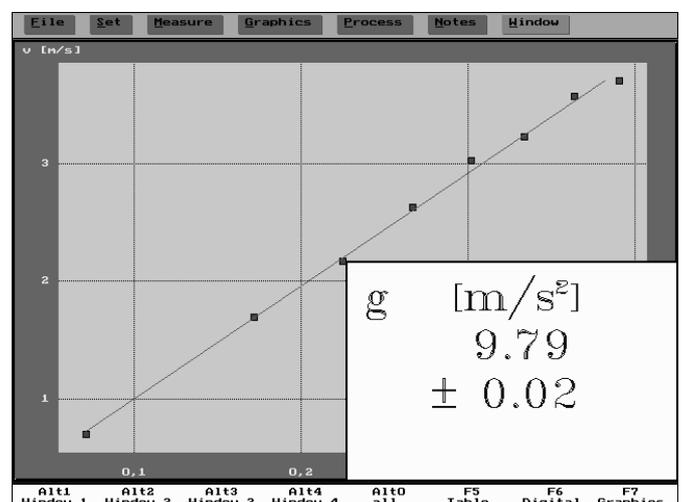Fig. 8: Example of a velocity-time diagram (autorange).

Fig. 9: Velocity-time diagram with theoretical curve and linear regression; the evaluated acceleration due to gravity is displayed.

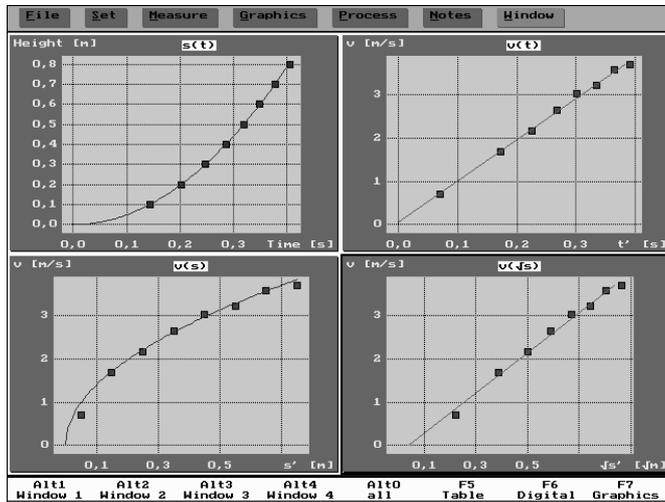Fig. 10: Different types of representations of the measurement example.

Fig. 11: Value table of the measurement example for representation $v(t)$.

No.	Time [s]	Height [m]	g [m/s²]	t' [s]	v [m/s]
1	0.143	0.100	9.82	0.071	0.701
2	0.202	0.200	9.77	0.172	1.695
3	0.248	0.300	9.78	0.225	2.174
4	0.286	0.400	9.81	0.267	2.632
5	0.319	0.500	9.84	0.302	3.030
6	0.350	0.600	9.79	0.334	3.226
7	0.378	0.700	9.77	0.364	3.571
8	0.405	0.800	9.77	0.391	3.704

This representation not only shows the regression line, but also the theoretical curve for $g = 9.81$ m/s^2 and the acceleration due to gravity which corresponds to the slope of the regression line. It can be seen that, as far as this example is concerned, the regression line is slightly flatter than the theoretical curve which exactly hits the origin of coordinates. Thus, g becomes slightly smaller than the given value.

In order to obtain figure 9 starting with figure 8 the following steps are necessary:
— Activate the program function <graphics> <zoom>.
— Enter 0 into the input fields of the coordinates Xmin and Ymin.
— Hit <ok> to close the window.
— Call <graphics> <curves> twice. First activate "curve with given g" and then "linear regression".
— Copy the g-value into window 4 by means of <process> <acceleration>.

Note

With the aid of the four-window-system the measurement series obtained can be represented in a comparative way. Figure 10 shows an example which can be obtained as follows:
— Select the representation <windows> <all>.
— Copy the measured values successively into the windows 2, 3 and 4 with the aid of <process> <new table>. Enter the corresponding names (e.g. "$v(t)$",…). Then copy the content of one of the three target windows back into window 1 and enter the name "$s(t)$".

Now the following steps must be carried out in each window:
— Use <graphics> <function> in order to select the representation which corresponds to the name of the window.
— Activate the program function <graphics> <zoom>.
— Enter 0 into the input fields of the coordinates Xmin and Ymin.
— Hit <ok> to close the window.
— Call <graphics> <curves> and plot "curve with given g". Attention: The curve must be plotted before selecting another representation by means of <graphics> <function>. The program always plots the theoretical curve which corresponds to the selected representation regardless of what type of diagram might currently be displayed in the window!

The method according to which the program calculates the velocity can be explained with the aid of the table corresponding to our measurement example. In order to obtain the table shown in figure 11, <graphics> <function> and representation "$v(t)$" must be selected. The table column before the last one indicates under t' that point of time which corresponds to the middle between the current measured value and the preceding value. The value which precedes the first measured value is: $t_0 = 0$; $s_0 = 0$. As the velocity increases in a linear way, the instantaneous velocity at the time t_n' corresponds exactly to the average velocity calculated by the program according to

$$v\,(t_n') = \frac{s_n - s_{n-1}}{t_n - t_{n-1}}$$

3) Precise measurement of the acceleration due to the gravity with a constant height of fall

Principle

Inaccuracate accelerations due to gravity are mainly due to errors which occur when measuring the distance of fall. The aim is to carry out the measurement as carefully as possible in order to limit this error regardless of the distance of fall s to 1 mm maximum. To keep this relative error as small as possible, the measurement must be carried out with a maximum height of fall. The acceleration due to gravity is then determined in a very precise way after a large number of measurements with a constant height of fall by forming the average value and eliminating (possibly by faulty releasings) faulty values.

Set-up and procedure

The experiment is set up as shown in part A.

— Start the "Free Fall" program.

— Select the program function <measure> <parameters>.

— If required, enter a name for the measurement. This name is t hen used for all graphical representations.

— Select "sequence – *s* constant" under <type of measurement>.

— Adjust a maximum height of fall (0.8 m) at the free fall unit.

— Determine the height of fall as precisely as possible (use a caliper gauge to measure the ball diameter) and enter it into the input field "height of fall". Use m as unit.

— Hit <ok> to close the window.

— Activate the program function <measure> <start> (in the most comfortable way simply by means of the F10 key). The window shows the height of fall entered which in this operating mode cannot be changed.

— Prepare the free fall unit so that it is ready for the measurement.
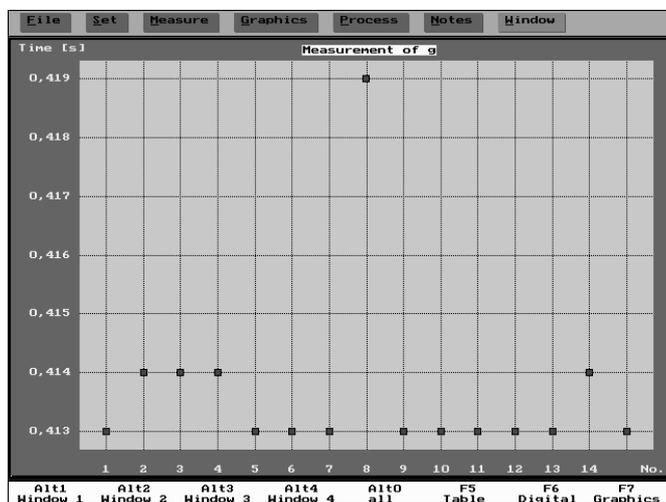
— Hit <ok> to trigger the measurement.

— When the program prompts "timer ready" in the status line on the screen, release the ball from the free fall unit.

— Carry out further measurements by means of <measure> <start> until the program has at least recorded 10 measurements. Attention: Do not use the menu function <measure> <parameters> since in this case the program would start a new series of measurements and overwrite all the preceding measurements.

— Select <file> <save> to save the measurement series on the hard disk.

When the measurement series is over, hit <F7> or click on the corresponding field in the lower screen line in order to switch to the display mode <graphics>. The fall times are represented as the function of the corresponding measurement number. The result is a graphical representation as shown in figure 12 for a measurement example comprising 15 measured values. Figure 13 shows the corresponding value table.

Theory and evaluation

Figure 12 as well as the table in figure 2 show that with one exception only fall times of 0.413 s or 0.414 s were measured. Thus, one can presume that the value number 8 is a releasing error. It is recommended to delete this value (<process> <delete>) before calculating the acceleration due to gravity with the aid of <process> <acceleration>. Figure 14 shows the revised measurement example.

In pratical application, of course, there might be a greater number of inaccurate values. However, one can always presume that this faulty measurements are due to the ball being improperly positioned in the fixing device. When, in this case, the ball is released, this might interrupt the releasing circuit for a short time, even if the ball is not falling at all. The resulting faulty fall times are always too big. Nevertheless, the program always provides are large number of measured values which do not deviate from their average value by more than 0.001 s. All the other values should be deleted before evaluation.

Fig. 12: Measurement example; 15 fall times measured with a height of fall of 0.839 m.

Fig. 13: Value table corresponding to the measurement example.

No.	Time [s]	Height [m]	g [m/s²]	t² [s²]
1	0.413	0.839	9.82	0.171
2	0.414	0.839	9.81	0.171
3	0.414	0.839	9.81	0.171
4	0.414	0.839	9.81	0.171
5	0.413	0.839	9.82	0.171
6	0.413	0.839	9.83	0.171
7	0.413	0.839	9.82	0.171
8	0.419	0.839	9.54	0.176
9	0.413	0.839	9.82	0.171
10	0.413	0.839	9.84	0.171
11	0.413	0.839	9.83	0.171
12	0.413	0.839	9.81	0.171
13	0.413	0.839	9.82	0.171
14	0.414	0.839	9.81	0.171
15	0.413	0.839	9.82	0.171

The result g = (9.81 ± 0.00) m/s^2 must not be considered as being the result of a calculation of error. The spread given stands for the standard deviation of 14 individual values taken to calculate the average value. Obviously, this standard deviation is smaller than 0.005 m/s^2. Apart from the spreading of the individual values also the systematic error s which occurs when measuring the height of fall must be taken into account. If one presumes that this error is 1 mm and bears in mind that g is a linear function of s ($g = 2s/t^2$), an additional relative error of g according to

$$\frac{\Delta g}{g} = \frac{\Delta s}{s} = \frac{1}{839} = 0.00119$$

must be taken into account. This means:

$$g = (9.81 \pm 0.01) \, m/s^2.$$

Fig. 14

Note

Make sure that the clamping screw at the right angle clamp holding the impact plate at the support rod is tightly fixed. Otherwise, the impact plate might be shifted downwards due to the impact of the ball. This is the case, when the fall times get longer and longer in the course of a measurement series. If, however, the acceleration due to gravity is always too small even with a precisely measured fall distance, please 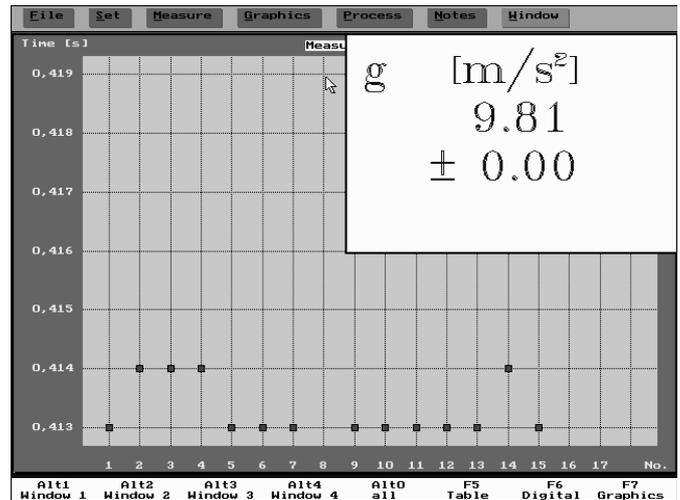check the circuit of the impact plate according to part A.