Homework Lecture 5 (Theory 3: Hadrons)

Exercises sent to brightspace on Wednesday 18 March, 12:00
Answers to be submitted individually by Tuesday 21 April 12:00
Please name file as "HW3_Name.pdf"
To: tuning@nikhef.nl
Format: .pdf

1 Completing the decuplet

Murray Gell-Mann proposed the quark-model to explain the large number of observed particles. From his scheme, he concluded that there must be a particle with quark content (s, s, s), the Ω^- particle. Let's have a look at the decay chain.

a) Gell-Mann predicted the mass of the Ω^-, by inspecting the masses in the baryon decuplet (containing the spin-$\frac{3}{2}$ baryons). What is the best guess for the mass of the Ω^-, using $m_\Delta = 1232$MeV, $m_{\Sigma^*} = 1385$MeV and $m_{\Xi^*} = 1532$MeV ? How close is the prediction to the observed value? (Browse to the webpage of the Particle Data Group, `http://pdglive.lbl.gov`.)

b) Next, look for the π^0 particle on the webpage of the Particle Data Group. How does it decay? What is its lifetime? If the π^0 has an energy of 1.4 GeV, how far will it fly? So, what is the signature of a π^0 in a bubble-chamber picture?

c) Look at the bubble-chamber picture which led to the discovery of the Ω^- at Brookhaven. How does the Ω^- decay? Also, specify the quark content of all particles involved.

d) Subsequently, how does the baryon with $S = 2$ decay? Also, specify the quark content of all particles involved.

e) Subsequently, how does the baryon with $S = 1$ decay? Also, specify the quark content of all particles involved.

2 $\pi^{\pm}p$ Scattering

We will inspect the cross-section for π^-p and π^+p scattering, as a function of the center-of-mass energy of the $\pi^{\pm}p$-system. If we consider π^-p scattering, the elastic process $\pi^-p \to \pi^-p$ is one of the possibilities. However, if (in the Yukawa picture) a charged pion is exchanged instead of a neutral pion, the quasi-elastic process $\pi^-p \to \pi^0 n$ can occur. Let's compare the following processes:
(A) $\pi^+p \to \pi^+p$
(B) $\pi^-p \to \pi^-p$
(C) $\pi^-p \to \pi^0 n$

a) Decompose $(\pi^+ p)$, $(\pi^- p)$ and $(\pi^0 n)$ in $I = 3/2$ and $I = 1/2$ components, using the Clebsch-Gordan coefficients.

b) Let's compare the transition amplitudes (or "matrix element" $\mathcal{M}$) of the three processes (A), (B) and (C), in terms of the $I = 3/2$ and $I = 1/2$ components:

$$(A)\mathcal{M}(\pi^+ p \to \pi^+ p) \quad =< \pi^+ p|\pi^+ p > \quad = \mathcal{M}_{3/2} \tag{1}$$

$$(B)\mathcal{M}(\pi^- p \to \pi^- p) \quad =< \pi^- p|\pi^- p > \quad = \frac{1}{3}\mathcal{M}_{3/2} + \frac{2}{3}\mathcal{M}_{1/2} \tag{2}$$

Write the equivalent decomposition for process (C).
(Hint: remember that $I = 3/2$ and $I = 1/2$ are orthogonal.)

c) Let's compare the cross section at $\sqrt{s} = 1232$ MeV, i.e. at the Δ resonance. The Δ particles (or "resonances") are *isospin-3/2*, $I = 3/2$, to explain the degeneracy of the four particles with (almost) equal mass. What is the relative contribution of the quasi-elastic contribution $\pi^- p \to \Delta^0 \to \pi^0 n$ compared to the elastic process $\pi^- p \to \Delta^0 \to \pi^- p$, in terms of $\mathcal{M}_{3/2}$ and $\mathcal{M}_{1/2}$?
(Note that the cross section is proportional to the square of the transition amplitude.)

d) So, how does the total $\pi^+ p$ cross section at the Δ resonance compare to the total $\pi^- p$ cross section ?

e) In the spectrum for $\pi^- p$ scattering there are peaks around 1520 MeV and 1680 MeV that are absent in $\pi^+ p$ scattering. What can you say about the isospin of these resonances?

3 Decay rates

We will use the Clebsch-Gordan tables to predict some decay rates, see
`http://pdg.lbl.gov/2012/reviews/rpp2012-rev-clebsch-gordan-coefs.pdf`.
The fractional decay rate to a specific final state is called "branching fraction" or "branching ratio". For a given particle, the sum of all branching fractions add up to 100%.

a) Let's consider the ρ particle. This is a meson with the same quark content as the pion, and also manifests itself as an isospin triplet. (The difference with the pion is that the ρ is heavier, about 770 MeV instead of 140 MeV, and that it has *spin-1*, and not spin-0 as the pion.) Decompose the ρ^+ ($|1, +1>$) in $I = 1$ isospin components (ie. look at the table (1×1)).

b) The ρ decays as $\rho \to \pi\pi$. What is the branching ratio for $\rho^+ \to \pi^+ \pi^0$?

c) Decompose the ρ^0 ($|1, 0>$) in $I = 1$ iso-spin components (ie. look at the table (1×1)). What is the branching ratio for $\rho^0 \to \pi^0 \pi^0$?